Homework 1

P10.1.5 Determine the response $V_O(j\omega)/V_{SRC}(j\omega)$, both in magnitude and phase, the passband gain, and the corner frequency in Figure P10.1.5.

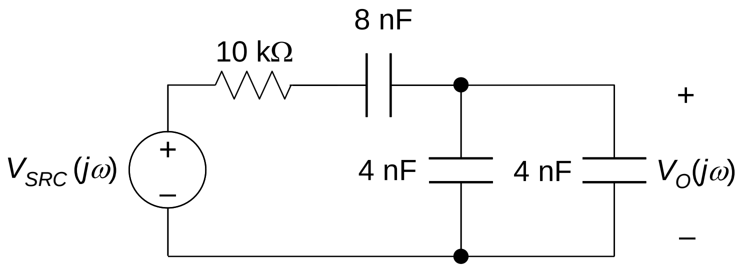
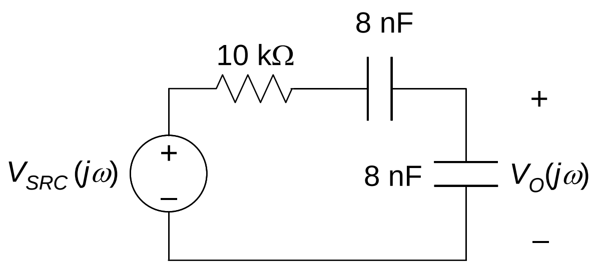


Figure P10.1.5

<u>Solution P10.1.5</u> The impedance of the 8 nF capacitor is $\frac{0.125}{j\omega}$ kΩ, where ω is in Mrad/s. Hence,



$$H(j\omega) = \frac{V_O(j\omega)}{V_{SRC}(j\omega)} = \frac{0.125/j\omega}{10 + 0.25/j\omega}$$

$= \frac{0.5}{1 + j40\omega}$. It follows that $|H(j\omega)| = \frac{0.5}{\sqrt{1 + (40\omega)^2}}$, $\angle H(j\omega) = -\tan^{-1}(40\omega)$. The response is lowpass; the passband gain, when $\omega \to 0$ is 0.5; the corner frequency is when $40\omega_{cl}$ = 1, or ω_{cl} = 25 krad/s.

P10.1.7 Determine the response $V_O(j\omega)/I_{SRC}(j\omega)$, both in magnitude and phase, the passband gain, and the corner frequency in Figure P10.1.7.

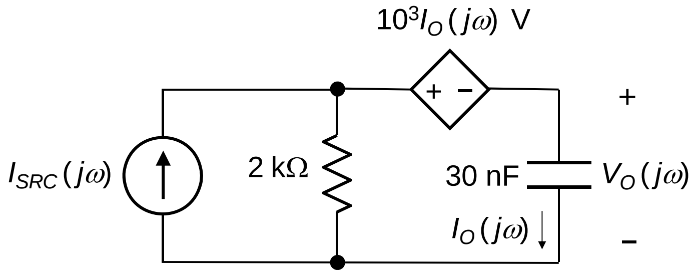


Figure P10.1.7

<u>Solution P10.1.7</u> The impedance of the capacitor is $\dfrac{100}{j3\omega}$ kΩ, where ω is in krad/s. The dependent source of voltage $10^3 I_O(j\omega)$, where $I_O(j\omega)$, is the current through the source, is equivalent to a resistance of $\dfrac{10^3 I_O(j\omega)}{I_O(j\omega)} \equiv 1$ kΩ. Hence,

$H(j\omega) = \dfrac{V_O(j\omega)}{I_{SRC}(j\omega)} = \dfrac{100 \times 10^3}{j3\omega} \times \dfrac{2}{3 + 100/j3\omega} = \dfrac{2 \times 10^3}{1 + j0.09\omega}$ V/A. It follows that

$\left|H(j\omega)\right| = \dfrac{2 \times 10^3}{\sqrt{1 + (0.09\omega)^2}}$ V/A, $\angle H(j\omega) = -\tan^{-1}(0.09\omega)$. The response is lowpass; the passband gain, when $\omega \rightarrow 0$ is 2×10^3; the corner frequency is when $0.09\omega_{cl} = 1$, or $\omega_{cl} = 100/9$ krad/s.

