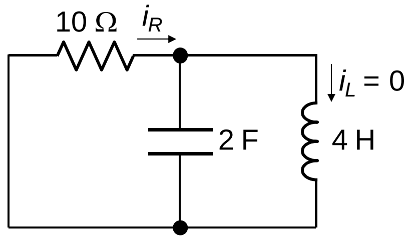
EECE 290 Analog Signal Processing – Quiz 2

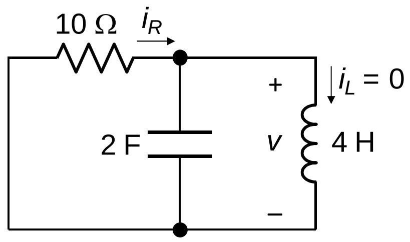
April 9, 2011

1. The initial values at t = 0 of i_L and i_R are: i_L = 0 and i_R = K A. Determine $\frac{di_L}{dt}$ at t = 0.



Solution: $v = L\frac{di_L}{dt}$, where v = -10K. Substituting,

$\frac{di_L}{dt} = \frac{v}{L} = -\frac{10K}{4} - 2.5K$ A/s.



Version 1: K = 1, $\frac{di_L}{dt} = -2.5K =$ -2.5 A/s

Version 2: K = 2, $\frac{di_L}{dt} = -2.5K =$ -5 A/s

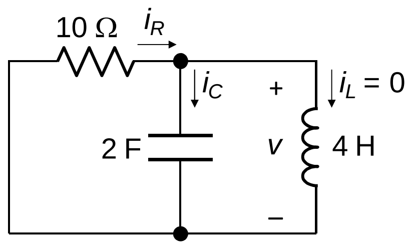
Version 3: K = 3, $\frac{di_L}{dt} = -2.5K =$ -7.5 A/s

Version 4: K = 4, $\frac{di_L}{dt} = -2.5K =$ -10 A/s

Version 5: K = 5, $\frac{di_L}{dt} = -2.5K =$ -12.5 A/s

2. Determine $\frac{di_R}{dt}$ at t = 0 in Problem 1.

Solution: v = -10i_R, $\frac{di_R}{dt} = -\frac{1}{10}\frac{dv}{dt}$; $\frac{dv}{dt} = \frac{i_C}{C} = \frac{i_R}{C} = \frac{K}{2}$.



Substituting, $\frac{di_R}{dt} = -\frac{K}{20}$ A/s = -50K mA/s.

Version 1: K = 1, $\frac{di_R}{dt} = -50K =$ -50 mA/s

Version 2: K = 2, $\frac{di_R}{dt} = -50K =$ -100 mA/s

Version 3: K = 3, $\frac{di_R}{dt} = -50K =$ -150 mA/s

Version 4: K = 4, $\frac{di_R}{dt} = -50K =$ -200 mA/s

Version 5: K = 5, $\frac{di_R}{dt} = -50K =$ -250 mA/s

3. Which of the following statements is true of the voltage v_C across C for large values of t in Problem 1?

Version 1: v_C approaches zero as an underdamped response

Version 2: v_C approaches a finite value as an underdamped response

Version 3: v_C approaches zero as an overdamped response

Version 4: v_C approaches a finite value as an overdamped response

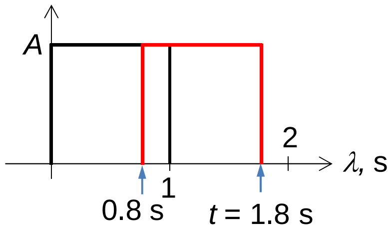
Version 5: v_C approaches zero as a critically damped response

Solution: The circuit is a parallel circuit; $\alpha_p = \dfrac{1}{2C_pR_p} = \dfrac{1}{2\times 2\times 10} = 0.025$;

$\omega_0 = \dfrac{1}{\sqrt{L_pC_p}} = \dfrac{1}{\sqrt{8}} = 0.354$. Since α_p is less than ω_0, the response is oscillatory. Because of the dissipating element, all responses in the circuit will eventually become zero. Hence, v_C approaches zero as an underdamped response.

4. A pulse of amplitude A units and 1 s duration, starting at the origin, is convolved with itself. Determine the value of the convolution integral at t = 1.8 s.

