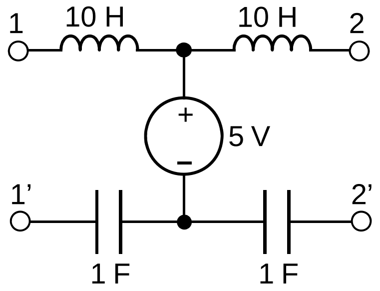
EECE 290

Quiz 3, May 7, 2011

1. Determine which of the following statements is or are true of the two-port circuit shown. (If a false statement is marked true, the question is considered incorrect).



A. The circuit is a symmetric two-port circuit.

B. The circuit is symmetric but not reciprocal.

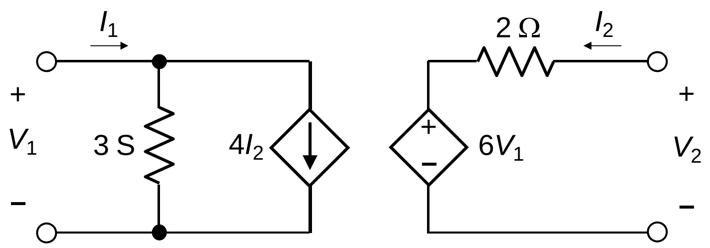
C. The circuit is not a valid two-port circuit.

D. The circuit is valid at some frequencies and invalid at other frequencies.

E. The circuit can be described in terms of *a* and *b* parameters.

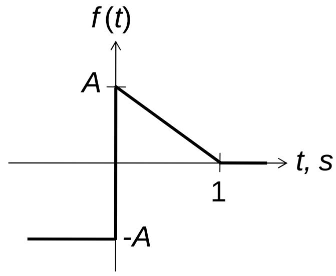
Answer. The only true statement is C.

2. Specify the two-port parameter equations that are directly represented by the circuit shown.



Solution: The circuit directly represents the *g*-parameter equations.

3. Determine the Laplace transform of the time integral of $f(t)$, $\int_{0^-}^{t} f(t)dt$, assuming A = 1. The Laplace transform should apply for $t \geq 2$ s.



Solution. *Method 1:* $\int_{0^-}^{2} f(t)dt$ is the area of the triangle, which $A/2$. The Laplace transform is therefore $A/2s$.

Method 2: $F(s) = \int_{0^-}^{1} A(1-t)e^{-st}dt = A\left[\left(-\frac{1}{s}e^{-st}\right)_{0^-}^{1} - \left(-\frac{t}{s}e^{-st}\right)_{0^-}^{1} - \frac{1}{s}\left(-\frac{1}{s}e^{-st}\right)_{0^-}^{1}\right] =$

$A\left[-\frac{e^{-s}}{s} + \frac{1}{s} + \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} - \frac{1}{s^2}\right] = A\left[\frac{1}{s} - \frac{1}{s^2} + \frac{e^{-s}}{s^2}\right]$. $\mathscr{L}\{f^{(-1)}(t)\} = A\left[\frac{1}{s^2} - \frac{1}{s^3} + \frac{e^{-s}}{s^3}\right]$; $f^{(-1)}(t)$ =

$= A\left[t - \frac{t^2}{2} + \frac{(t-1)^2}{2}u(t-1)\right]$. For $t \geq 2$ s, $f^{(-1)}(t) = A/2$, and the Laplace transform is $A/2s$.

Version 1: $A = 1$, $A/2s = 0.5/s$

Version 2: $A = 2$, $A/2s = 1/s$

Version 3: A = 3, $A/2s = 1.5/s$

Version 4: A = 4, $A/2s = 2/s$

Version 5: A = 5, $A/2s = 2.5/s$

4. Given that the Laplace transform of $A\left(e^{-t}/\sqrt{t}\right)u(t)$ is $\dfrac{A\sqrt{\pi}}{\sqrt{s+1}}$, determine the Laplace transform of $\left(Ae^{-t}\sqrt{t}\right)u(t)$, assuming A = 1.

