Homework 2

P10.2.4 In the circuit of Figure P10.2.4, the magnitude of the transfer function is unity at 1 Mrad/s and zero at 0.5 Mrad/s. Determine L_1 and L_2.

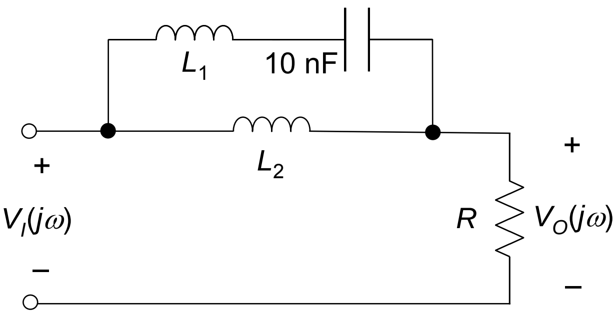


Figure P10.2.4

<u>Solution P10.2.4</u> At 1 Mrad/s, L_1 is in series resonance with the 10 nF capacitor; hence $L_1 = \frac{1}{10^{-8}\times 10^{12}} = 10^{-4}$ H, so $L_1 =$ 0.1 mH. At 0.5 Mrad/s, the reactance of L_1 and C is capacitive and is in parallel resonance with L_2. The reactance of L_1 and C at 0.5 Mrad/s is $0.5\times10^6\times10^{-4}$ - $\frac{1}{0.5\times10^6\times10^{-8}}$ = 50 – 200 = -150; hence, $0.5\times10^6\times L_2$ = 150, or $L_2 = \frac{150}{0.5\times10^6}$ = 300×10^{-6} = 0.3 mH.

P10.2.7 Determine: (a) the transfer function of the circuit of Figure P6.10.2.7; (b) the resonant frequency; (c) the Q; and (d) the bandwidth.

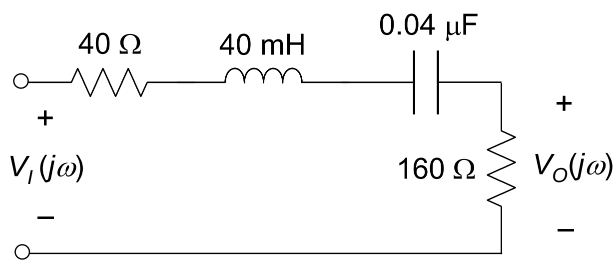


Figure P10.2.7

<u>Solution P10.2.7</u> (a) $H(s) = \dfrac{160}{0.04s + 200 + \dfrac{1}{s\times 0.04\times 10^{-6}}} = \dfrac{4000s}{s^2 + 5000s + 625\times 10^6} =$

$\dfrac{4s}{s^2 + 5s + 625}$, where s is in krad/s.

(b) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{40\times10^{-3}\times 4\times 10^{-8}}} = 25$ krad/s.

(c) $Q = \dfrac{\omega_0 L}{R} = \dfrac{25\times10^3\times 0.04}{200} = 5$; (d) BW = $\dfrac{\omega_o}{Q} = \dfrac{25}{5} = 5$ krad/s.

P10.2.9 Given the circuit of Figure P10.2.9, where k is a positive constant. Determine Q when: (a) $k = 1$; (b) k is very large. What is the interpretation of the value of Q when k is large?

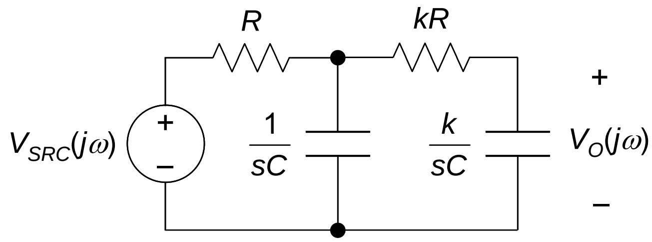
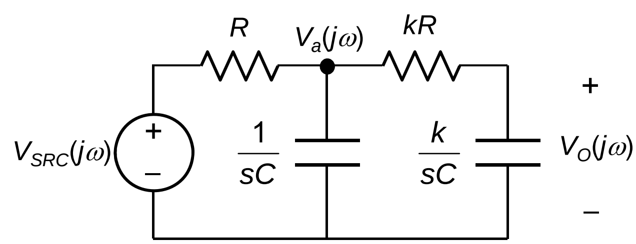


Figure P10.2.9

<u>Solution P10.2.9</u> Q can be determined from the denominator in standard form of the transfer function $V_O(j\omega)/V_{SRC}(j\omega)$. The transfer function can be determined from node-voltage analysis, mesh-current analysis, or voltage division.