Solution: Let the amplitude of the pulse be A. When one pulse is folded about the vertical axis and shifted by t = 1.8 s, the graphical construction becomes as shown. The integration area is $y(t) = A\times A\times 0.2 = 0.2A^2$.



Version 1: $A = 1$, $y(t) = 0.2A^2 = 0.2\times 1 = 0.2$

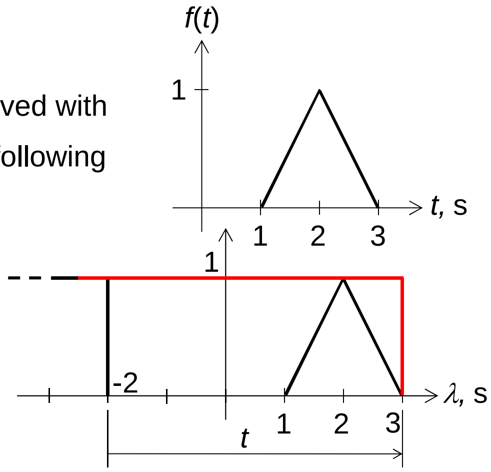
Version 2: $A = 1.5$, $y(t) = 0.2A^2 = 0.2\times 2.25 = 0.45$

Version 3: $A = 2$, $y(t) = 0.2A^2 = 0.2\times 4 = 0.8$

Version 4: $A = 2.5$, $y(t) = 0.2A^2 = 0.2\times 6.25 = 1.25$

Version 5: $A = 3$, $y(t) = 0.2A^2 = 0.2\times 9 = 1.8$

5. The delayed triangular pulse of unit height is convolved with the step function $Au(t-2)$. Determine which of the following choices is a valid convolution integral for $t \geq 5$ s.



Solution: Let the amplitude of the step be A. When the step is folded about the vertical axis and shifted by t = 5 s, the graphical construction becomes as shown for A = 1. The integration area is A times the area of the triangle, which is 1. Thus, $y(t) = Au(t-5)$, since this value is valid for $t \geq 5$.

Version 1: $A = 1$, $y(t) = Au(t-5) = u(t-5)$

Version 2: $A = 2$, $y(t) = Au(t-5) = 2u(t-5)$

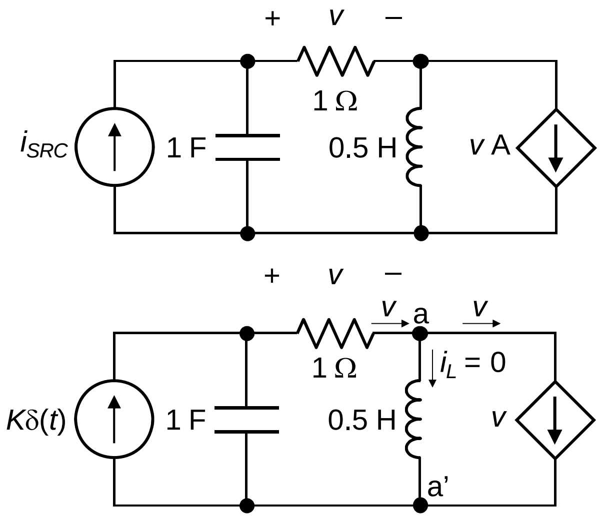
Version 3: $A = 3$, $y(t) = Au(t-5) = 3u(t-5)$

Version 4: $A = 4$, $y(t) = Au(t-5) = 4u(t-5)$

Version 5: $A = 5$, $y(t) = Au(t-5) = 5u(t-5)$

4%

6. Given that $i_{SRC} = K\delta(t)$ As. Determine v as a function of t for $t \geq 0^+$.



Solution: The current through R is numerically equal to v, the same as the current of the dependent source. This makes $i_L = 0$ and node a at the same voltage as a'. A current impulse $K\delta(t)$ is thus applied to 1 Ω in parallel with 1 F. At $t = 0^+$, the impulse deposits a charge K coulombs, which gives an initial voltage $v(0) = K/1$ V across the capacitor. It follows that for $t \geq 0^+$, $v(t) = Ke^{-t}$ V.

