

## Quiz 1

*Instructor: Fatima Abu Salem***Name:****Duration: 1 hour****This exam is closed notes.****Question 1 (On Propositional Equivalences) (18%)**

Determine whether or not the following is a tautology

$$(p \wedge q) \rightarrow r \quad \leftrightarrow \quad (p \rightarrow r) \wedge (q \rightarrow r)$$

- (a) (8%) Using a Truth Table.
- (b) (10%) Using Symbolic Derivation.

**Question 2 (On Nested Quantifiers) (48%)**

Are the following statements true or false? Justify your answer entirely; else, no credit will be given.

In all of the following,  $\mathbb{Z}$  denotes the set of integers,  $\mathbb{Q}$  denotes the set of rationals, and  $\mathbb{R}$  denotes the set of reals.

- (a) (8%)  $\forall x \in \mathbb{Z} \quad \exists y \in \mathbb{Z} \quad (x = 2y \vee x = x + 1)$ .
- (b) (8%)  $\exists x \in \mathbb{Z} \quad \forall y \in \mathbb{Z} \quad (x = 2y \vee x = x + 1)$ .
- (c) (8%)  $\forall x \in \mathbb{Q} \quad \exists y \in \mathbb{Z} \quad \exists z \in \mathbb{Z} \quad (x = z/y)$ .
- (d) (8%)  $\forall x \in \mathbb{R} \quad \exists y \in \mathbb{Z} \quad \exists z \in \mathbb{Z} \quad (x = z/y)$ .

Express the NEGATIONS of these statements using quantifiers:

- (e) (8%) There is a tourist in this group who has never been to a zoo.
- (f) (8%) There is a tourist in this group who has been in at least one museum of every town.

### Question 3 (On Sets and Set Operations) (16%)

- (a) (8%) Given two sets  $A$  and  $B$ , show that if  $A \cap B = \emptyset$ , then  $A - B = A$ .
- (b) (8%) Find the sets  $A$  and  $B$  if  $A - B = \{a, b, c, d\}$ ,  $B - A = \{e, f, g\}$ , and  $A \cap B = \{h, i\}$ .

### Question 4 (On Rules of Inference) (18%)

- (a) (10%) Give a direct proof that if  $(p \vee q) \wedge (q \rightarrow r)$  then  $(\neg(p \vee q) \vee q) \rightarrow ((p \vee q) \wedge r)$ .
- (b) (8%) Assume that if you exercise daily, you will shed some extra weight. Is it correct to assume that you did not shed extra weight if you did not exercise daily? Justify your answer using a logical argument (involving propositional variables, statements, connectives, and laws).

# Answer Sheet 1

## Answer Sheet 2

# Answer Sheet 3

# Answer Sheet 4

# Answer Sheet 5

# Answer Sheet 6



# Answer Sheet 7

# Answer Sheet 8

# Answer Sheet 9

# Answer Sheet 10