Solution. The function whose Laplace transform is required is t times the given function. Hence, its Laplace transform is $-\dfrac{dF(s)}{ds}$, where $F(s) = \dfrac{A\sqrt{\pi}}{\sqrt{s+1}}$. Thus,

$$-\frac{dF(s)}{ds} = \frac{A\sqrt{\pi}}{2(s+1)^{3/2}}.$$

Version 1: A = 1, $\dfrac{A\sqrt{\pi}}{2(s+1)^{3/2}} = \dfrac{0.5\sqrt{\pi}}{(s+1)^{3/2}}$

Version 2: A = 2, $\dfrac{A\sqrt{\pi}}{2(s+1)^{3/2}} = \dfrac{\sqrt{\pi}}{(s+1)^{3/2}}$

Version 3: A = 3, $\dfrac{A\sqrt{\pi}}{2(s+1)^{3/2}} = \dfrac{1.5\sqrt{\pi}}{(s+1)^{3/2}}$

Version 4: A = 4, $\dfrac{A\sqrt{\pi}}{2(s+1)^{3/2}} = \dfrac{2\sqrt{\pi}}{(s+1)^{3/2}}$

Version 5: A = 5, $\dfrac{A\sqrt{\pi}}{2(s+1)^{3/2}} = \dfrac{2.5\sqrt{\pi}}{(s+1)^{3/2}}$

5. Determine which of the following statements is or are true. (If a false statement is marked true, the question is considered incorrect).

A. The response of the same circuit can be bounded for a certain input and unbounded for another input.

B. The poles of a first-order circuit must always occur in complex conjugate pairs.

C. A second-order circuit cannot have a double pole.

D. A pole of a voltage source at s = -2 corresponds to dc voltage of -2 V.

E. A circuit is unstable if the poles of the transfer function are in the right half of the s plane including the imaginary axis.

Answer. The only true statement is A.

6. Determine the value of the convolution integral $y(t)$ at t = 0.5 s, where $y(t) = f(t)*g(t)$, with $f(t) = \sin t$, $g(t) = 2\delta(t) + \delta^{(2)}(t)$, and $\delta^{(2)}(t)$ being the second derivative of $\delta(t)$.

Solution. $\mathfrak{L}\{\sin t\} = \dfrac{1}{s^2+1}$, and $\mathfrak{L}\{2\delta(t) + \delta^{(2)}\} = 2 + s^2$; $Y(s) = \dfrac{s^2+2}{s^2+1} = 1 + \dfrac{1}{s^2+1}$;

$y(t) = \delta(t) + \sin t$. At $t = T$ s, $y(T) = \sin T$.

Version 1: T = 0.5, sin(0.5) = 0.48

Version 2: T = 1, sin(1) = 0.84

Version 3: T = 1.5, sin(1.5) = 1.00

Version 4: T = 2, sin(2) = 0.91

Version 5: T = 2.5, sin(2.5) = 0.60

7. Derive an expression for the Laplace transform of $f(t-b)u(t-a)$, where a and b are positive constants.

Solution. $\mathfrak{L}\{f(t-b)u(t-a)\} = \int_{0^-}^{\infty} f(t-b)u(t-a)e^{-st}dt = \int_{a}^{\infty} f(t-b)e^{-st}dt$. Substituting, $t' = t - a$,

$\mathfrak{L}\{f(t-b)u(t-a)\} = \int_{0^-}^{\infty} f(t'+a-b)e^{-s(t'+a)}dt' = e^{-as}\,\mathfrak{L}\{f(t+a-b)\}$.