Version 1: $K = 0.5$, $v(t) = Ke^{-t} = 0.5e^{-t}$ V

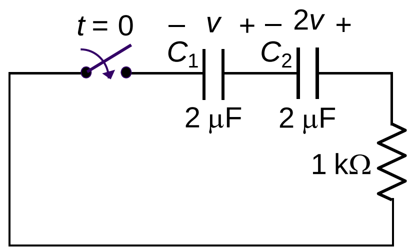
Version 2: $K = 1$, $v(t) = Ke^{-t} = e^{-t}$ V

Version 3: $K = 1.5$, $v(t) = Ke^{-t} = 1.5e^{-t}$ V

Version 4: $K = 2$, $v(t) = Ke^{-t} = 2e^{-t}$ V

Version 5: $K = 2.5$, $v(t) = Ke^{-t} = 2.5e^{-t}$ V

7. Determine the quantity of charge that flows through the 1 kΩ resistor, and its direction of flow, from when the switch is closed at $t = 0$, to $t \to \infty$, assuming $v = 1$ V.



Solution: $C_{eqs} = 1$ μF; the charge that flows through the 1 kΩ resistor is the charge on this capacitor, which is $q = 1 \times 3v$ μC. As a check, the initial charges on C_1 and C_2 are, respectively, $2 \times v$ and $2 \times 2v$ μC. The final charges are $2v - 3v = -v$ μC and $4v - 3v = v$ μC, so the voltages are equal and opposite at $-v/2$ and $+v/2$.

Version 1: $v = 1$, $q = 3v = 3$ μC

Version 2: $v = 2$, $q = 3v = 6$ μC

Version 3: $v = 3$, $q = 3v = 9$ μC

Version 4: $v = 4$, $q = 3v = 12$ μC

Version 5: $v = 5$, $q = 3v = 15$ μC

8. Determine the energy dissipated in the resistor from $t = 0$ to $t \to \infty$ in Problem 7.

Solution: *Method 1:* The power dissipated is that stored in C_{eqs}, where $C_{eqs} = 1$ μF, the initial voltage is $3v$ and the charge on this capacitor is $3v$ μC. The stored energy is $w = qv/2 = 4.5v^2$ μJ.

Method 2: The initial current through the resistor is $3v/1 = 3v$ mA. As a function of time, the current is $3ve^{-t/\tau}$ mA, where $\tau = (1\ \mu\text{F})\times(1\ \text{k}\Omega) = 1$ ms, so that t is in ms. The energy dissipated is $\int_0^\infty 9v^2 \times 1 \times e^{-2t/\tau} dt$. The dimensions of the integrand are $(\text{mA})^2\times\text{k}\Omega\times\text{ms}$, which is μJ. The value is $\left[-4.5v^2 e^{-2t/\tau}\right]_0^\infty = 4.5v^2$ μJ.

Version 1: $v = 1$, $w = 4.5v^2 = 4.5$ μJ

Version 2: $v = 2$, $w = 4.5v^2 = 18$ μJ

Version 3: $v = 3$, $w = 4.5v^2 = 40.5$ μJ

Version 4: $v = 4$, $w = 4.5v^2 = 72$ μJ

Version 5: $v = 5$, $w = 4.5v^2 = 112.5$ μJ

9. The response of a circuit to a unit step is $kte^{-t/10}$, where t is in s. Determine the response at $t = 10$ s to an impulse applied at $t = 5$ s.

Solution: The response to an impulse at $t = 0$ is the time derivative of the response to a unit impulse and is: $ke^{-t/10} - (k/10)te^{-t/10}$. The response to an impulse at $t = 5$ s is $ke^{-(t-5)/10}\left[1-(t-5)/10\right]$. The response at $t = 10$ s is $ke^{-(10-5)/10}\left[1-(10-5)/10\right] = 0.5ke^{-0.5}$.

