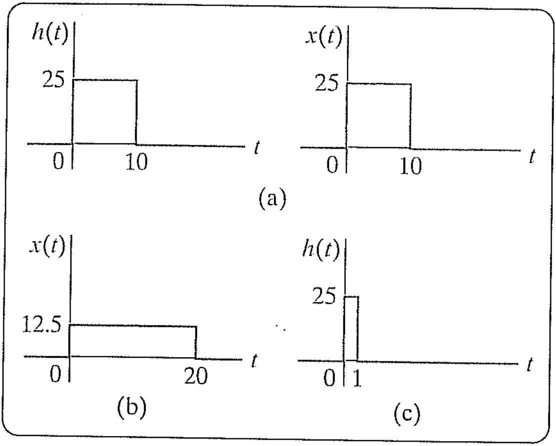
13.59 a) Find $h(t) * x(t)$ when $h(t)$ and $x(t)$ are the rectangular pulses shown in Fig. P13.59(a).

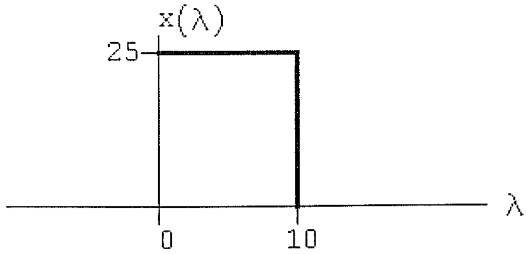
b) Repeat (a) when $x(t)$ changes to the rectangular pulse shown in Fig. P13.59(b).

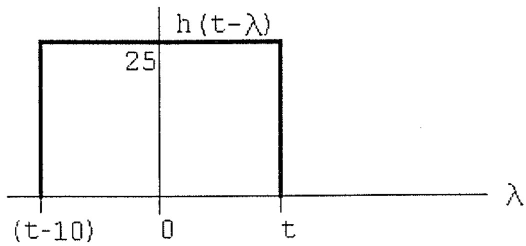
c) Repeat (a) when $h(t)$ changes to the rectangular pulse shown in Fig. P13.59(c).

Figure P13.59



P 13.59 **[a]**



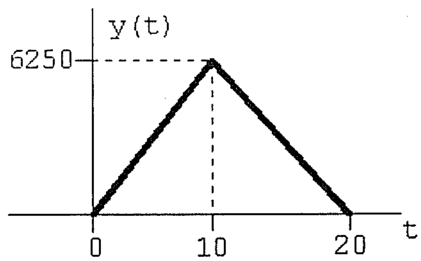


$t < 0:$ $\quad y(t) = 0$

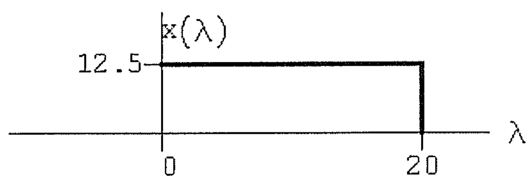
$0 \leq t \leq 10:$ $\quad y(t) = \int_0^t 625\, d\lambda = 625t$

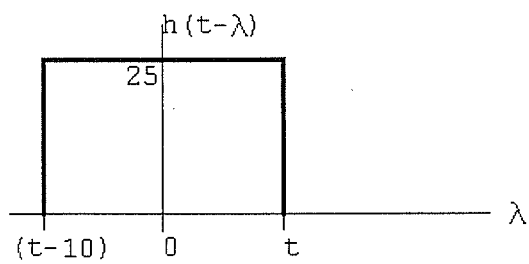
$10 \leq t \leq 20:$ $\quad y(t) = \int_{t-10}^{10} 625\, d\lambda = 625(10 - t + 10) = 625(20 - t)$

$20 \leq t < \infty:$ $\quad y(t) = 0$



[b]





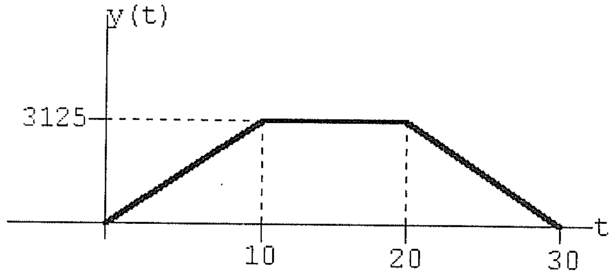
$t < 0:$ $\quad y(t) = 0$

$0 \leq t \leq 10:$ $\quad y(t) = \int_0^t 312.5\, d\lambda = 312.5t$

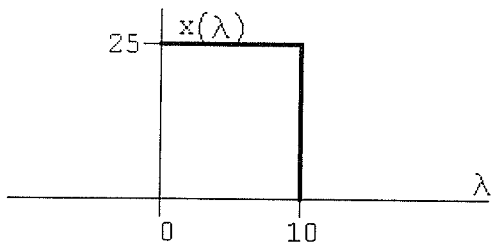
$10 \le t \le 20:\qquad y(t) = \int_{t-10}^{t} 312.5\,d\lambda = 3125$

