EECE 290 – Quiz 1
March 10, 2012

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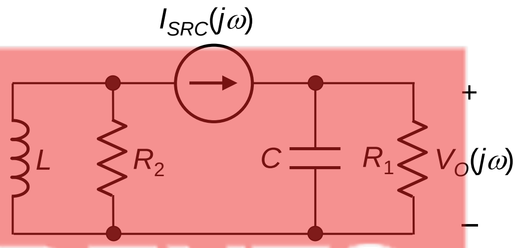
1. Specify the type of response $V_O(j\omega)/I_{SRC}(j\omega)$.



A. First-order lowpass
B. First-order highpass
C. Second-order lowpass
D. Second-order highpass
E. Second-order bandpass

Solution: The current source isolates the right-half of the circuit from the left half. $V_O(j\omega) \to 0$ as $\omega \to \infty$, and $V_O(j\omega) \to R_1 I_{SRC}(j\omega)$ as $\omega \to 0$ independently of the left half. The response is first-order lowpass. The transfer function is $\dfrac{V_O(j\omega)}{I_{SRC}(j\omega)} = \dfrac{R_1}{1 + j\omega CR_1}$.

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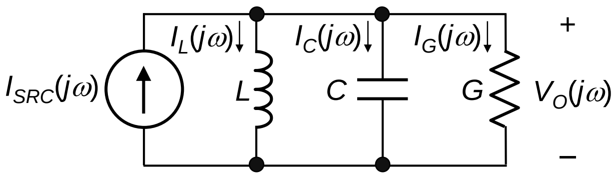
2. Specify which combination of variables gives a bandstop response with respect to $I_{SRC}(j\omega)$.



A. $I_L(j\omega) + I_C(j\omega)$
B. $I_L(j\omega) + I_G(j\omega)$
C. $I_G(j\omega) + I_C(j\omega)$
D. $I_G(j\omega)$
E. $V_O(j\omega)$

Solution: The correct combination is $I_L(j\omega) + I_C(j\omega)$, the dual of $V_C(j\omega) + V_L(j\omega)$ in the series circuit. The transfer function is $\dfrac{1/j\omega L}{G + 1/j\omega L + j\omega C} + \dfrac{j\omega C}{G + 1/j\omega L + j\omega C} =$

$$\frac{1-\omega^2 LC}{1-\omega^2 LC + j\omega LG} = \frac{1-\omega^2/\omega_0^2}{1-\omega^2/\omega_0^2 + j\omega LG} = \frac{\omega_0^2-\omega^2}{\omega_0^2-\omega^2 + j\omega\omega_0/Q} = \frac{s^2+\omega_0^2}{s^2+(\omega_0/Q)s+\omega_0^2}.$$

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3. Specify which of the following statements is or are true. (If more than one statement is true, no credit is given for a partially correct answer).

A. In a series *RLC* circuit, the magnitude of the peak of the lowpass response can exceed the magnitude of the applied excitation.
B. In a series *RLC* circuit, the magnitude of the peak of the bandpass response can exceed the magnitude of the applied excitation.

C. In a second-order, lowpass filter, the corner frequency is not equal to the half-power frequency except in the case of a Butterworth response.

D. In applying frequency and magnitude scaling, Q stays the same.

E. In a parallel RLC tuned circuit, increasing R, with L and C constant, increases the bandwidth.

Solution: A is true; B is false; C is true, D is true, and E is false.

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4. Determine the frequency at which $V_O(j\omega) = 0$, assuming $C = 1$ μF.