Version 1: $k = 1$, $0.5ke^{-0.5} = 0.303$

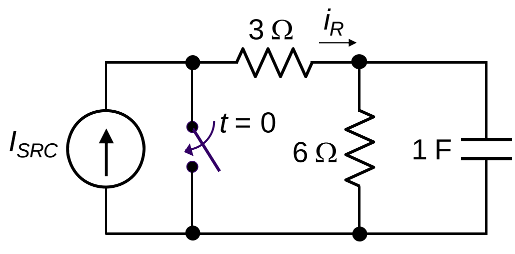
Version 2: $k = 2$, $0.5ke^{-0.5} = 0.607$

Version 3: $k = 3$, $0.5ke^{-0.5} = 0.910$

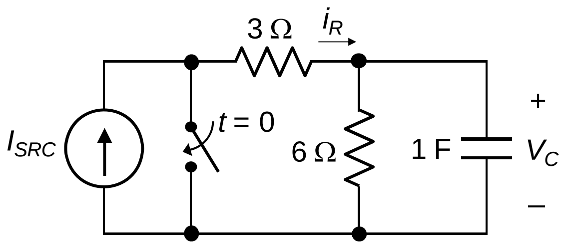
Version 4: $k = 4$, $0.5ke^{-0.5} = 1.21$

Version 5: $k = 5$, $0.5ke^{-0.5} = 1.52$

10. The switch is closed at $t = 0$ after being open for a long time. Determine i_R as a function of time, assuming $I_{SRC} = 1$ A.



Solution: After the switch has been open for a long time, $V_C = 6I_{SRC}$. After the switch is closed, the initial value of i_R is $-2I_{SRC}$. The final value of i_R is zero. The time constant, with the switch closed, is (3||6)×1 = 2 s. Hence, $i_R = -2I_{SRC}e^{-t/2}$



Version 1: I_{SRC} = 1 A, $i_R = -2I_{SRC}e^{-t/2} = -2e^{-t/2}$ A

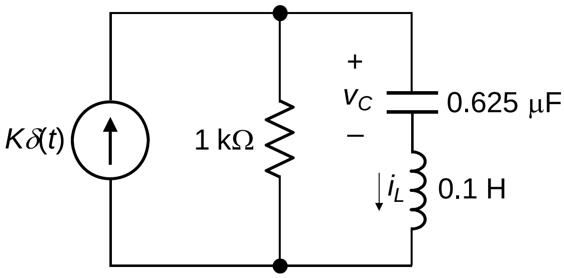
Version 2: I_{SRC} = 2 A, $i_R = -2I_{SRC}e^{-t/2} = -4e^{-t/2}$ A

Version 3: I_{SRC} = 3 A, $i_R = -2I_{SRC}e^{-t/2} = -6e^{-t/2}$ A

Version 4: I_{SRC} = 4 A, $i_R = -2I_{SRC}e^{-t/2} = -8e^{-t/2}$ A

Version 5: I_{SRC} = 5 A, $i_R = -2I_{SRC}e^{-t/2} = -10e^{-t/2}$ A

11. A current impulse of K mAs is applied at t = 0, with zero initial conditions.



6% (a) Determine the values of v_C and i_L at $t = 0^+$.

7% (b) Derive the form of the expression for i_L as a function of time for $t > 0^+$ in terms of the two arbitrary constants of the differential equation. Assume that these arbitrary constants are unknown but specify the values of all other quantities in the expression for i_L.

7% (c) Determine the values of the arbitrary constants assuming, v_C = 200 V and i_L = 1 A at $t = 0^+$.

Solution: (a) The current source impulse in parallel with R Ω can be transformed to a voltage source impulse of $(KR)\delta(t)$ Vs in series with R. As argued in the book for a series RLC circuit, the voltage impulse appears across L and produces a jump in i_L equal to $KR/L = K\times10^{-3}\times10^{3}/(0.1) = 10K$ A. The finite jump in i_L does not change v_C, which remains at zero. Alternatively, it can be argued that the current impulse will flow through R and not through L and C. If the current impulse flows through L and C, v_C will jump in value by a finite amount, and the voltage across L will change proportionately to $\delta^{(1)}(t)$. The voltage across L and C will be inconsistent with the voltage across R, which can be only be proportional to either $\delta(t)$ or to a finite quantity.

The current impulse through R will cause a voltage impulse $KR\delta(t)$ across R. This impulse will appear across L and cause a jump in current of KR/L.

(b) For $t > 0^+$, the source is zero, and the circuit reduces to a series RLC circuit.

$\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.1 \times 0.625 \times 10^{-6}}} = 4$ krad/s; $\alpha = \dfrac{R}{2L} = \dfrac{1000}{2 \times 0.1} = 5$ krad/s. Since $\alpha > \omega_0$, the response is overdamped, $s_1 = -5 + \sqrt{5^2 - 4^2} = -2$ krad/s and $s_2 = -5 - \sqrt{5^2 - 4^2} = -8$ krad/s. It follows that $i_L = Ae^{-2t} + Be^{-8t}$ A, where t is in ms.

