AUB-CMPS 211: *Discrete Structures*	June 3rd, 2009
Final Exam	
Instructor: Fatima Abu Salem	**Name:**

Duration: 90 minutes

This exam is closed notes.

Question 1 (On Logic and Rules of inference) (10%)

Convert each of the following into a symbolic proof, and justify each step of the proof leading up to the conclusion.

(a) (5%) For me to wear my summer clothes it is necessary that it be sunny. When it is sunny I always wear my sunscreen. Today I did not wear my sunscreen. Therefore, it must not be sunny, and so I am not wearing my summer clothes.

(b) (5%) For me to wear my summer clothes, it is sufficient that it be sunny. For me to wear my sunscreen, it is necessary that it be sunny. I am wearing my sunscreen. Therefore, it must be sunny, and so I must have been wearing my summer clothes.

Question 2 (On Iterative Algorithms) (18%)

In this question we examine a sorting algorithm known as *Selection* sort. Selection sort works iteratively as follows. A list of n integers is initially given in an array $A[0, \ldots, n-1]$. In the first iteration, the minimum element in the list $A[0, \ldots, n-1]$ is found then swapped with the element positioned in $A[0]$. In the second iteration, the second minimal element in the list $A[1, \ldots, n-1]$ is found then swapped with the element in $A[1]$. Generally, in the i'th iteration, the i'th minimal element in the list $A[i-1, \ldots, n-1]$ is found then swapped with the element in $A[i-1]$.

(a) (5%) Develop the pseudo-code for Selection Sort.

(b) (5%) Prove that Selection Sort is correct.

(c) (4%) Determine the asymptotic run-time of Selection Sort.

(d) (4%) Is Selection Sort asymptotically faster or slower than Merge Sort? Justify your answer.

Question 3 (On Recursive Algorithms) (19%)

Given a list of n positive integers in an array $A[0, \ldots, n-1]$, our goal is to find the greatest common divisor of all integers in this list. Assume that n is a power of two.

(a) (5%) Suggest and develop the pseudo-code for a divide and conquer algorithm to achieve this goal.

(a) (5%) Prove that your algorithm is correct.

(a) (5%) Write down the recurrence associated with the run-time of this algorithm.

(a) (4%) Use the Master Theorem to determine the run-time of this algorithm.

Question 4 (On Recurrences) (14%)

A string that contains only the characters 0, 1, and 2 is called a *ternary* string.

(a) (5%) Find a recurrence relation for the number of ternary strings that do not contain two consecutive 2's.

(b) (4%) What are the initial conditions for this recurrence?

(c) (5%) How many ternary strings of lenth five do not contain two consecutive 2's?

Question 5 (On Counting) (29%)

In how many ways can an organiser of a tournament arrange six players in a row, including the first prize winner and the second prize winner, if

(a) (5%) the first prize winner must be next to the second prize winner?

(b) (5%) the first prize winner is not next to the second prize winner?

(c) (5%) the first prize winner is standing somewhere to the right of the second prize winner?

A computer programming class consists of 14 math students and 12 computer science students. The course instructor chooses students randomly to serve on a programming contest.

(d) (5%) How many students must the course instructor select to be sure that at least two students of the same major are participating in the contest?

(e) (4%) How many students must the course instructor select to be sure that at least two computer science students are participating in the contest?

(f) (5%) Suppose there should be six students to participate in this contest. How many ways are there to form a group of contestants if the group must have the same number of math and computer science students?

Question 6 (On Equivalence relations and classes) (10%)

Let R denote the relation on the set of all bit strings such that two bit strings s and t are related by R if and only if the difference in absolute value of the number of zeros in s and t respectively is even or is equal to 0.

(a) (5%) Show that R is an equivalence relation.

(b) (5%) What is the equivalence class under R of the bit string $s = 0$?

Answer Sheet 1

Answer Sheet 2

Answer Sheet 3

Answer Sheet 4

Answer Sheet 5

Answer Sheet 6

Answer Sheet 7

Answer Sheet 8

Answer Sheet 9

Answer Sheet 10