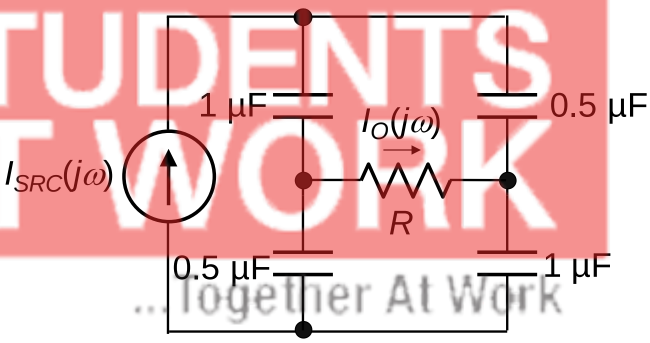
A. 707.1 krad/s

B. 577.4 krad/s

C. 500 krad/s

D. 447.2 krad/s

E. 408.2 krad/s



Solution: At the required frequency ω_Z, the impedance of the parallel LC circuit is inductive and produces series resonance with C, resulting in $V_O(j\omega) = 0$. The impedance of the parallel LC circuit is: $\dfrac{\left(j\omega'_Z \times 10^{-6}\right)\left(1/\left(j\omega'_Z \times 10^{-6}\right)\right)}{j\omega'_Z \times 10^{-6} + 1/\left(j\omega'_Z \times 10^{-6}\right)} = \dfrac{j\omega'_Z \times 10^{-6}}{1-(\omega'_Z)^2 \times 10^{-12}} = \dfrac{j\omega_Z}{1-\omega_Z^2}$, where ω'_Z is in rad/s and ω_Z is in Mrad/s. It follows that for series resonance, $\dfrac{j\omega_Z}{1-\omega_Z^2} + \dfrac{1}{j\omega_Z C} = 0$, or $\omega_Z^2 C = 1-\omega_Z^2$, which gives $\omega_Z = 1/\sqrt{1+C}$ Mrad/s.

Version 1: $C = 1$, $\omega_Z = 1/\sqrt{1+C} = 1/\sqrt{2} \equiv 707.1$ krad/s

Version 2: $C = 2$, $\omega_Z = 1/\sqrt{1+C} = 1/\sqrt{3} \equiv 577.4$ krad/s

Version 3: $C = 3$, $\omega_Z = 1/\sqrt{1+C} = 1/\sqrt{4} \equiv 500$ krad/s

Version 4: $C = 4$, $\omega_Z = 1/\sqrt{1+C} = 1/\sqrt{5} \equiv 447.2$ krad/s

Version 5: $C = 5$, $\omega_Z = 1/\sqrt{1+C} = 1/\sqrt{6} \equiv 408.2$ krad/s.

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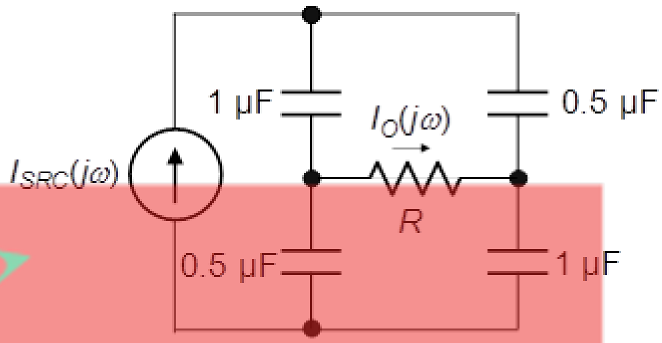
5. Determine the 3-dB cutoff frequency for the response $I_O(j\omega)/I_{SRC}(j\omega)$, assuming R = 1 kΩ.



A. 1500 rad/s
B. 750 rad/s
C. 500 rad/s
D. 375 rad/s
E. 300 rad/s

Solution: ω_c is determined as the effective capacitance C_{eff} seen across R when the current source is set to zero, that is, replaced by an open circuit. Under these conditions, the upper capacitors are in series and will have a capacitance of $\dfrac{0.5\times 1}{1.5}=\dfrac{1}{3}$ μF. Similarly the lower capacitors will have a capacitance of 1/3 μF. C_{eff} will then be that of two 1/3 μF capacitors in parallel, or 2/3 μF. $\omega_c = \dfrac{1}{C_{eff}R}=\dfrac{1}{(2/3)\times 10^{-6}\times R\times 10^{3}}=\dfrac{1500}{R}$, where R is in kΩ.

Version 1: R = 1 kΩ, ω_c = 1500/1 = 1500 rad/s

Version 2: R = 2 kΩ, ω_c = 1500/2 = 750 rad/s

Version 3: R = 3 kΩ, ω_c = 1500/3 = 500 rad/s

Version 4: R = 4 kΩ, ω_c = 1500/4 = 375 rad/s

Version 5: R = 5 kΩ, ω_c = 1500/5 = 300 rad/s.