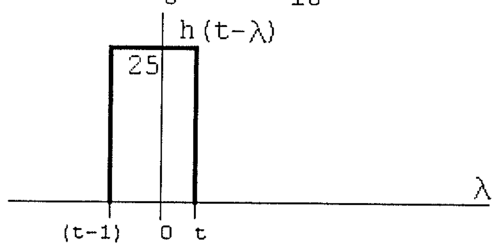
$20 \le t \le 30:\qquad y(t) = \int_{t-10}^{20} 312.5\,d\lambda = 312.5(30-t)$

$30 \le t < \infty:\qquad y(t) = 0$



[c]





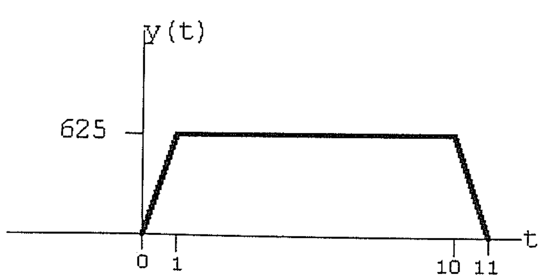
$t < 0:\qquad y(t) = 0$

$0 \le t \le 1:\qquad y(t) = \int_{0}^{t} 625\,d\lambda = 625t$

$1 \le t \le 10:\qquad y(t) = \int_{t-1}^{t} 625\,d\lambda = 625$

$10 \le t \le 11:\qquad y(t) = \int_{t-1}^{10} 625\,d\lambda = 625(11-t)$

$11 \le t < \infty:\qquad y(t) = 0$

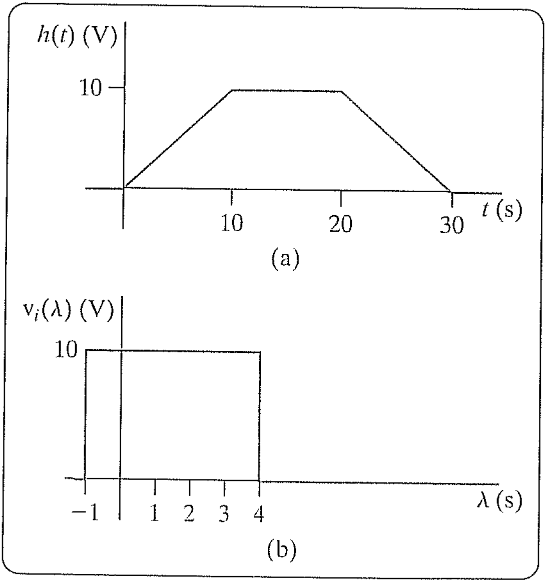


13.61 The voltage impulse response of a circuit is shown in Fig. P13.61(a). The input signal to the circuit is the rectangular voltage pulse shown in Fig. P13.61(b).

a) Derive the equations for the output voltage. Note the range of time for which each equation is applicable.

b) Sketch v_o for $-1 \leq t \leq 34$ s.

Figure P13.61



P 13.61 **[a]** $-1 \le t \le 4$:

$$v_o = \int_0^{t+1} 10\lambda\, d\lambda = 5\lambda^2 \Big|_0^{t+1} = 5t^2 + 10t + 5\text{ V}$$

$4 \le t \le 9$:

$$v_o = \int_{t-4}^{t+1} 10\lambda\, d\lambda = 5\lambda^2 \Big|_{t-4}^{t+1} = 50t - 75\text{ V}$$

$9 \le t \le 14$:

$$v_o = 10\int_{t-4}^{10} \lambda\, d\lambda + 10\int_{10}^{t+1} 10\, d\lambda$$

$$= 5\lambda^2 \Big|_{t-4}^{10} + 100\lambda \Big|_{10}^{t+1} = -5t^2 + 140t - 480\text{ V}$$

