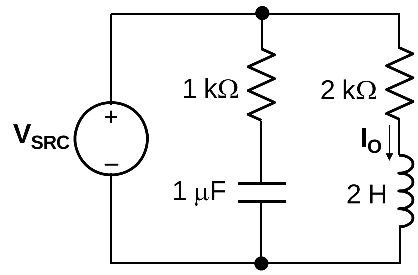
**EECE 290 Analog Signal Processing – Quiz 1**

**March 12, 2011**

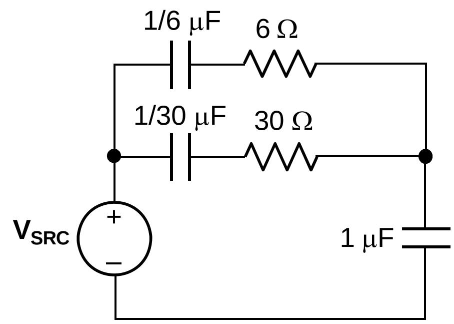
**1.** Specify the type of response $\mathbf{I_O}/\mathbf{V_{SRC}}$ in the circuit shown.




**Solution:** The transfer function is $\dfrac{\mathbf{I_O}}{\mathbf{V_{SCR}}} = \dfrac{1}{R + j\omega L}$,

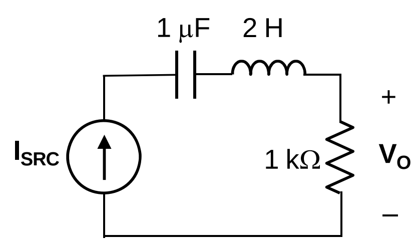
which is first-order lowpass.

**2.** Specify the order of the frequency-selective circuit shown.




**Solution:** The impedance of one *RC* branch is five times the other. The parallel impedance is therefore 5/6 of the lower impedance branch, that is, 5 Ω in series with a capacitance 1/5 μF. This capacitance can then be combined with the 1 μF capacitance to give a first-order circuit.

**3.** Specify the type of response $\mathbf{V_O}$ in the circuit shown over the frequency range 1 rad/s to 1 Mrad/s.




**Solution:** Over the specified frequency range, the impedances of *L* and *C* are finite, current flows in the circuit, and the response $\dfrac{\mathbf{V_O}}{\mathbf{I_{SCR}}} = R$ is independent of frequency.

**4.** The magnitude of a transfer function is $\dfrac{8\omega^2}{\sqrt{\omega^4 + K^4}}$, where $\omega$ is in rad/s. Determine the frequency at which the response is 3 dB less than the maximum value.

**Solution:** The magnitude of the transfer function is $\dfrac{8}{\sqrt{1 + (K/\omega)^4}}$. This is a second order, highpass, Butterworth filter of high-frequency gain of 8. The response is 3 dB less than the maximum value at $\omega = K$ rad/s

**Version 1**: $K = 1$, $\omega_{3\mathrm{dB}} = 1$ rad/s

**Version 2**: $K = 2$, $\omega_{3dB} = 2$ rad/s

**Version 3**: $K = 3$, $\omega_{3dB} = 3$ rad/s

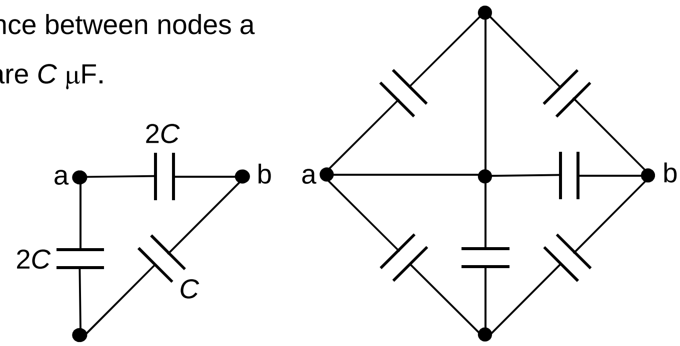
**Version 4**: $K = 4$, $\omega_{3dB} = 4$ rad/s

**Version 5**: $K = 5$, $\omega_{3dB} = 5$ rad/s

**5.** Determine the slope of the low-frequency asymptote of the transfer function of the preceding problem.

**Solution:** At low frequencies, the magnitude of the transfer function is $\frac{8}{K^2}\omega^2$. The slope is +40 dB/decade.