P10.1.10 Determine the response $V_O(j\omega)/V_{SRC}(j\omega)$, both in magnitude and phase, the passband gain, and the corner frequency in Figure P10.1.10.

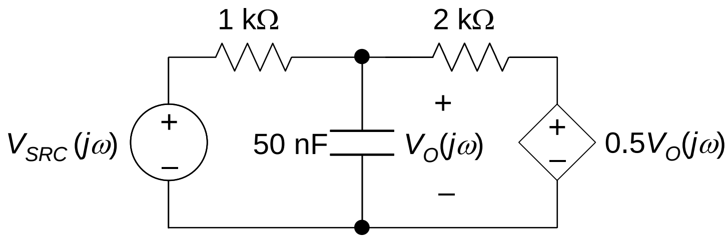


Figure P10.1.10

Solution P10.1.10 Let the current in the capacitor be $I_O(j\omega)$. Then $V_O(j\omega) = I_O(j\omega)/j\omega C$.

The current in the 2 kΩ resistor is $\frac{0.5V_O(j\omega)}{2000}$, and the total current is $I_O(j\omega)$ + $\frac{0.5V_O(j\omega)}{2000}$ = $V_O(j\omega)[j\omega C + 1/4000)]$. It is seen that a 4 kΩ resistor appears in parallel with the capacitor. TEC as seen by the capacitor will have

$V_{Th}(j\omega) = \frac{4}{5}V_{SRC}(j\omega) = 0.8V_{SRC}(j\omega)$; R_{Th} = $\frac{4}{5} = 0.8$ kΩ.

TEC could have been derived directly. Thus, on open circuit, the current is

$\frac{V_O(j\omega) - 0.5V_O(j\omega)}{2}$ mA, so that $V_{SRC}(j\omega) = V_O(j\omega) + \frac{V_O(j\omega)}{4}$, so that

$V_{Th}(j\omega) = \frac{4}{5}V_{SRC}(j\omega) = 0.8V_{SRC}(j\omega)$. On short circuit, $I_{sc}(j\omega) = \frac{V_{SRC}(j\omega)}{1}$ mA.

Hence, $R_{Th} = \frac{V_{Th}(j\omega)}{I_{sc}} = 0.8$ kΩ.

The impedance of the capacitor is $\frac{20}{j\omega}$ kΩ, where ω is in krad/s. Hence,

$\frac{V_O(j\omega)}{0.8V_{SRC}(j\omega)} = \frac{20/j\omega}{0.8 + 20/j\omega}$, or $H(j\omega) = \frac{V_O(j\omega)}{V_{SRC}(j\omega)} = \frac{0.8}{1 + 0.04j\omega}$. It follows that

$|H(j\omega)| = \frac{0.8}{\sqrt{1 + (0.04\omega)^2}}$. $\angle H(j\omega) = -\tan^{-1}(0.04\omega)$. The response is lowpass; the passband gain, when $\omega \to 0$ is 0.8. The corner frequency is when $0.04\omega_{cl}$ = 1, or ω_{cl} = 25 krad/s.

P10.1.12 Determine the response $V_O(j\omega)/V_{SRC}(j\omega)$, both in magnitude and phase, the passband gain, and the corner frequency in Figure P10.1.12.