$14 \le t \le 19$:

$$v_o = 100\int_{t-4}^{t+1} d\lambda = 500\text{ V}$$

$19 \le t \le 24$:

$$v_o = \int_{t-4}^{20} 100\, d\lambda + \int_{20}^{t+1} 10(30-\lambda)\, d\lambda$$

$$= 100\lambda \Big|_{t-4}^{20} + 300\lambda \Big|_{20}^{t+1} - 5\lambda^2 \Big|_{20}^{t+1}$$

$$= -5t^2 + 190t - 1305\text{ V}$$

$24 \le t \le 29$:

$$v_o = 10\int_{t-4}^{t+1} (30-\lambda)\, d\lambda = 300\lambda \Big|_{t-4}^{t+1} - 5\lambda^2 \Big|_{t-4}^{t+1}$$

$$= 1575 - 50t\text{ V}$$

$29 \le t \le 34$:

$$v_o = 10\int_{t-4}^{30} (30-\lambda)\, d\lambda = 300\lambda \Big|_{t-4}^{30} - 5\lambda^2 \Big|_{t-4}^{30}$$

$$= 5t^2 - 340t + 5780\text{ V}$$

Summary:

$v_o = 0 \qquad -\infty \le t \le -1$

$v_o = 5t^2 + 10t + 5\text{ V} \qquad -1 \le t \le 4$

$v_o = 50t - 75\text{ V} \qquad 4 \le t \le 9$

$v_o = -5t^2 + 140t - 480\text{ V} \qquad 9 \le t \le 14$

$v_o = 500\,\text{V} \qquad 14 \le t \le 19$

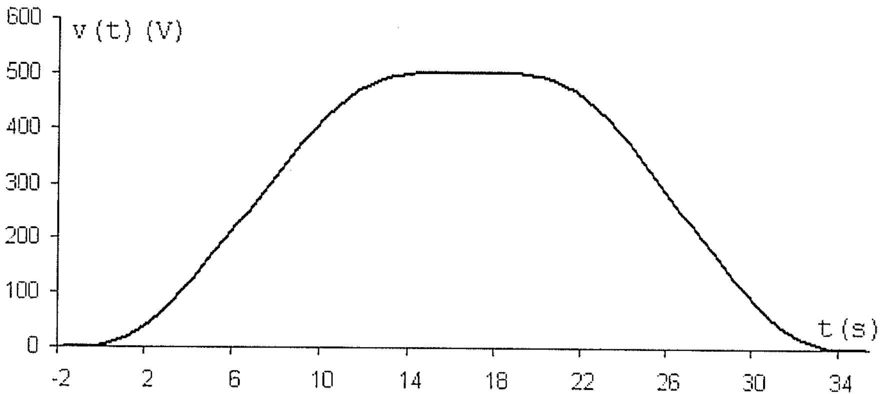
$v_o = -5t^2 + 190t - 1305\,\text{V} \qquad 19 \le t \le 24$

$v_o = 1575 - 50t\,\text{V} \qquad 24 \le t \le 29$

$v_o = 5t^2 - 340t + 5780\,\text{V} \qquad 29 \le t \le 34$

$v_o = 0\,\text{V} \qquad 34 \le t \le \infty$

[b]



13.62 Assume the voltage impulse response of a circuit can be modeled by the triangular waveform shown in Fig. P13.62. The voltage input signal to this circuit is the step function $10u(t)$ V.

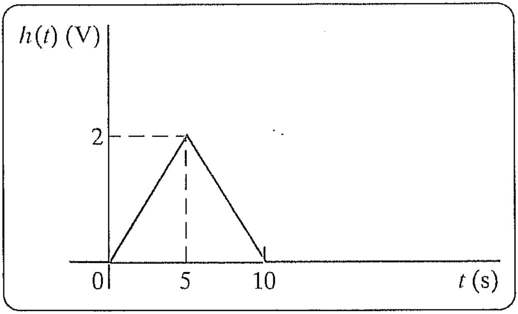
a) Use the convolution integral to derive the expressions for the output voltage.

b) Sketch the output voltage over the interval 0 to 15 s.

c) Repeat parts (a) and (b) if the area under the voltage impulse response stays the same but the width of the impulse response narrows to 4 s.

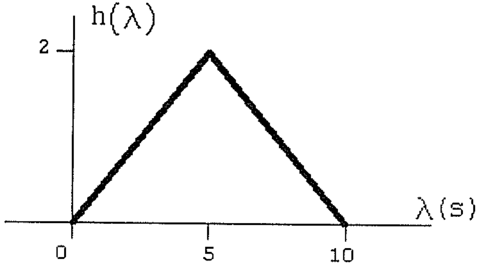
d) Which output waveform is closer to replicating the input waveform: (b) or (c)? Explain.