To apply voltage division, the parallel impedance of the three branches between V_a and ground is:



$$\frac{1/sC\left(kR+k/sC\right)}{kR+(k+1)/sC}=$$

$$\frac{1}{sC}\frac{\left(1+sCR\right)}{(1+1/k)+sCR}$$. It follows that: $$\frac{V_a(j\omega)}{V_{SRC}(j\omega)}=\frac{\frac{1}{sC}\frac{\left(1+sCR\right)}{(1+1/k)+sCR}}{R+\frac{1}{sC}\frac{\left(1+sCR\right)}{(1+1/k)+sCR}}=$$

$$\frac{\left(1+sCR\right)}{1+\left(2+1/k\right)sCR+s^2C^2R^2};\ \frac{V_O(j\omega)}{V_a(j\omega)}=\frac{k/sC}{kR+k/sC}=\frac{1}{1+sCR}$$. Hence,

$$\frac{V_O(j\omega)}{V_{SRC}(j\omega)}=\frac{1}{s^2C^2R^2+\left(2+1/k\right)sCR+1}=\frac{1/C^2R^2}{s^2+\left(2+1/k\right)s/CR+1/C^2R^2}$$. This is of the standard for for a second-order low-pass filter $\frac{\omega_n^2}{s^2+\left(\omega_n/Q\right)s+\omega_n^2}$, where $\omega_n=1/CR$ and $Q=1/(2+1/k)$.

(a) If $k=1$, $Q=1/3$.

(b) if k is very large, $Q = 1/2$. The interpretation is that when k is large, the circuit composed of kR and C/k has a negligible loading effect on the first circuit. The two circuits, of identical time constant RC, are cascaded but effectively isolated. The overall transfer function becomes:

$$\frac{V_O(j\omega)}{V_{SRC}(j\omega)}=\frac{1}{\left(1+sCR\right)^2}=\frac{1}{1+2sCR+s^2C^2R^2}=\frac{1/C^2R^2}{s^2+2s/CR+1/C^2R^2},\ Q = 1/2.$$

P10.2.10 In Figure P10.2.10, the response $I_C(j\omega)/I_{SRC}(j\omega)$ is to remain at -3db at the same frequency, whether the switch is opened or closed. (a) Determine R and the corner frequency, assuming L = 20 mH. (b) Calculate the response in dB at one-tenth the corner frequency for the two cases when the switch is open or closed.

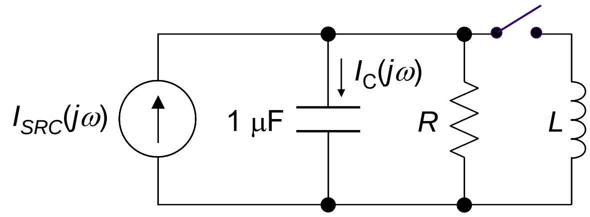
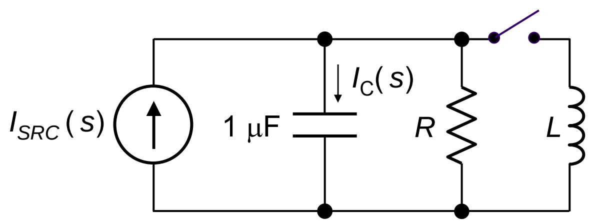


Figure P10.2.10

<u>**Solution P10.2.10**</u> (a) When the switch is open, $\dfrac{I_C(s)}{I_{SRC}(s)} = \dfrac{sCR}{1+sCR} = \dfrac{1}{1+1/sCR}$, and $\omega_{ch} = \dfrac{1}{CR} = \dfrac{10^6}{R}$. When the switch is closed,