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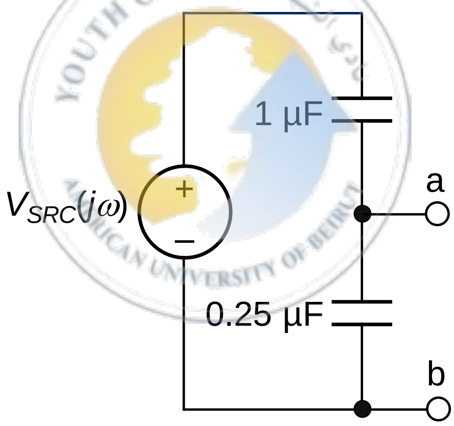
6. Determine Thevenin's voltage looking into terminals ab, assuming $V_{SRC}(j\omega)$ = 1 V.



A. 1.6 V
B. 0.4 V
C. 0.8 V
D. 0.2 V
E. 0.6 V

Solution: The impedance of the 1 μF capacitor is $1/(j\omega\times 1)$ Ω, where ω is in Mrad/s and the impedance of the 0.25 μF capacitor is $1/(j\omega\times 0.25)$ Ω = $4/j\omega$. It follows from voltage division that $V_{Th} = \dfrac{4/j\omega}{4/j\omega+1/j\omega}V_{SRC}(j\omega)=0.8V_{SRC}(j\omega)$.

Version 1: $V_{SRC}(j\omega)$ = 1 V; V_{Th} = 0.8×1 = 0.8 V

Version 2: $V_{SRC}(j\omega)$ = 2 V; V_{Th} = 0.8×2 = 1.6 V

Version 3: $V_{SRC}(j\omega) = 3$ V; $V_{Th} = 0.8\times3 = 2.4$ V

Version 4: $V_{SRC}(j\omega) = 4$ V; $V_{Th} = 0.8\times4 = 3.2$ V

Version 5: $V_{SRC}(j\omega) = 5$ V; $V_{Th} = 0.8\times5 = 4$ V.

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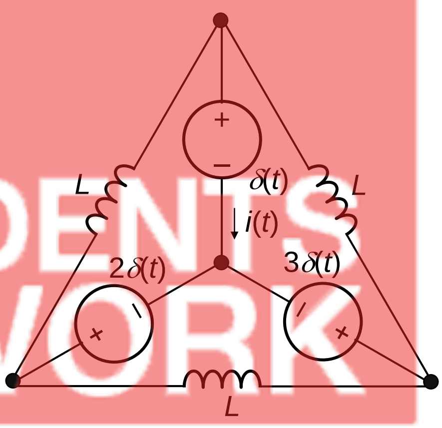
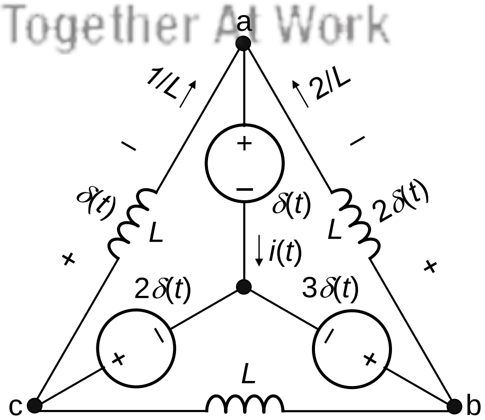
7. Determine $i(t)$, $t > 0$, assuming zero initial energy storage, unit impulses are in μVs, and $L = 1$ μH.



A. 3 A
B. 1.5 A
C. 1 A
D. 0.75 A
E. 0.6 A

Solution: From KVL, $v_{ba} = 3\delta(t) - \delta(t)$, and $i_{ba} = \frac{1}{L}\int_{0^-}^{0^+}[3\delta(t)-\delta(t)]dt = \frac{3}{L}-\frac{1}{L}=\frac{2}{L}$ A, $t > 0$. Similarly, $v_{ca} = 2\delta(t) - \delta(t)$, and $i_{ca} = \frac{1}{L}\int_{0^-}^{0^+}[2\delta(t)-\delta(t)]dt = \frac{2}{L}-\frac{1}{L}$ $= \frac{1}{L}$ A, $t > 0$. From KCL, $i(t) = i_{ba} + i_{ca} = 3/L$ A, $t > 0$.



