$A \cap (B-A) = \phi$ $\frac{2.9}{N=12}$

$\Leftrightarrow \begin{cases} A \cap (B-A) \subseteq \phi \\ \phi \subseteq A \cap (B-A) \end{cases}$

$\Leftrightarrow \begin{cases} \forall x\,(x \in A \cap (B-A) \longrightarrow x \in \phi) \wedge \\ \forall x\,(x \in \phi \longrightarrow x \in A \cap (B-A)) \end{cases}$

③ (1) take random $x \in A \cap (B-A)$

$\Leftrightarrow x \in A \cap (B-A)$

$\Leftrightarrow x \in A \wedge (x \in (B-A))$

$\Leftrightarrow x \in A \wedge x \in B \wedge x \notin A$

$\Leftrightarrow x \in A \wedge x \notin A \wedge x \in B$

$\Leftrightarrow$ False $\wedge\, x \in B$

$\Leftrightarrow$ False

$\Leftrightarrow x \in \phi$

$\therefore \forall x\,(x \in A \cap (B-A) \to x \in \phi) \wedge$
$\forall x\,(x \in \phi \longrightarrow x \in A \cap (B-A))$

$\therefore A \cap (B-A = \phi$

(e) $A \cup (B-A) = A \cup B$

$\Leftrightarrow \begin{cases} A \cup B - A \subseteq A \cup B \\ A \cup B \subseteq A \cup (B-A \end{cases}$

$\Leftrightarrow \begin{cases} \forall x\,(x \in A \cup B - A) \to x \in A \cup B) \\ \forall x\,(x \in A \cup B \to x \in A \cup (B-A)) \end{cases} \wedge$

(1) take $x \in A \cup (B-A)$

$\Leftrightarrow x \in A \cup (B-A)$

$\Leftrightarrow x \in A \vee (x \in B \wedge x \notin A)$

$\Leftrightarrow (x \in A \vee x \notin A) \wedge (x \in A \vee x \in B)$

$\Leftrightarrow$ True $\wedge\, (x \in A \vee x \in B)$

$\Leftrightarrow x \in A \vee x \in B$

$\Leftrightarrow x \in (A \cup B)$

$\therefore \forall x\,(x \in A \cup (B-A) \to x \in A \cup B) \wedge \forall x\,(x \in A \cup B \to x \in A \cup (B-A))$

$\therefore A \cup B - A = A \cup B$

(16)

(a) $(A \cap B) \subseteq A$

$\Leftrightarrow \{ \forall x \, (x \in A \cap B \longrightarrow x \in A)) \}$

take random $x \in A \cap B$

$\Rightarrow x \in A \cap B$

$\Rightarrow x \in A \wedge x \in B$ ~~[illegible]~~

$\Rightarrow x \in A$

$\therefore \forall x \, (x \in A \cap B) \rightarrow x \in A) \therefore (A \cap B) \subseteq A$

(b) $A \subseteq (A \cup B)$

$\Leftrightarrow \{ \forall x \, (x \in A \longrightarrow x \in A \cup B) \}$

take random $x \in A$

$\Rightarrow x \in A$

$\Rightarrow x \in A \vee x \in B$ is also true

$\Rightarrow x \in A \cup B$

$\therefore \forall x \, (x \in A \longrightarrow x \in A \cup B)$

$\therefore A \subseteq (A \cup B)$

(c) $A - B \subseteq A$

$\{ \forall x \, (x \in A - B \longrightarrow x \in A \}$

take random $x \in A - B$

$\Rightarrow x \in A - B$

$\Rightarrow x \in A \wedge x \notin B$

$\Rightarrow x \in A$

$\therefore \forall x \, (x \in A - B \longrightarrow x \in A)$

$\therefore A - B \subseteq A$

2.2

3 $A = \{1,2,3,4,5\}$ $B = \{0,3,6\}$

(a) $A \cup B = \{x \mid x \in A \vee x \in B\}$

$= \{0,1,2,3,4,5,6\}$

(b) $A \cap B = \{x \mid x \in A \wedge x \in B\}$

$= \{3\}$

(c) $A - B = \{x \mid x \in A \wedge x \notin B\}$

$= \{1,2,\cancel{3},4,5\}$

(d) $B - A = \{x \mid x \in B \wedge x \notin A\}$

$= \{0,6\}$

Section 2.3

N°29) a $A \xrightarrow{g} B \quad B \xrightarrow{f} C$

a) if f & g are one to one then $f \circ g$ is also one to one?

let f is onto one be P.

g is one to one be q $\qquad$ $f \circ g$ is one to one be R

So we need to proove that: $p \wedge q \Rightarrow R$

$\underline{P}$: $\forall x \in B$, $\exists$ only one $f(x)$ in $B \Leftrightarrow \neg \exists x \neq y \,/\, f(x) = f(y)$

$\underline{q}$: $\forall y \in A \quad \neg \exists \, y \neq z \,/\, g(y) \neq g(z)$

$x \in B \Leftrightarrow \exists$ only one $y \in A \,/\, x = g(y)$

$x \in B \Leftrightarrow \exists$ one $z \in C \,/\, z = f(x)$

$\Leftrightarrow z = f(g(y))$ for only one z

$\Leftrightarrow z = (f \circ g)(y) \quad y \in A$ and $z \in C$

Second Method; By contradiction

Suppose $\exists x, y \, (x \neq y) \in A \,/\, (f \circ g)(x) = (f \circ g)(y)$

$\Leftrightarrow f(g(x)) = f(g(y))$

$\exists \, z, w \left\{ \begin{matrix} z = g(x) \\ w = g(y) \end{matrix} \right\} \in B \,/\, f(z) = f(w)$ which is $z \neq w$

a contradiction because $f(x)$ is a one to one function, So $(f \circ g)$ must be also one to one.

N°30) $\underbrace{(f \text{ is onto one})}_{P} \wedge \underbrace{(f \circ g \text{ is one to one})}_{q} \Rightarrow \underbrace{g \text{ is one to one}}_{Q}$

$P \Leftrightarrow \neg \exists x, y_{x \neq y} \mid f(x) = f(y)$

$q \Leftrightarrow \neg \exists x, y_{x \neq y} \mid (f \circ g)(x) = (f \circ g)(y) \Leftrightarrow f(\underbrace{g(x)}_{X}) = f(\underbrace{g(y)}_{Y})$

So $\Leftrightarrow \neg \exists X, Y \; X \neq Y \mid f(X) = f(Y)$ $\quad X = g(x)$ $\quad Y = g(y)$

$\Leftrightarrow \neg \exists x, y_{x \neq y} \mid g(x) = g(y)$

So g is also one to one

N°36) b) $f(S \cap T) \subseteq f(S) \cap f(T)$??

$\forall x \, (x \in f(S \cap T) \rightarrow (x \in f(S)) \cap (x \in f(T)))$

Take random ~~[illegible]~~ $x \in S \cap T$

$\Leftrightarrow x \in S \wedge x \in T$

$\Leftrightarrow (\exists y \mid f(x) = y \in f(S)) \wedge (\exists y^* \mid f(x) = y^* \in f(T))$

$\Leftrightarrow (f(x) \in f(S)) \wedge (f(x) \in f(T))$

$\Leftrightarrow f(x) \in f(S) \cap f(T)$

So $f(S \cap T) \subseteq f(S) \cap f(T)$.