**6.** Determine the equivalent capacitance between nodes a and b, assuming all capacitances are $C$ μF.




**Solution:** the circuit reduces to that shown. The equivalent capacitance between a and b is $2C + 2C/3 = 8C/3$

**Version 1**: $C = 1$ μF, $C_{ab} = 8/3 = 2.67$ μF

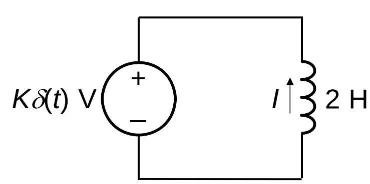
**Version 2**: $C = 2$ μF, $C_{ab} = 16/3 = 5.33$ μF

**Version 3**: $C = 3$ μF, $C_{ab} = 24/3 = 8$ μF

**Version 4**: $C = 4$ μF, $C_{ab} = 32/3 = 10.67$ μF

**Version 5**: $C = 5$ μF, $C_{ab} = 40/3 = 13.33$ μF

**7.** A constant current $I$ A has been flowing through the inductor for a long time. Determine the magnitude and polarity of the impulse $K\delta(t)$ that will reduce $I$ to zero.




**Solution:** The initial flux linkage is $2I$ Wb-turns. The impulse has to be of this strength and of the polarity of the source.

**Version 1**: $I = 0.5$ A, $K = 2I = 1$ Wb-turn

**Version 2**: $I = 1$ A, $K = 2I = 2$ Wb-turns

**Version 3**: $I = 1.5$ A, $K = 2I = 3$ Wb-turns

**Version 4**: $I = 2$ A, $K = 2I = 4$ Wb-turns

**Version 5**: $I = 2.5$ A, $K = 2I = 5$ Wb-turns

**8.** Evaluate the integral $\int_{-\infty}^{\infty} K\delta(t)e^{-2t}\cos(t)dt$ .

**Solution:** The integrand is zero everywhere except between $0^-$ and $0^+$. Both $e^{-2t}$ and cos($t$) are equal to 1 and are continuous at $t$ = 0. The integral reduces to $\int_{0^-}^{0^+} K\delta(t)dt = K$ .

**Version 1**: $K$ = 2, integral = 2

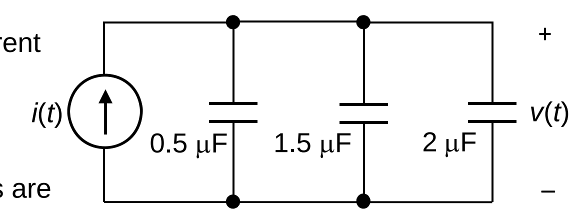
**Version 2**: $K$ = 3, integral = 3

**Version 3**: $K$ = 4, integral = 4

**Version 4**: $K$ = 5, integral = 5

**Version 5**: $K$ = 6, integral = 6

**9.** For $t$ < 0, $i(t)$ = 0 and $v(t)$ = 4 V. At $t$ = 0, a current pulse of amplitude $I$ μA and 1 s duration is applied. Determine $v(t)$ for $t$ > 1 s.




**Solution:** For $t$ < 0, the charges on the capacitors are 0.5×4 = 2 μC, 1.5×4 = 6 μC, and 2×4 = 8 μC. The value of the equivalent parallel capacitor is 0.5 + 1.5 + 2 = 4 μC, its charge is 2 + 6 + 8 = 16 μC, and the voltage across it is 16/4 = 4 V. The current pulse will add $I$×1 μC, so the total charge across the equivalent capacitor will be (16 + $I$) μC, and the final voltage will be (16 + $I$)/4 = (4 + $I$/4) V.

