Quiz 2

Instructor: Fatima Abu Salem **Name:**

Duration: 1 hour

This exam is closed notes.

Question 1 (On Proofs) (15%)

Let x denote an integer. Show that the following statements are equivalent:

1. $3x - 2$ is odd.
2. $x + 8$ is odd.
3. $x^2 + 2$ is even.

Question 2 (On Induction) (30%)

(a) (10%) Show that $\exists\, k > 0$ such that $x^2 - 3x - 4 \geq 0 \quad \forall\, x \geq k$.
Let n denote a positive integer which is a power of 2 and let $f(n)$ denote the recursive function defined by

$$f(n) = \begin{cases} 1 & \text{if} \quad n = 1 \\ f(\frac{n}{2}) + n & \text{if} \quad n > 1. \end{cases}$$

We wish to show that the predicate $f(n) \leq 2n \log n$ is true $\forall\, n \geq 2$.

(b) (10%) Can one use mathematical (weak) induction to establish the predicate? Justify why or why not.

(c) (10%) Use strong induction to establish the predicate.

Question 3 (On Iterative Algorithms) (25%)

(a) (10%) Devise, and write pseudo-code for, an iterative algorithm which takes a list of integers and returns the sum of all integers in this list that are greater than 10.

(b) (15%) Prove that your algorithm is correct.

Question 4 (On Recursive Algorithms) (30%)

Consider the following test which checks whether a given integer k is even or odd:

$$k \bmod 2 = \begin{cases} 1 & \text{if} \quad k \text{ is odd} \\ 0 & \text{if} \quad k \text{ is even} \end{cases}$$

where mod denotes the operator yielding the remainder of division of k by 2.

(a) (10%) Devise, and write pseudo-code for, a recursive algorithm which returns whether or not a list of integers contains an odd number.

(b) (10%) Trace your algorithm on the list: $\{2, 14, 6, 8\,1\}$.

(c) (10%) Prove that your algorithm is correct.

Answer Sheet 1

Answer Sheet 2

Answer Sheet 3

Answer Sheet 4

Answer Sheet 5

Answer Sheet 6

Answer Sheet 7

Answer Sheet 8

Answer Sheet 9

Answer Sheet 10