Version 1: $L = 1$ μH, $i(t) = 3/1 = 3$ A

Version 2: $L = 2$ μH, $i(t) = 3/2 = 1.5$ A

Version 3: $L = 3$ μH, $i(t) = 3/3 = 1$ A

Version 4: $L = 4$ μH, $i(t) = 3/4 = 0.75$ A

Version 5: $L = 5$ μH, $i(t) = 3/5 = 0.6$ A.

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8. Evaluate $A\int_{-\infty}^{\infty}(\cos t)\delta\left(2t-\frac{\pi}{2}\right)dt$, where $A = 1$.

A. 0.35
B. 0.71
C. 1.06
D. 1.41
E. 1.77

Solution: The coefficient of t in the impulse function should be unity. Hence a new variable t' is defined, where $t' = 2t$. The integral becomes: $\frac{A}{2}\int_{-\infty}^{\infty}\cos\frac{t'}{2}\delta\left(t'-\frac{\pi}{2}\right)dt'$. The integrand is zero everywhere except from $t' = (\pi/2)^-$ to $t' = (\pi/2)^+$, when the impulse occurs. At this value of t', $\cos\left(\frac{t'}{2}\right)$ is continuous and equals $\cos(\pi/4) = 1/\sqrt{2}$. The integral becomes:

$$\frac{A}{2\sqrt{2}}\int_{(\pi/2)^-}^{(\pi/2)^+}\delta\left(t' - \frac{\pi}{2}\right)dt' = \frac{A}{2\sqrt{2}}.$$

Version 1: $A = 1$, $\frac{1}{2\sqrt{2}} = 0.35$

Version 2: $A = 2$, $\frac{1}{\sqrt{2}} = 0.71$

Version 3: $A = 3$, $\frac{3}{2\sqrt{2}} = 1.06$

Version 4: $A = 4$, $\frac{2}{\sqrt{2}} = 1.41$

Version 5: $A = 5$, $\frac{5}{2\sqrt{2}} = 1.77$.

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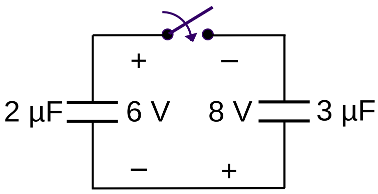
9. A 2 μF capacitor is connected at $t = 0$ in parallel with a 3 μF, the initial voltages being 6 V and 8 V, respectively, with the polarities being as shown. Determine the amount of charge transferred between the two capacitors.



A. 16.8 μC
B. 8.4 μC
C. 25.2 μC
D. 21 μC
E. 12.6 μC

Solution: The initial charges are +12 μC on C_1 and -24 μC on C_2. C_{eqp} = 2 + 3 = 5 μF and will have a charge of +12 – 24 = -12 μC. The voltage on C_{eqp}, and hence on C_1 and C_2, is -12/5 = -2.4 V. The final charges are -4.8 μC on C_1 and -7.2 μC on C_2. The charge transferred the 2 μF capacitor to the 3 μF capacitor is 12 – (-4.8) = 16.8 μC.

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10. A voltage pulse of 1 V amplitude and 2 μs duration is applied to two uncharged inductors of 2 μH and 3 μH connected in series. Determine the final flux linkage in the 2 μH inductor in μWb-turns.

A. 0.8

B. 1.6

C. 2.4

D. 3.2

E. 4

Solution: L_{eqs} = 5 μH and the flux linkage due to the voltage pulse is 2*B* μC, where *B* is the amplitude of the pulse. The current in L_{eqs}, and hence in L_1 and L_2, is 2*B*/5 A. The final flux linkage in the 2 μH inductor is λ = 4*B*/5 μWb-turns.

Version 1: *B* = 1, λ = 4/5 = 0.8 μWb-turns

Version 2: *B* = 2, λ = 8/5 = 1.6 μWb-turns

Version 3: *B* = 3, λ =12/5 = 2.4 μWb-turns

Version 4: *B* = 4, λ = 16/5 = 3.2 μWb-turns

Version 5: *B* = 5, λ = 20/5 = 4 μWb-turns.

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11. It is required to design a bandpass filter having a resonant frequency of 10^6 rad/s, a bandwidth of 10^4 rad/s, and a maximum magnitude of response of 0.5.

7% a. Specify the required transfer function in terms of $s = j\omega$.