**Version 1**: $I$ = 1 μA, $v(\infty)$ = 4 + 1/4 = 4.25 V

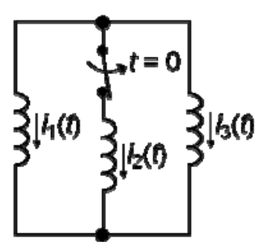
**Version 2**: $I$ = 2 μA, $v(\infty)$ = 4 + 2/4 = 4.5 V

**Version 3**: $I$ = 3 μA, $v(\infty)$ = 4 + 3/4 = 4.75 V

**Version 4**: $I$ = 4 μA, $v(\infty)$ = 4 + 4/4 = 5 V

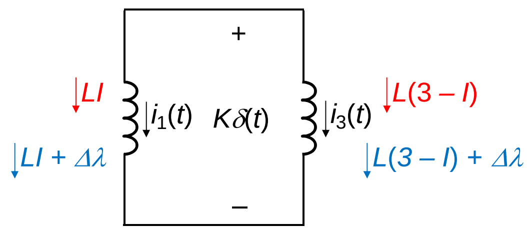
**Version 5**: $I$ = 5 μA, $v(\infty)$ = 4 + 5/4 = 5.25 V

10. In the circuit shown, all inductances are $L$ H, where $L$ need not be specified, $i_1(t) = I$ A, $i_2(t)$ = -3 A, and $i_3(t) = 3 - I$ A, for $t$ < 0. If the switch is opened at $t$ = 0, determine $i_1(t)$ for $t$ > 0.



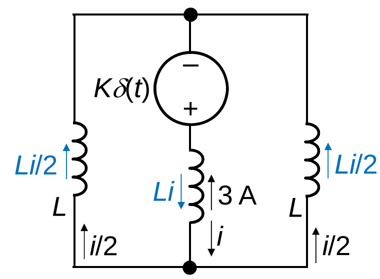

**Solution.** *Method 1*: After the switch is opened the only closed path remaining is that of $i_1(t)$ and $i_3(t)$. Hence, flux linkage in this closed path must be conserved after the switch is opened. Going in the counterclockwise direction, the total flux linkage before the switch is opened, is $L \times I - L(3 - I) = 2LI - 3L$ Wb-turns. After the switch is opened, the total flux linkage in the counterclockwise direction in the same closed path is $L \times i_1(t) + L \times i_1(t) = 2Li_1(t)$, where $i_1(t)$ flows upwards in the rightmost branch. Equating these, $i_1(t) = I - 1.5$ A.

*Method 2*: The currents $i_1(t)$ and $i_3(t)$ can only be changed by a voltage impulse. Let a voltage impulse $K\delta(t)$ arise of the polarity shown, this polarity being quite arbitrary. The impulse will add a flux linkage $\Delta\lambda$ as shown,




where the initial flux linkages are shown in red and the final flux linkages in blue, the direction of flux linkage being the same as that of current. The final currents in the two branches must add to zero. Thus, $\frac{LI+\Delta\lambda}{L}+\frac{L(3-I)+\Delta\lambda}{L}=0$. This gives $\Delta\lambda=-1.5L$ Wb-turns. The final value of $i_1(t)$ is $\frac{LI-1.5L}{L}=I-1.5$ A, as before.

*Method 3*: The switch can be replaced by a voltage source that applies an impulse $K\delta(t)$ at $t = 0$. For $t < 0$, the source acts as a short circuit, and the impulse at $t = 0$ reduces $i_2(t)$ to zero. The flux linkage applied to the middle branch is $Li = 3L$ Wb-turns in the direction shown, so as to reduce the current to zero. The total flux linkage




is $Li + (L/2)i$, where $L/2$ is the total inductance of the two branches in parallel and $i$ is the total current in the two branches. The flux linkage in each of the two parallel branches is $Li/2$ Wb-Turns directed upward, the same as in the parallel combination. Since, $Li = 3L$, $Li/2 = 1.5\,L$ directed upward, as determined in Method 2.