8. Determine $f(t)$ if $F(s) = \dfrac{Ks}{s^2+4s+20}$, assuming K = 1.

Solution. $F(s) = \dfrac{Ks}{s^2+4s+20} = \dfrac{Ks}{(s+2)^2+16} = \dfrac{K(s+2)-2K}{(s+2)^2+16}$

$f(t) = Ke^{-2t}\cos 4t - \dfrac{K}{2}e^{-2t}\sin 4t$

Version 1: K = 1, $f(t) = e^{-2t}\cos 4t - 0.5e^{-2t}\sin 4t$

Version 2: K = 2, $f(t) = 2e^{-2t}\cos 4t - e^{-2t}\sin 4t$

Version 3: K = 3, $f(t) = 3e^{-2t}\cos 4t - 1.5e^{-2t}\sin 4t$

Version 4: K = 4, $f(t) = 4e^{-2t}\cos 4t - 2e^{-2t}\sin 4t$

Version 5: K = 5, $f(t) = 5e^{-2t}\cos 4t - 2.5e^{-2t}\sin 4t$

9. If $F(s) = K\dfrac{s^2+s+1}{s^3+s^2+s+1}$, determine $f(t)$ for large values of t, assuming K = 1.

Solution. $K\dfrac{s^2+s+1}{s^3+s^2+s+1} = K\dfrac{s^2+s+1}{(s^2+1)(s+1)} = \dfrac{K}{2}\left[\dfrac{1}{s+1} + \dfrac{s+1}{s^2+1}\right]$. $f(t) = \dfrac{K}{2}\left[e^{-t} + \cos t + \sin t\right]$.

For large values of t, $f(t) \to \dfrac{K}{2}\left[\cos t + \sin t\right] = \dfrac{K}{\sqrt{2}}\cos(t - \pi/4)$. The final-value theorem does not apply because of the poles at $s = \pm j$.

Version 1: $K = 1$, $\frac{K}{\sqrt{2}}\cos(t-\pi/4) = \frac{1}{\sqrt{2}}\cos(t-\pi/4)$

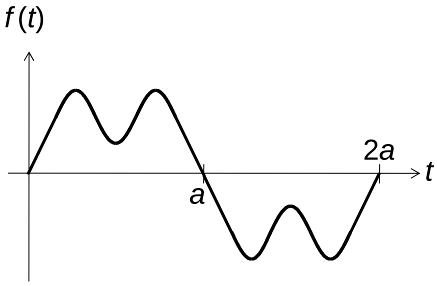
Version 2: $K = 2$, $\frac{K}{\sqrt{2}}\cos(t-\pi/4) = \sqrt{2}\cos(t-\pi/4)$

Version 3: $K = 3$, $\frac{K}{\sqrt{2}}\cos(t-\pi/4) = \frac{3}{\sqrt{2}}\cos(t-\pi/4)$

Version 4: $K = 4$, $\frac{K}{\sqrt{2}}\cos(t-\pi/4) = 2\sqrt{2}\cos(t-\pi/4)$

Version 5: $K = 5$, $\frac{K}{\sqrt{2}}\cos(t-\pi/4) = \frac{5}{\sqrt{2}}\cos(t-\pi/4)$

10. The figure shows two identical, consecutive pulses, each of duration a, the second pulse being inverted with respect to the first. If $F(s)$ is the Laplace transform of the **two pulses shown**, determine, in terms of $F(s)$, the Laplace transform of $f(t)$ shifted to the **left** by a. Assume $a = 1$.



Solution. Let $G(s)$ be the Laplace transform of a single pulse that extends from $t = 0$ to $t = a$. Then, $F(s) = G(s) - G(s)e^{-as}$, or $G(s) = \frac{F(s)}{1-e^{-as}}$. When shifted to the left, the Laplace transform of $f(t + a)$ is $-G(s) = \frac{-F(s)}{1-e^{-as}}$