7% b. Determine *R* and *C*, assuming a series circuit and *L* = 1 mH.

6% c. Draw the required circuit using 5 Ω resistors, 2 mH inductors, and 2 nF capacitors, and specify how the output voltage is taken.

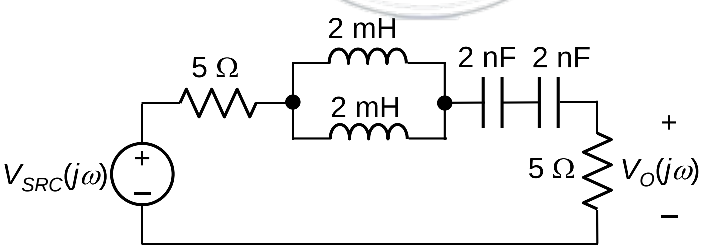
Solution: (a) The required bandpass transfer function in standard form is:

$H(s) = \dfrac{(\omega_0/Q)s}{s^2 + (\omega_0/Q)s + \omega_0^2}$, where (ω_0/Q) is the bandwidth, and the maximum magnitude of response is 1. It follows that the required transfer function is:

$H(s) = \dfrac{5000s}{s^2 + 10^4 s + 10^6}$.

(b) $\omega_0^2 = \dfrac{1}{LC}$, $C = \dfrac{1}{\omega_0^2 L} = \dfrac{1}{10^{12} \times 10^{-3}} = 10^{-9} \equiv 1$ nF. BW $= \dfrac{\omega_0}{Q} = \dfrac{R}{L}$. Hence $R = 10^{-3} \times 10^4 = 10$ Ω.

(c) The circuit will have two 2 mH inductors in parallel, two 2 nF capacitors in series, and two 5 Ω resistors in series, the output voltage being taken across one of the resistors.



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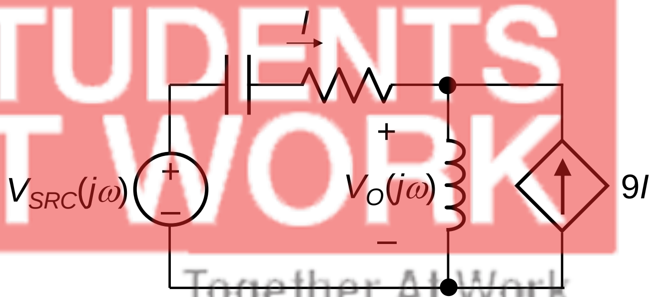
12. It is required to design a highpass, second order Butterworth filter having ω_0 = 10 krad/s, using a 0.1 μF capacitor, given that the normalized second-order Butterworth function is $s^2+\sqrt{2}s+1$.

5% a. Specify the required transfer function in terms of the second-order normalized Butterworth function.

10% b. Derive the required actual values of R and L using scaling and assuming a series RLC circuit.

5% c. If the filter circuit is that shown, specify the values of resistance, capacitance and inductance that will implement the required transfer function.



Solution: (a) The required highpass transfer function is that having s^2 in the numerator and the normalized second-order Butterworth function in the denominator, that is:

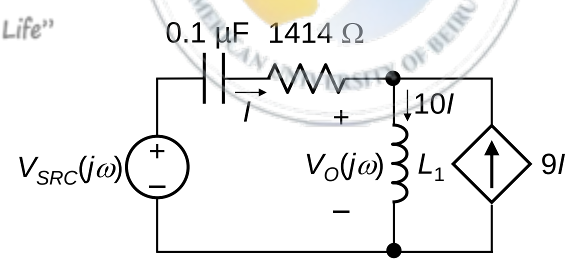
$$H(s)=\frac{s^2}{s^2+\sqrt{2}s+1}$$, where ω_0 = 1 rad/s, L = 1 H, C = 1 F, $\sqrt{2}=\frac{\omega_0}{Q}=\frac{\omega_0 R}{\omega_0 L}=R$.