**Version 1**: $I = 0.5$ A, $i_1(\infty) = 0.5 – 1.5 = -1$ A

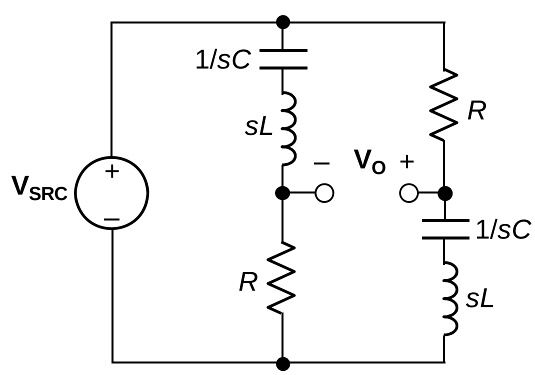
**Version 2**: $I = 1$ A, $i_1(\infty) = 1 – 1.5 = -0.5$ A

**Version 3**: $I = 1.5$ A, $i_1(\infty) = 1.5 – 1.5 = 0$

**Version 4**: $I = 2$ A, $i_1(\infty) = 2 – 1.5 = 0.5$ A

**Version 5**: $I = 2.5$ A, $i_1(\infty) = 2.5 – 1.5 = 1$ A

**11.** In the circuit shown, $R = 10$ kΩ, $L = 1$ μH, and $C = 1$ μF.




12% (a) Derive the transfer function

$$H(s)=\frac{\mathbf{V_O}}{\mathbf{V_{SRC}}}$$ in terms of $R$, $L$ , $C$, and $s = j\omega$.

3% (b) Specify the type of response represented by $H(s)$.

5% (c) Determine the frequency at which the phase shift is 180°.

**Solution:** (a) The transfer function for the response across $L$ and $C$ is:

$H_1(s) = \dfrac{sL + 1/sC}{R + sL + 1/sC} = \dfrac{s^2 + \omega_0^2}{s^2 + sR/L + \omega_0^2}$. The transfer function for the response across $R$ is:

$H_2(s) = \dfrac{R}{R + sL + 1/sC} = \dfrac{sR/L}{s^2 + sR/L + \omega_0^2}$. The transfer function $H(s) = H_1(s) - H_2(s) =$

$\dfrac{s^2 - sR/L + \omega_0^2}{s^2 + sR/L + \omega_0^2}$, where $\omega_0 = \dfrac{1}{\sqrt{LC}}$.

(b) The response is allpass.

(c) The phase shift is 180° at $\omega = \omega_0$, when $H(s) = -1$. $\omega_0 = \dfrac{10^6}{\sqrt{C}}$, where $C$ is the value in μF.

Alternatively, if the phase shift is to be determined directly from the transfer function,

$\angle H(j\omega) = -\tan^{-1}\left(\dfrac{\omega R/L}{\omega_0^2 - \omega^2}\right) - \tan^{-1}\left(\dfrac{\omega R/L}{\omega_0^2 - \omega^2}\right) = -2\tan^{-1}\left(\dfrac{\omega R/L}{\omega_0^2 - \omega^2}\right)$; hence,

$\tan^{-1}\left(\dfrac{\omega R/L}{\omega_0^2 - \omega^2}\right) = 90^\circ$; $\left(\dfrac{\omega R/L}{\omega_0^2 - \omega^2}\right) \to \infty$; $\omega = \omega_0$.

**Version 1**: $C$ = 1 μF, $\omega_0 = \dfrac{10^6}{\sqrt{C}}$ = 1 Mrad/s

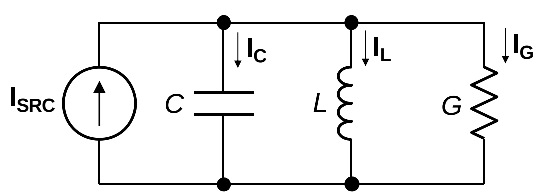
**Version 2**: $C$ = 1/4 μF, $\omega_0 = \dfrac{10^6}{\sqrt{C}}$ = 2 Mrad/s

**Version 3**: $C$ = 1/9 μF, $\omega_0 = \dfrac{10^6}{\sqrt{C}}$ = 3 Mrad/s

**Version 4**: $C$ = 1/16 μF, $\omega_0 = \dfrac{10^6}{\sqrt{C}}$ = 4 Mrad/s

**Version 5**: $C$ = 1/25 μF, $\omega_0 = \dfrac{10^6}{\sqrt{C}}$ = 5 Mrad/s

**12.** Given the parallel circuit shown.




10% (a) Select a branch current to give a highpass response with respect to $I_{SRC}$ and determine the transfer function in terms of $G$, $L$, $C$, and $s = j\omega$.