(c) At $t = 0^+$, $A + B = 1$; $q = -\dfrac{A}{2000}e^{-2000t} - \dfrac{B}{8000}e^{-8000t}$ C, where the constant of integration is zero since $q \to 0$ as $t \to \infty$; at $t = 0^+$, $v_C = \dfrac{q}{C} - \dfrac{A \times 10^3}{1.25} -, \dfrac{B \times 10^3}{5}$ or

$200 = -\dfrac{A \times 10^3}{1.25} - \dfrac{B \times 10^3}{5}$, or $4A + B = -1$. This gives $A = -2/3$ A and $B = 5/3$ A.

Alternatively, $v_L = L\dfrac{d_{i_L}}{dt} = 0.1\left(-2Ae^{-2t} - 8Be^{-8t}\right)$ H×A/ms, or $v_L = 100\left(-2Ae^{-2t} - 8Be^{-8t}\right)$ V, or $-v_L/100 = 2A + 8B$ at $t = 0^+$. But $v_L + v_C + v_R = 0$, or $-v_L = v_C + v_R$. This gives, $2A + 8B = (200 + 1000)/100$. Hence, $A + 4B = 6$, which gives the same values for A and B.

Version 1: (a) $K = 0.1$; $i_L(0^+) = 10K = 1$ A, $v_C(0^+) = 0$

Version 2: (a) $K = 0.2$; $i_L(0^+) = 10K = 2$ A, $v_C(0^+) = 0$

Version 3: (a) $K = 0.3$; $i_L(0^+) = 10K = 3$ A, $v_C(0^+) = 0$

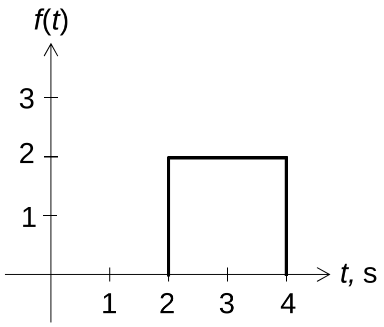
Version 4: (a) $K = 0.4$; $i_L(0^+) = 10K = 4$ A, $v_C(0^+) = 0$

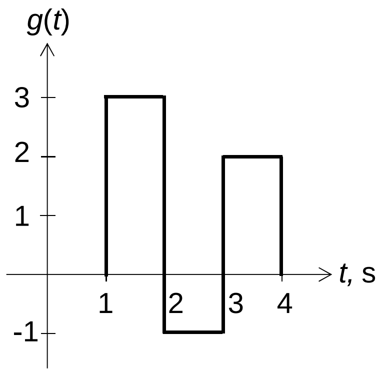
Version 5: (a) $K = 0.5$; $i_L(0^+) = 10K = 5$ A, $v_C(0^+) = 0$

12. Given the two functions $f(t)$ and $g(t)$ shown.

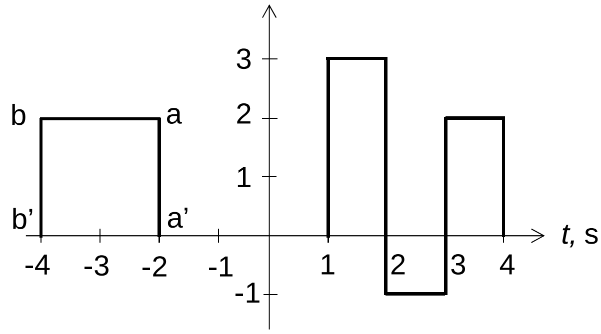
15% (a) Sketch $y(t) = f(t)*g(t)$ as function of t.

5% (b) Verify $y(t)$ from the product of two polynomials.