(b) k_f = 10 (krad/s)/1 (rad/s) = 10^4; $0.1\mu\text{F}=\frac{1}{k_f k_m}(1\text{F})$, or $k_m=\frac{1}{10^4\times10^{-7}}=10^3$. This

gives $L=\frac{k_m}{k_f}(1\text{H})=0.1\text{H}$ and $R=k_m\sqrt{2}=1000\sqrt{2}=1414$ Ω. As a check,

$$\omega_0=\frac{1}{\sqrt{0.1\times10^{-7}}}=10^4 \text{ rad/s}, Q=\frac{\omega_0 L}{R}=\frac{10^4\times0.1}{1000\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{1}{\omega_0 CR}=\frac{1}{10^4\times10^{-7}\times1000\sqrt{2}}.$$

(c) From the circuit shown, $V_O(j\omega)=j\omega 10L_1 I$, or $\frac{V_O(j\omega)}{I}=j\omega 10L_1$. The effective inductance is therefore $10L_1$. Equating this to 0.1H, gives L_1 = 10 mH. C = 0.1 μF as given, and R = 1414 Ω as calculated in (b).



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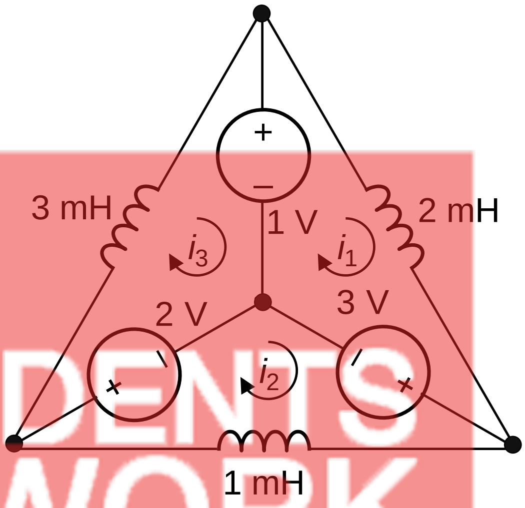
13. It is required to derive the dual of the circuit shown.



5% a. Derive the mesh-current equations using the mesh currents shown.

10% b. Deduce the node-voltage equations of the dual circuit.

5% c. Derive the circuit that will give these node-voltage equations with respect to a specified reference node.

Solution: (a) The mesh-current equations are:

$$j\omega 2\times10^{-3} i_1 = 1\text{ V} - 3\text{ V}$$

$$j\omega 1\times10^{-3} i_2 = 3\text{ V} - 2\text{ V}$$

$$j\omega 3\times10^{-3} i_3 = 2\text{ V} - 1\text{ V}$$

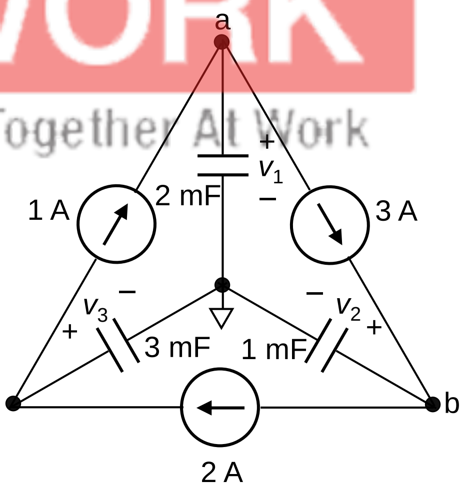
(b) The node-voltage equations of the dual circuit are:

$$j\omega 2\times10^{-3} v_1 = 1\text{ A} - 3\text{ A}$$

$$j\omega 1\times10^{-3} v_2 = 3\text{ A} - 2\text{ A}$$

$$j\omega 3\times10^{-3} v_3 = 2\text{ A} - 1\text{ A}$$

(c) The circuit that gives these node-voltage equations is as shown.



Note that if a current K is added to each of the three current sources in the clockwise direction, so they become 1 + K, 3 + K, and 2 + K, the node-voltage equations are not affected. By choosing a suitable value of K, one of the current sources can be eliminated. Thus, K = -1, for example, eliminates the source on the left, the source on the right becomes 2 A, and the bottom source becomes 1 A. This corresponds to subtracting 1 V from each of the Y-connected sources in the original circuit, which would reduce the 1 V source to zero.

Connection-wise, the dual of three voltage source in wye is three current sources in delta, and the dual of three inductors in delta is three capacitors in wye. The dual circuit can be derived by means of the graphical procedure as shown. The polarity of a current source is towards a node if the voltage source it replaces is a voltage rise in the direction of the clockwise mesh current.