10% (b) It is desired to have the highpass response of the Butterworth form, with $\omega_0$ = 100 krad/s and $C$ = 100 nF. Determine the required values of $L$ and $R = 1/G$. (Note that the normalized second-order Butterworth polynomial is $s^2 + \sqrt{2}s + 1$, and that the magnitude scaling factor $k_m$ applies to scaling impedances).

**Solution:** (a) $\mathbf{I_C}$ is zero at low frequencies and equals $\mathbf{I_{SRC}}$ at infinite frequencies. It produces a highpass response. The transfer function is $H(s) = \frac{\mathbf{I_C}}{\mathbf{I_{SRC}}} = \frac{sC}{G + sC + 1/sL} =$

$$\frac{s^2}{s^2 + sG/C + 1/LC} = \frac{s^2}{s^2 + s(\omega_0/Q) + \omega_0^2}.$$

(b) For the required filter, $\omega_0 = \frac{1}{\sqrt{LC}}$, $L = \frac{1}{\omega_0^2 C}$. $Q = \frac{1}{\sqrt{2}}$ for a highpass Butterworth response, scaled or not scaled. For the required filter $\frac{\omega_0}{Q} = \frac{G}{C}$, so $G = \omega_0 C\sqrt{2}$ and $R = \frac{1}{\omega_0 C\sqrt{2}}$. If scaling is to be used, $k_f = \omega_0$, $10^{-7} = \frac{1}{k_m k_f} \times 1$, $10^{-7} = \frac{1}{k_m \omega_0}$, $k_m = \frac{1}{10^{-7}\omega_0}$, $L = \frac{k_m}{k_f} \times 1 = \frac{1}{10^{-7}\omega_0^2}$, $R = \frac{k_m}{G} = \frac{1}{10^{-7}\omega_0\sqrt{2}}$, where for the normalized Butterworth filter, $G = \sqrt{2}$, and $k_m$ is uses as a scaling factor for $R$.

**Version 1**: $\omega_0$ = 100 krad/s, $L = \frac{1}{\omega_0^2 C} = \frac{1}{10^{10} \times 10^{-7}} \equiv 1$ mH, $R = \frac{1}{\omega_0 C\sqrt{2}} = \frac{1}{10^5 \times 10^{-7}\sqrt{2}} = \frac{100}{\sqrt{2}} = 50\sqrt{2}\ \Omega$

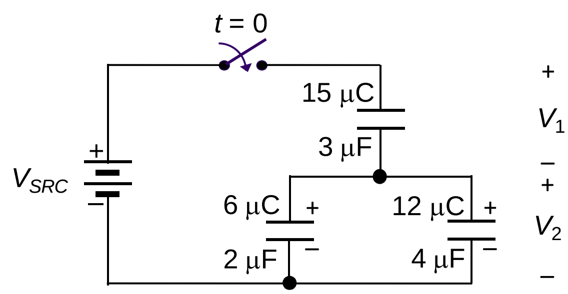
**Version 2**: $\omega_0$ = 50 krad/s, $L = \frac{1}{\omega_0^2 C} = \frac{1}{25 \times 10^8 \times 10^{-7}} \equiv 4$ mH, $R = \frac{1}{\omega_0 C\sqrt{2}} = \frac{1}{5 \times 10^4 \times 10^{-7}\sqrt{2}} = \frac{200}{\sqrt{2}} = 100\sqrt{2}\ \Omega$

**Version 3**: $\omega_0$ = 40 krad/s, $L = \frac{1}{\omega_0^2 C} = \frac{1}{16 \times 10^8 \times 10^{-7}} \equiv 6.25$ mH, $R = \frac{1}{\omega_0 C\sqrt{2}} = \frac{1}{4 \times 10^4 \times 10^{-7}\sqrt{2}} = \frac{250}{\sqrt{2}} = 125\sqrt{2}\ \Omega$