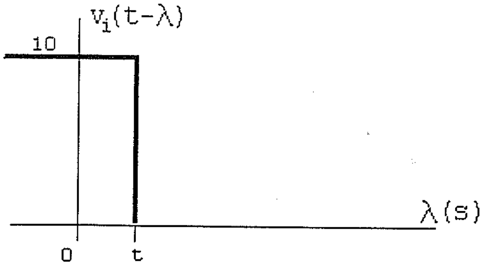
Figure P13.62



P 13.62 **[a]** $h(\lambda) = \frac{2}{5}\lambda \qquad 0 \leq \lambda \leq 5$

$$h(\lambda) = \left(4 - \frac{2}{5}\lambda\right) \qquad 5 \leq \lambda \leq 10$$





$0 \leq t \leq 5$:

$$v_o = 10\int_0^t \frac{2}{5}\lambda\,d\lambda = 2t^2$$

$5 \le t \le 10$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\, d\lambda + 10\int_5^t \left(4 - \frac{2}{5}\lambda\right)\, d\lambda$$

$$= \frac{4\lambda^2}{2}\Big|_0^5 + 40\lambda\Big|_5^t - \frac{4\lambda^2}{2}\Big|_5^t$$

$$= -100 + 40t - 2t^2$$

$10 \le t \le \infty$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\, d\lambda + 10\int_5^{10} \left(4 - \frac{2}{5}\lambda\right)\, d\lambda$$

$$= \frac{4\lambda^2}{2}\Big|_0^5 + 40\lambda\Big|_5^{10} - \frac{4\lambda^2}{2}\Big|_5^{10}$$

$$= 50 + 200 - 150 = 100$$

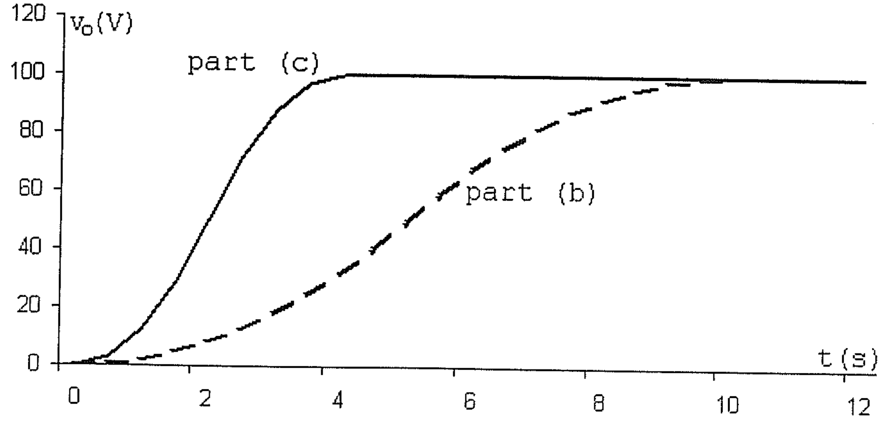
Summary:

$v_o = 2t^2\,\text{V} \qquad 0 \le t \le 5$

$v_o = 40t - 100 - 2t^2\,\text{V} \qquad 5 \le t \le 10$

$v_o = 100\,\text{V} \qquad 10 \le t \le \infty$

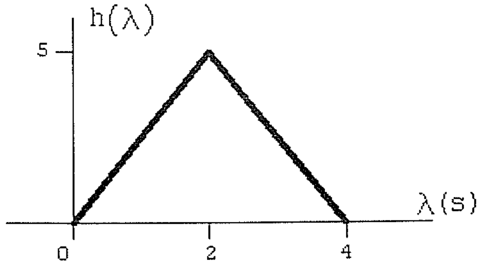
[b]

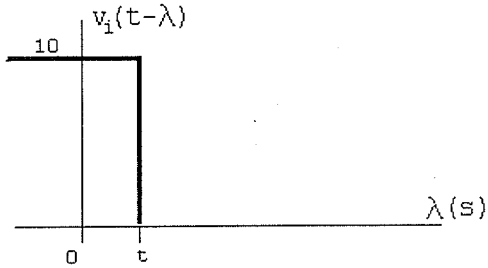


[c] Area $= \frac{1}{2}(10)(2) = 10 \quad \therefore \quad \frac{1}{2}(4)h = 10 \quad$ so $\quad h = 5$