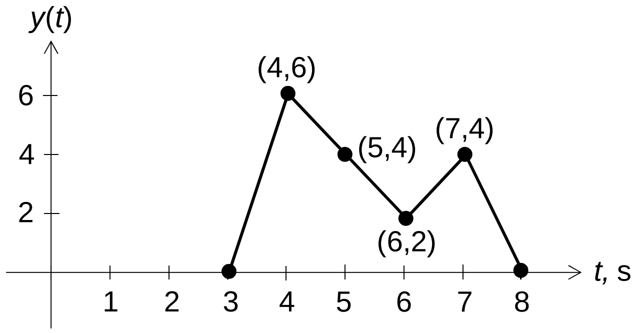




Solution: (a) When $f(t)$ is folded, it becomes as shown. The convolution integral is zero until $t = 3$, when side aa' coincides with $g(1)$. At $t = 4$, aa' coincides with $g(2)$, and the area under the product of the two functions is $3 \times 2 = 6$. At $t = 5$, aa' coincides with $g(3)$, and the area under the product of the two functions is $3 \times 2 - 1 \times 2 = 4$. At $t = 6$, aa' coincides with $g(4)$, and the area



under the product of the two functions is -1×2 + 2×2 = 2. At t = 7, bb' coincides with $g(3)$, and the area under the product of the two functions is 2×2 = 4. At t = 8, bb' coincides with $g(4)$, and the area under the product of the two functions is back to zero. The convolution function $y(t)$ is as shown.

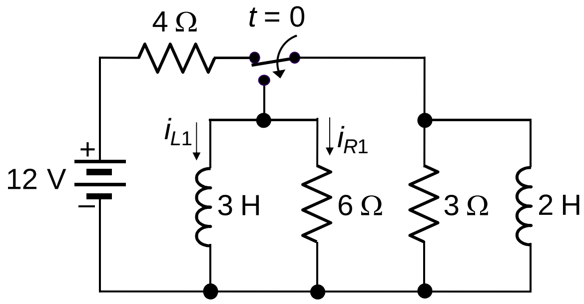


(b) To derive $y(t)$ In terms of a polynomial, $f(t)$ and $g(t)$ are assumed to both start at $t = 0$. $f(t)$ is divided into two successive 1 s intervals, each of amplitude 2 units. Thus, as polynomials in x, $f(x) = 2x + 2$, and $g(x) = 3x^2 - x + 2$. The product $f(x)g(x) = 6x^3 + 4x^2 + 2x + 4$. The non-zero breakpoints are thus at (1,6), (2,4), (3,2), and (4,4). For the given functions, the total shift in time from the origin is 3 s. Because the system is linear and time-invariant, as assumed in the derivation of the convolution integral, the 3 s is added to the time coordinates of the breakpoints from the product of polynomials to give the non-zero breakpoints of $y(t)$ in the figure.

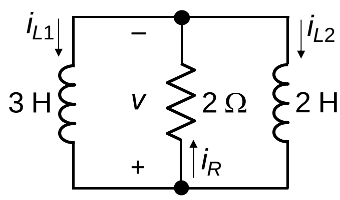
13. The switch is moved at $t = 0$ after having been in the first position for a long time, with i_{L1} initially zero. Determine, as a function of t, for $t \geq 0^+$:

10% (a) i_{R1}

10% (b) i_{L1}



Solution: After switching, the circuit becomes as shown, where at $t = 0^+$: $i_{L2} = 12/4 = 3$ A, $i_{L1} = 0$, $i_R = 3$ A, $v = 3×2 = 6$ V. As $t \to \infty$, $v = 0$. The time constant is (2||3)/2 = 1.2/2 = 0.6 s. Hence $v = 6e^{-t/0.6}$ V.



(a) It follows that $i_{R1} = -\frac{v}{6} = -e^{-t/0.6}$ A

(b) i_{L1} can be determined from conservation of flux linkage in the loop formed by the two inductors. At t = 0^+, going clockwise around the loop, the total flux linkage is 3×2 + 0 = 6 Wb-T. As $t \to \infty$, the currents in the two inductors are equal in magnitude. Going clockwise around the loop, i_{L2}×5 = 6 Wb-T, so that $i_{L1} = -i_{L2} = -1.2$ A. It follows that $i_{L1} = -1.2 + (0 + 1.2)e^{-t/0.6} = -1.2(1 - e^{-t/0.6})$ A.

Alternatively, $i_{L1} = -\frac{1}{L}\int_0^t v dt + i_{L1}(0) = -\frac{1}{3}\int_0^t 6e^{-t/0.6} dt = -2\left[-e^{-t/0.6}\right]_0^t = -1.2\left(1 - e^{-t/0.6}\right)$ A.