**Version 4**: $\omega_0$ = 25 krad/s, $L = \frac{1}{\omega_0^2 C} = \frac{1}{6.25 \times 10^8 \times 10^{-7}} \equiv 16$ mH, $R = \frac{1}{\omega_0 C\sqrt{2}} = \frac{1}{2.5 \times 10^4 \times 10^{-7}\sqrt{2}} = \frac{400}{\sqrt{2}} = 200\sqrt{2}\ \Omega$

**Version 5**: $\omega_0$ = 20 krad/s, $L = \frac{1}{\omega_0^2 C} = \frac{1}{4 \times 10^8 \times 10^{-7}} \equiv 25$ mH, $R = \frac{1}{\omega_0 C\sqrt{2}} = \frac{1}{2 \times 10^4 \times 10^{-7}\sqrt{2}} = \frac{500}{\sqrt{2}} = 250\sqrt{2}\ \Omega$

**13.** The three capacitors are initially charged as shown in the figure. The switch is closed at $t$ = 0. Determine:

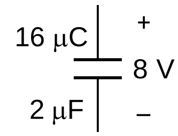



8% (a) The value, voltage and charge on the capacitor that is equivalent to the three capacitors.

6% (b) The additional charge on this capacitor after the switch is closed.

6%  (c) The final values of $V_1$ and $V_2$. Check that $V_1 + V_2 = V_{SRC}$.

**Solution:** (a) The two parallel capacitors can be combined in a 6 μF capacitor having a voltage of 3 V. The 3 μF capacitor has a voltage of 5 V. The equivalent capacitor $C_{eq}$ will have a capacitance of $\frac{3 \times 6}{3+6} = 2$ μF and a voltage of 8 V. The charge on this capacitor is therefore 16 μC.




(b) When connected to $V_{SRC}$, the charge on $C_{eq}$ μC will be $2V_{SRC}$ and the additional charge is $(2V_{SRC} - 16)$ μC.

(c) This charge will be added to each of the 3 μF capacitor and the 6 μF in series with it. The 3 μF capacitor will have a charge of $(2V_{SRC} - 16 + 15) = (2V_{SRC} - 1)$ μC, so $V_1 = (2V_{SRC} - 1)/3$ V. The 6 μF capacitor will have a charge of $(2V_{SRC} - 16 + 18) = (2V_{SRC} + 2)$ μC, and $V_2 = (V_{SRC} + 1)/3$ V. As a check, $V_1 + V_2 = V_{SRC}$.

**Version 1**: $V_{SRC} = 10$ V, $\Delta q = (2V_{SRC} - 16) = 4$ μC; $V_1 = (2V_{SRC} - 1)/3$ V = 6.33 V, $V_2 = (V_{SRC} + 1)/3$ V = 3.67 V

**Version 2**: $V_{SRC} = 12$ V, $\Delta q = (2V_{SRC} - 16) = 8$ μC; $V_1 = (2V_{SRC} - 1)/3$ V = 7.67 V, $V_2 = (V_{SRC} + 1)/3$ V = 4.33 V

**Version 3**: $V_{SRC} = 14$ V, $\Delta q = (2V_{SRC} - 16) = 12$ μC; $V_1 = (2V_{SRC} - 1)/3$ V = 9 V, $V_2 = (V_{SRC} + 1)/3$ V = 5 V

**Version 4**: $V_{SRC} = 16$ V, $\Delta q = (2V_{SRC} - 16) = 16$ μC; $V_1 = (2V_{SRC} - 1)/3$ V = 10.33 V, $V_2 = (V_{SRC} + 1)/3$ V = 5.67 V

**Version 5**: $V_{SRC} = 18$ V, $\Delta q = (2V_{SRC} - 16) = 20$ μC; $V_1 = (2V_{SRC} - 1)/3$ V = 11.67 V, $V_2 = (V_{SRC} + 1)/3$ V = 6.33 V