Version 1: $a = 1$, $-G(s) = \frac{-F(s)}{1-e^{-s}}$

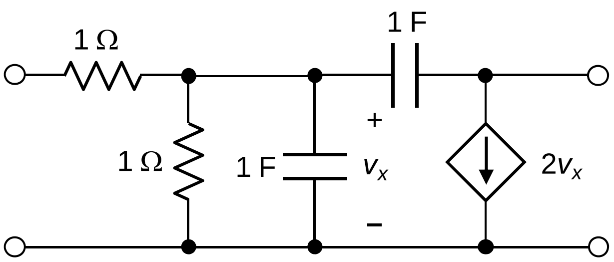
Version 2: $a = 2$, $-G(s) = \frac{-F(s)}{1-e^{-2s}}$

Version 3: $a = 3$, $-G(s) = \frac{-F(s)}{1-e^{-3s}}$

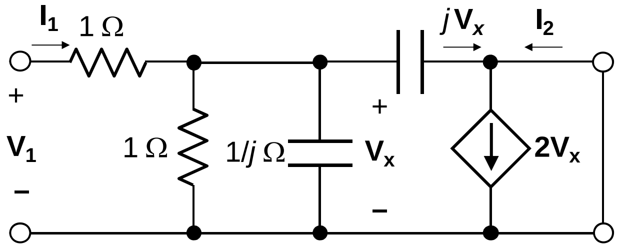
Version 4: $a = 4$, $-G(s) = \frac{-F(s)}{1-e^{-4s}}$

Version 5: $a = 5$, $-G(s) = \frac{-F(s)}{1-e^{-5s}}$

11. Determine the h parameters for the circuit shown, assuming $\omega = 1$ rad/s (5% for each parameter).

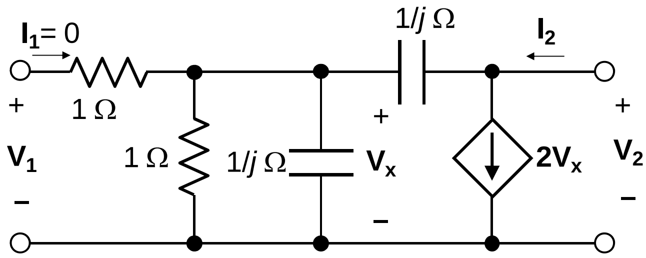


Solution. With the output short circuited, the impedance of the paralleled capacitors and resistor is



$\frac{1/j2}{1+1/j2} = \frac{1}{1+j2}$; h_{11} = $\frac{\mathbf{V_1}}{\mathbf{I_1}} =$ $1+\frac{1}{1+j2} = \frac{2(1+j)}{1+j2} = \frac{2(3-j)}{5}$ Ω; $\mathbf{V}_x$ = $\frac{1}{1+j2}\mathbf{I_1}$, and $2\mathbf{V}_x = \mathbf{I_2} + j\mathbf{V_x}$, or $\mathbf{V}_x = \frac{1}{2-j}\mathbf{I_2}$. Equating $\mathbf{V_x}$, $\frac{1}{1+j2}\mathbf{I_1} = \frac{1}{2-j}\mathbf{I_2}$; $h_{21} = \frac{\mathbf{I_2}}{\mathbf{I_1}} = \frac{2-j}{1+j2} = -j$.

With the input open circuited, $\mathbf{V}_\pi = \mathbf{V_1} =$ $\frac{1/(1+j)}{1/j+1/(1+j)}\mathbf{V_2}$; hence,



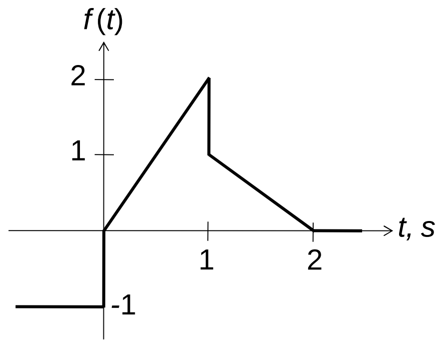
$h_{12} = \frac{\mathbf{V_1}}{\mathbf{V_2}} = \frac{j}{1+j2} = \frac{2+j}{5}$.