$h(\lambda) = \frac{5}{2}\lambda \qquad 0 \le \lambda \le 2$

$h(\lambda) = \left(10 - \frac{5}{2}\lambda\right) \qquad 2 \le \lambda \le 4$





$0 \leq t \leq 2$:

$$v_o = 10\int_0^t \frac{5}{2}\lambda\,d\lambda = 12.5t^2$$

$2 \leq t \leq 4$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\,d\lambda + 10\int_2^t \left(10 - \frac{5}{2}\lambda\right)\,d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^t - \frac{25\lambda^2}{2}\Big|_2^t$$

$$= -100 + 100t - 12.5t^2$$

$4 \leq t \leq \infty$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\,d\lambda + 10\int_2^4 \left(10 - \frac{5}{2}\lambda\right)\,d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^4 - \frac{25\lambda^2}{2}\Big|_2^4$$

$$= 50 + 200 - 150 = 100$$

$v_o = 12.5t^2$ V $\qquad 0 \leq t \leq 2$

$v_o = 100t - 100 - 12.5t^2$ V $\qquad 2 \leq t \leq 4$

$v_o = 100$ V $\qquad 4 \leq t \leq \infty$

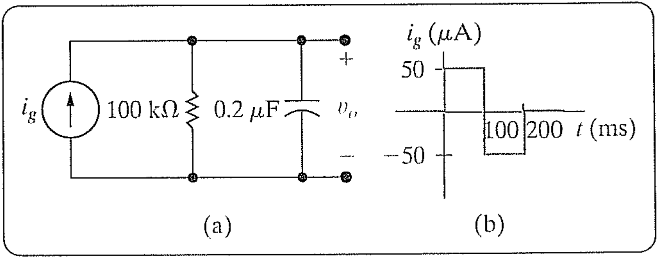
[d] The waveform in part (c) is closer to replicating the input waveform because i part (c) $h(\lambda)$ is closer to being an ideal impulse response. That is, the area wa preserved as the base was shortened.

13.66 a) Use the convolution integral to find v_o in the circuit in Fig. P13.66(a) if i_g is the pulse shown in Fig. P13.66(b).

b) Use the convolution integral to find i_o.

c) Show that your solutions for v_o and i_o are consistent by calculating i_o at 100^- ms, 100^+ ms, 200^- ms, and 200^+ ms.

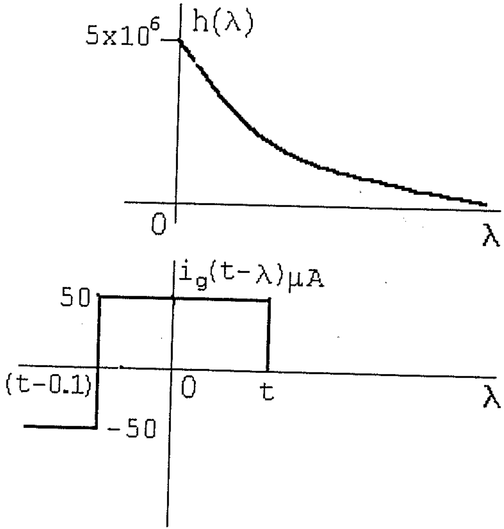
Figure P13.66



P 13.66 **[a]** $I_g = \frac{V_o}{10^5} + \frac{V_o s}{5 \times 10^6} = \frac{V_o(s+50)}{5 \times 10^6}$

$$\frac{V_o}{I_g} = H(s) = \frac{5 \times 10^6}{s + 50}$$

$$h(\lambda) = 5 \times 10^6 e^{-50\lambda} u(\lambda)$$

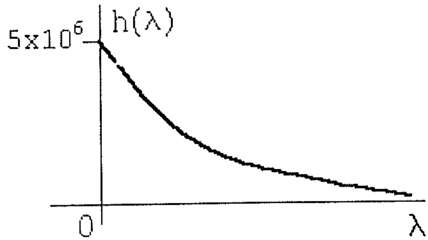


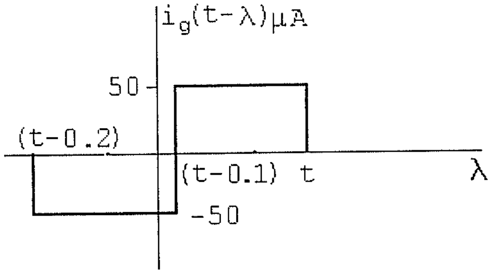
$0 \le t \le 0.1$ s:

$$v_o = \int_0^t (50 \times 10^{-6})(5 \times 10^6)e^{-50\lambda}\, d\lambda = 250\frac{e^{-50\lambda}}{-50}\Big|_0^t$$

$$= 5(1 - e^{-50t})\text{ V}$$

$0.1\text{ s} \le t \le 0.2$ s:





$$v_o = \int_0^{t-0.1} (-50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda}\, d\lambda)$$

