Discrete Mathematics

Topic 2 – Logic: Apps. On Prop. Logic

(Ch 1.2)*

CMPS 211 – American University of Beirut

* Extracted from *Discrete Mathematics and It's Applications* book slides

Translating English Sentences

- Translating English sentences into expressions involving propositional variables and connectives is important
  - Remove the ambiguity of English language
  - Be able to logically reason about the sentences using valid rules of inference
- Example:
  - You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old"
- Solution:
  - $q =$ ``You can ride the roller coaster''
  - $r =$ ``You are under 4 feet tall''
  - $s =$ ``You are older than 16 years old''
  - $(r \wedge \neg s) \rightarrow \neg q$

System Specifications

- System and Software engineers take requirements in English and express them in a precise manner using logic
- Example:
  - Express in propositional logic "the automated reply cannot be sent when the file system is full"
- One possible solution:
  - Let p denote "The automated reply can be sent" and q denote "The file system is full"
  - $q \rightarrow \neg p$

Testing Consistency of System Specifications

- Definition:
  - A list of specifications is consistent if it is possible to assign truth values (true or false) to the propositional variables so that each specification is true

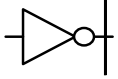
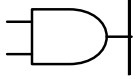
Testing Consistency Exercise

- Are these specifications consistent?
  - "The diagnostic message is stored in the buffer or it is retransmitted"
  - "The diagnostic message is not stored in the buffer"
  - "If the diagnostic message is stored in the buffer, then it is retransmitted"
- Solution:
  - Let p denote "The diagnostic message is stored in the buffer"
  - Let *q* denote "The diagnostic message is retransmitted"
  - The specification can be written as: $p \vee q$, $\neg p$, $p \rightarrow q$
  - When p is false and q is true all three statements are true
  - So the specifications are consistent

Testing Consistency Exercise 2

- Are these specifications consistent?
  - "The diagnostic message is stored in the buffer or it is retransmitted"
  - "The diagnostic message is not stored in the buffer"
  - "If the diagnostic message is stored in the buffer, then it is retransmitted"
  - "The diagnostic message is not retransmitted"
- Solution:
  - Now we are adding $\neg q$ and there is no satisfying assignment
  - So the specifications are no longer consistent

Alternative Notations for Logic Operators

Name:	not	and	or	xor	implies	iff
Propositional logic:	$\neg$	$\wedge$	$\vee$	$\oplus$	$\rightarrow$	$\leftrightarrow$
Boolean algebra:	$\overline{p}$	pq	$+$	$\oplus$		
Logic gates:						

Any Questions?