$\mathbf{I_2} = \frac{1}{1/j+1/(1+j)}\mathbf{V_2} + 2\mathbf{V_x} = \frac{j(1+j)}{1+j2}\mathbf{V_2}$ $+ \frac{j2}{1+j2}\mathbf{V_2}$, so $h_{22} = \frac{\mathbf{I_2}}{\mathbf{V_2}} = \frac{-1+j3}{1+j2} = (j+1)$ S.

12. Determine the Laplace transform of:

10%(a) $f(t)$.

10%(b) $df(t)/dt$.

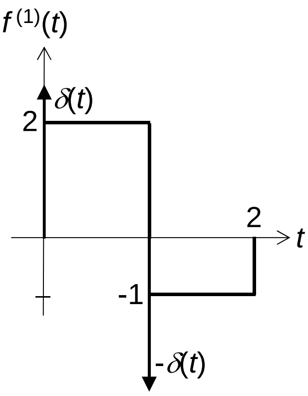


Solution. (a) *Method 1:* $f(t) = 2tu(t) - 2(t-1)u(t-1) - u(t-1) - (t-1)u(t-1) + (t-2)u(t-2)$; hence,

$$F(s) = \frac{2}{s^2} - \left(\frac{1}{s} + \frac{3}{s^2}\right)e^{-s} + \frac{1}{s^2}e^{-2s}.$$

Method 2: $F(s) = \int_{0^-}^{1} 2te^{-st}dt + \int_{1}^{2}(2-t)e^{-st}dt = 2\left[-\frac{t}{s}e^{-st}\right]_{0^-}^{1} + \frac{2}{s}\left[-\frac{1}{s}e^{-st}\right]_{0^-}^{1} + 2\left[-\frac{1}{s}e^{-st}\right]_{1}^{2} - \left[-\frac{t}{s}e^{-st}\right]_{1}^{2} - \frac{1}{s}\left[-\frac{1}{s}e^{-st}\right]_{1}^{2} =$

$$-\frac{2}{s}e^{-s} - \frac{2}{s^2}e^{-s} + \frac{2}{s^2} - \frac{2}{s}e^{-2s} + \frac{2}{s}e^{-s} + \frac{2}{s}e^{-2s} - \frac{1}{s}e^{-s} + \frac{1}{s^2}e^{-2s} - \frac{1}{s^2}e^{-s} = \frac{2}{s^2} - \left(\frac{1}{s} + \frac{3}{s^2}\right)e^{-s} + \frac{1}{s^2}e^{-2s}.$$



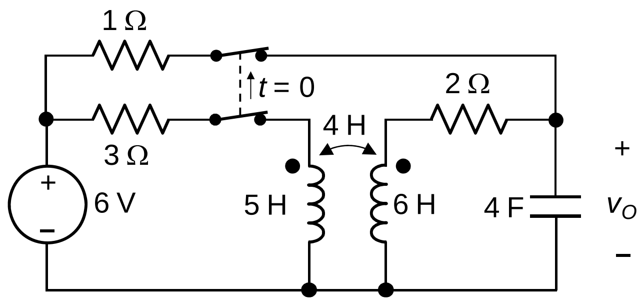
(b) $\mathscr{L}\{df/dt\} = sF(s) - f(0^-) = 1 + \frac{2}{s} - \left(1 + \frac{3}{s}\right)e^{-s} + \frac{1}{s}e^{-2s}$.

As a check, $f^{(1)}(t) = \delta(t) + 2u(t) - 3u(t-1) - \delta(t-1) + u(t-2)$.

It follows that $\mathcal{L}\{df/dt\} = 1 + \frac{2}{s} - \frac{3}{s}e^{-s} - e^{-s} + \frac{1}{s}e^{-2s}$.