$\dfrac{I_C(s)}{I_{SRC}(s)} = \dfrac{sC}{sC + \dfrac{1}{R} + \dfrac{1}{sL}} = \dfrac{s^2}{s^2 + \dfrac{s}{CR} + \dfrac{1}{LC}}$; $|H(j\omega)| = \dfrac{\omega^2}{\sqrt{\left(\omega^2 - \dfrac{1}{LC}\right)^2 + \left(\dfrac{\omega}{CR}\right)^2}}$. At $\omega = \omega_{ch}$, $|H(j\omega)| = \dfrac{1}{\sqrt{2}}$ or

$2\omega_{ch}^4 = \omega_{ch}^4 - \dfrac{2\omega_{ch}^2}{LC} + \dfrac{1}{L^2C^2} + \dfrac{\omega_{ch}^2}{C^2R^2}$. Substituting $\omega_{ch}^2 = \dfrac{1}{C^2R^2}$ gives: $\omega_{ch}^2 = \dfrac{1}{2LC}$.

With $C = 10^{-6}$ F and $L = 2\times10^{-2}$ H, $\omega_{ch} = 5$ krad/s and $R = \dfrac{10^6}{\omega_{ch}} = \dfrac{10^6}{5\times10^3} = 200\ \Omega$.

(b) At $\omega_{ch}/10$, the response when the switch is open is $20\log_{10}\dfrac{1}{\sqrt{1+(\omega_{ch}/\omega)^2}} =$

$20\log_{10}\dfrac{1}{\sqrt{101}} =$ -20.04 dB. When the switch is closed, $\dfrac{1}{LC} = 2\omega_{ch}^2$ and $\dfrac{1}{CR} = \omega_{ch}$.

This gives: $|H(j0.1\omega_{ch})| = \dfrac{1}{\sqrt{\left(1 - \dfrac{1}{\omega^2 LC}\right)^2 + \left(\dfrac{1}{\omega CR}\right)^2}} =$

$\dfrac{1}{\sqrt{\left(1 - \dfrac{2\omega_{ch}^2}{(0.1\omega_{ch})^2}\right)^2 + \left(\dfrac{\omega_{ch}}{0.1\omega_{ch}}\right)^2}} = \dfrac{1}{199.25}$, and $20\log_{10}\dfrac{1}{199.25} =$ -46 dB.

P10.3.3 Derive the transfer function of the circuit shown in Figure P10.3.3, and determine L and C so as to have a second-order, lowpass, normalized, Butterworth response. Scale the parameters so as to have R_L = 10 kΩ and the 3-dB frequency f_0 = 10 kHz.

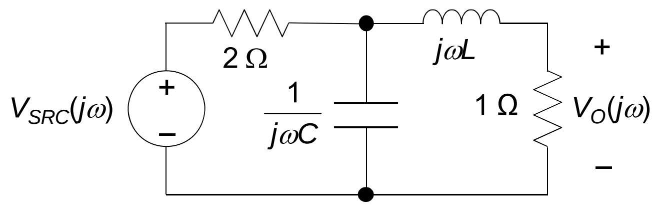


Figure P10.3.3

Solution P10.3.3 The impedance of C in parallel with L and R_2 is: $Z_p = \dfrac{(sL+R_2)/sC}{sL+R_2+1/sC} = \dfrac{sL+R_2}{s^2LC+sCR_2+1}$. It follows that: $\dfrac{V_O}{V_{SRC}} = \dfrac{Z_p}{R_1+Z_p}\dfrac{R_2}{sL+R_2} = \dfrac{R_2}{LCR_1}\dfrac{1}{s^2+s\left(\dfrac{1}{CR_1}+\dfrac{R_2}{L}\right)+\dfrac{1}{LC}\dfrac{R_1+R_2}{R_1}}$ Substituting R_1 = 2 Ω and R_2 = 1 Ω, the denominator becomes: $s^2+s\left(\dfrac{1}{2C}+\dfrac{1}{L}\right)+\dfrac{3}{2LC}$. For a Butterworth response, $LC = \dfrac{3}{2}$ and $\dfrac{1}{2C}+\dfrac{1}{L}=\sqrt{2}$. Substituting $\dfrac{1}{2C}=\dfrac{L}{3}$, gives $\dfrac{L}{3}+\dfrac{1}{L}=\sqrt{2}$, or $L^2-3\sqrt{2}L+3=0$.