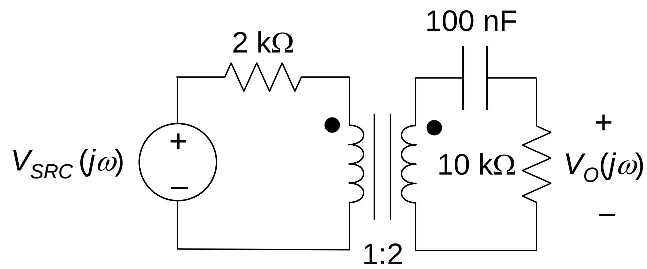
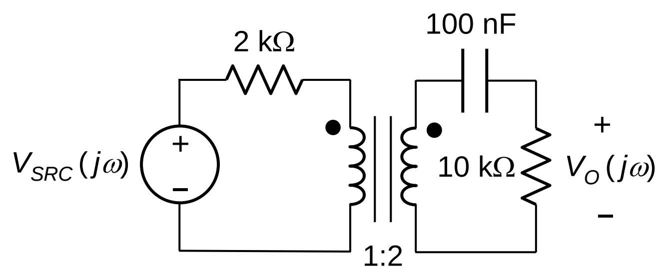


Figure P10.1.12

<u>**Solution P10.1.12**</u> The impedance of the capacitor is

$$\frac{1}{j\omega 100\times 10^{-9}} \equiv \frac{10}{j\omega}\ \text{k}\Omega,$$

where ω is in krad/s.



When the source and 2 kΩ resistor are reflected to the secondary side, they become a source of $2V_{SRC}(j\omega)$ in series with 8 kΩ. Hence, $\frac{V_O(j\omega)}{2V_{SRC}(j\omega)} = \frac{10}{18+10/j\omega}$, or

$H(j\omega) = \frac{V_O(j\omega)}{V_{SRC}(j\omega)} = \frac{20}{18+10/j\omega} = \frac{2j\omega}{1+j1.8\omega}$. It follows that

$|H(j\omega)| = \frac{2\omega}{\sqrt{1+(1.8\omega)^2}}$, $\angle H(j\omega) = 90^\circ - \tan^{-1}(1.8\omega)$. The response is highpass; the passband gain, when $\omega \to \infty$ is 10/9 ≡ 0.915 dB; the corner frequency is $\omega_{cl} = \frac{5}{9}$ krad/s ≡ 88.4 Hz

P10.2.2 Determine R, L, and C in Figure P10.2.2 such that: (a) the maximum response of $V_O(s)$ is 1 V; (b) the bandwidth is 4 krad/s; and (c) the center frequency is 100 krad/s.

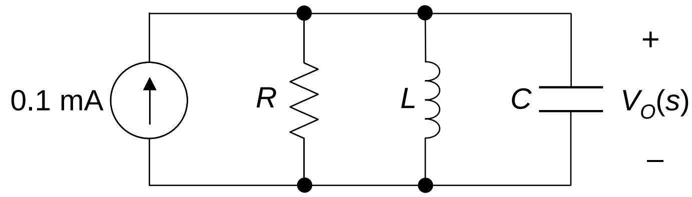


Figure P10.2.2

<u>Solution P10.2.2</u> Maximum response, which occurs at ω_0, is RI; hence R = 10 kΩ. Q = ω_0/BW = 100/4 = 25; it follows that $C = \dfrac{Q}{\omega_0 R} = \dfrac{25}{10^5 \times 10^4} \equiv 25$ nF, and $L = \dfrac{R}{\omega_0 Q} = \dfrac{10^4}{25 \times 10^5} \equiv 4$ mH.

P10.2.3 Given the transfer function $H(s) = \dfrac{4s^2 + 1000s + 100}{5s^2 + 500s + 125}$. Determine the maximum response in dB.

<u>Solution P10.2.3</u>. $H(s) = \dfrac{4}{5}\left[\dfrac{s^2 + 250s + 25}{s^2 + 100s + 25}\right] = 0.8\left[\dfrac{s^2 + 25}{s^2 + 100s + 25} + \dfrac{250s}{s^2 + 100s + 25}\right]$.

The first term is a bandstop response of magnitude 0.8 at low and high frequencies and zero at ω = 5 rad/ s. The second term is a bandpass response of zero at low and high frequencies and magnitude of 2 at ω = 5 rad/ s. The sum will have a maximum magnitude of 2, or $20\log_{10}2$ = 6 dB, at ω = 5 rad/s.