Note that in order to obtain the Laplace transform of $f(t)$ from $f^{(1)}(t)$, then because $f(t)$ has an initial value of -1 at $t = 0^-$, the correct expression is: $F(s) = \frac{1}{s}\mathcal{L}\{df/dt\} + \frac{f(0^-)}{s}$. Thus,

$$\frac{1}{s} + \frac{2}{s^2} - \left(\frac{1}{s} + \frac{3}{s^2}\right)e^{-s} + \frac{1}{s^2}e^{-2s} - \frac{1}{s}.$$

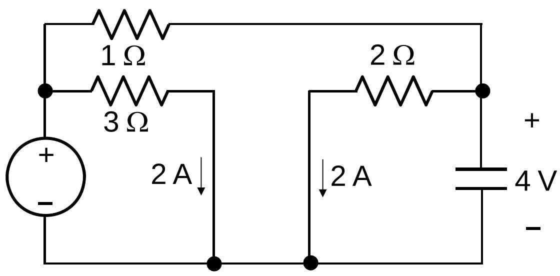


13. The double switch is opened at $t = 0$ after having been closed for a long time.

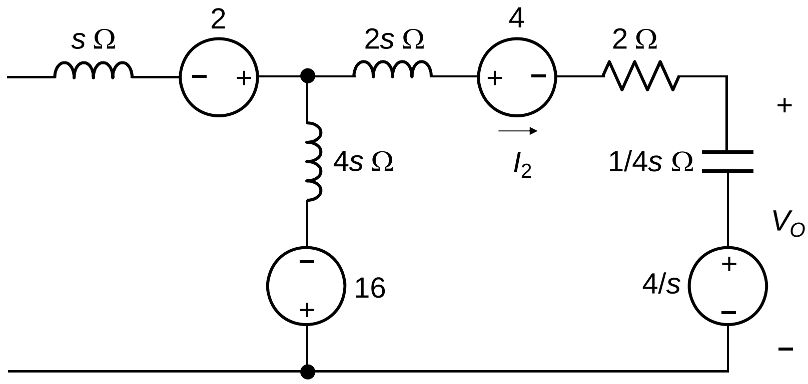
8% (a) Draw the circuit in the s domain for $t \geq 0^+$.

6% (b) Derive $V_O(s)$.

6% (c) Derive $v_O(t)$ for $t \geq 0^+$.



Solution. (a) At $t = 0^-$, there will be an initial current of 2 A in each winding and an initial voltage of 4 V across the capacitor as shown. The circuit in the s domain is shown for $t \geq 0^+$. The secondary inductor is $L_2 - M = 2$ H and the source is (2 H)×(-2 A) = -4. The current in the 4 H mutual inductor is 2 + 2 = 4 A directed downwards, and the source is (4 H)×(4 A) = 16.



(b) From KVL: $(6s + 2 + 1/4s)I_2 + 16 + 4 + 4/s = 0$, which gives:

$$I_2 = -\frac{4(5s+1)}{6s^2 + 2s + 0.25};$$

$$V_O = \frac{I_2}{4s} + \frac{4}{s} = \frac{1}{s}\left[\frac{I_2}{4} + 4\right] = \frac{1}{s}\left[4 - \frac{5s+1}{6s^2 + 2s + 0.25}\right] = \frac{1}{s}\left[\frac{24s^2 + 3s}{6s^2 + 2s + 0.25}\right] = \frac{3(8s+1)}{6s^2 + 2s + 0.25}.$$

(c) $V_O = \frac{4s + 0.5}{s^2 + s/3 + 1/24} = \frac{4(s + 1/6)}{(s + 1/6)^2 + \left(1/6\sqrt{2}\right)^2} - \sqrt{2}\frac{1/6\sqrt{2}}{(s + 1/6)^2 + \left(1/6\sqrt{2}\right)^2}$. It follows that

$$v_O(t) = 4e^{-t/6}\cos\left(6t/\sqrt{2}\right) - \sqrt{2}e^{-t/6}\sin\left(6t/\sqrt{2}\right) \text{ V}.$$