Hence, $L=\dfrac{\sqrt{6}}{2}\left(\sqrt{3}\pm 1\right)$ H and $C=\dfrac{\sqrt{6}}{4}\left(\sqrt{3}\mp 1\right)$ F.

To scale, $R_L = R_2$ to 10 kΩ, $k_m = 10^4$. For a normalized butterworth response, the 3-dB cut-off frequency is 1 rad/s. To scale f_n = 10 kHz, $k_f = 2\pi\times 10^4$; hence,

$L' = \dfrac{k_m}{k_f}L \equiv \dfrac{\sqrt{6}}{4\pi}\left(\sqrt{3}\pm 1\right)$ H and $C' = \dfrac{1}{k_m k_f}C \equiv \dfrac{5\sqrt{6}}{4\pi}\left(\sqrt{3}\mp 1\right)$ nF.

Note that for a Butterworth filter, the corner frequency is the same as the -3dB cutoff frequency, the latter being always the same as the half-power frequency.

P10.4.4 Given the straight-line, Bode plot approximation of Figure P10.4.4. Determine: (a) the *X* dB level; (b) the transfer function approximated by this Bode plot.

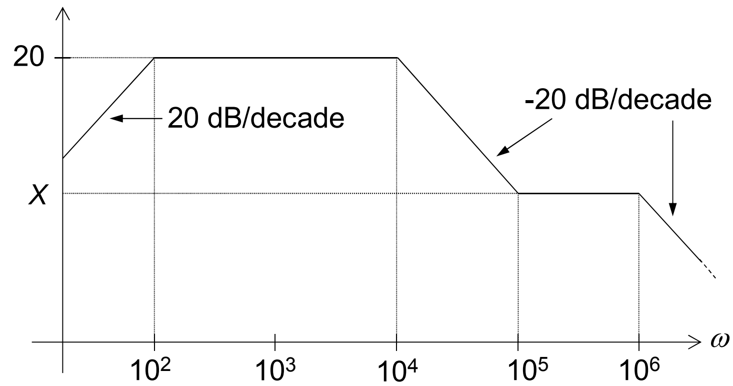


Figure P10.4.4

Solution P10.4.4 (a) Since the plot drops 20 dB in a decade from the 20 dB level, *X* must be at 0 dB.

(b) The response from very low frequencies up to $\omega = 10^4$ rad/s is a first-order high-pass response having $\omega_{ch} = 10^2$ rad/s. The transfer function up to $\omega = 10^4$ rad/s is of the form $K\dfrac{j\omega}{j\omega+\omega_{ch}} = K\dfrac{s}{s+10^2}$. At $\omega = 10^4$ rad/s there is a response of the form $\dfrac{1}{j\omega+10^4} = \dfrac{1}{s+10^4}$ so that a corner frequency is obtained at $\omega = 10^4$ rad/s, the response is 0 dB for $\omega < 10^4$ rad/s and the response decreases at a rate of -20 dB/decade for $\omega > 10^4$ rad/s. At $\omega = 10^5$ rad/s there is a response of the form $j\omega + 10^5 = s + 10^5$, so that a corner frequency is obtained at $\omega = 10^5$ rad/s, the response is 0 dB for $\omega < 10^5$ rad/s and the response increases at a rate of +20 dB/decade for $\omega > 10^5$ rad/s so as to give a horizontal asymptote $\omega > 10^5$ rad/s. At $\omega = 10^6$ rad/s there is a response of the form $\dfrac{1}{j\omega+10^6} = \dfrac{1}{s+10^6}$ so that a corner frequency is obtained at $\omega = 10^6$ rad/s, the response is 0 dB for $\omega < 10^6$ rad/s and the response decreases at a rate of -20 dB/decade for $\omega > 10^6$ rad/s. The transfer function is of the form: $H(s) = \dfrac{Ks(s+10^5)}{(s+100)(s+10^4)(s+10^6)}$. To determine *K*, we note that at very low frequencies, $H(s) \approx \dfrac{Ks\times10^5}{10^2\times10^4\times10^6}$. The line passes through the point (100 rad/s, 20 dB) so that $|H(s)| = 10$ at $\omega = 100$ rad/s. This gives $K = \dfrac{10\times10^{12}}{10^2\times10^5} = \dfrac{10^{13}}{10^7} = 10^6$.