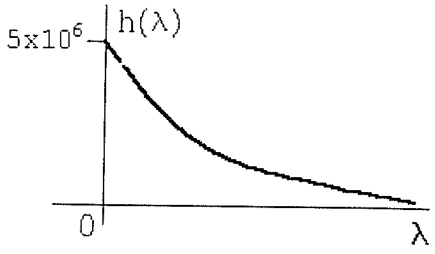
$$+\int_{t-0.1}^{t} (50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda}\, d\lambda)$$

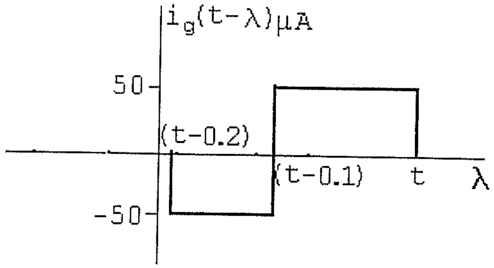
$$= -250\frac{e^{-50\lambda}}{-50}\Big|_0^{t-0.1} + 250\frac{e^{-50\lambda}}{-50}\Big|_{t-0.1}^{t}$$

$$= 5\left[e^{-50(t-0.1)} - 1\right] - 5\left[e^{-50t} - e^{-50(t-0.1)}\right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50t} - 5]\text{ V}$$

$0.2\,\text{s} \leq t \leq \infty$:





$$v_o = \int_{t-0.2}^{t-0.1} -250e^{-50\lambda}\,d\lambda + \int_{t-0.1}^{t} 250e^{-50\lambda}\,d\lambda$$

$$= 5e^{-50\lambda}\Big|_{t-0.2}^{t-0.1} - 5e^{-50\lambda}\Big|_{t-0.1}^{t}$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}]\,\text{V}$$

Summary:

$$v_o = 5(1 - e^{-50t})\,\text{V} \qquad 0 \leq t \leq 0.1\,\text{s}$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50t} - 5]\,\text{V} \qquad 0.1\,\text{s} \leq t \leq 0.2\,\text{s}$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}]\,\text{V} \qquad 0.2\,\text{s} \leq t \leq \infty$$

[b] $I_o = \dfrac{V_o s}{5\times 10^6} = \dfrac{s}{5\times 10^6}\cdot\dfrac{5\times 10^6 I_g}{s+50}$

$$\frac{I_o}{I_g} = H(s) = \frac{s}{s+50} = 1 - \frac{50}{s+50}$$

$$h(\lambda) = \delta(\lambda) - 50e^{-50\lambda}$$

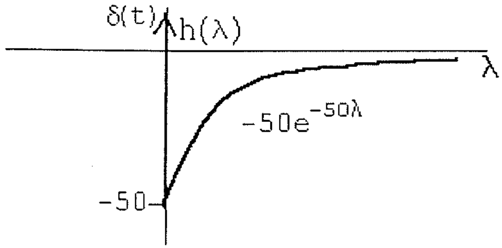
$0 < t < 0.1\,\text{s}$:

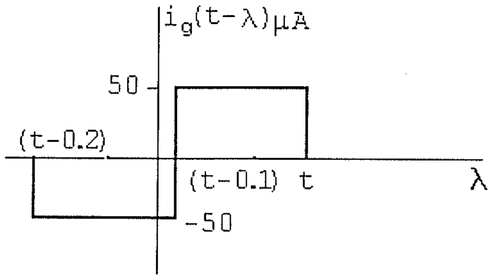
$$i_o = \int_0^t (50\times 10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

$$= 50\times 10^{-6} - \left[50\times 50\times 10^{-6}\frac{e^{-50\lambda}}{-50}\right]\Big|_0^t$$

$$= 50\times 10^{-6} + 50\times 10^{-6}[e^{-50t} - 1] = 50e^{-50t}\,\mu\text{A}$$

$0.1\,\text{s} < t < 0.2\,\text{s}$:





$$i_o = \int_0^{t-0.1} (-50 \times 10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

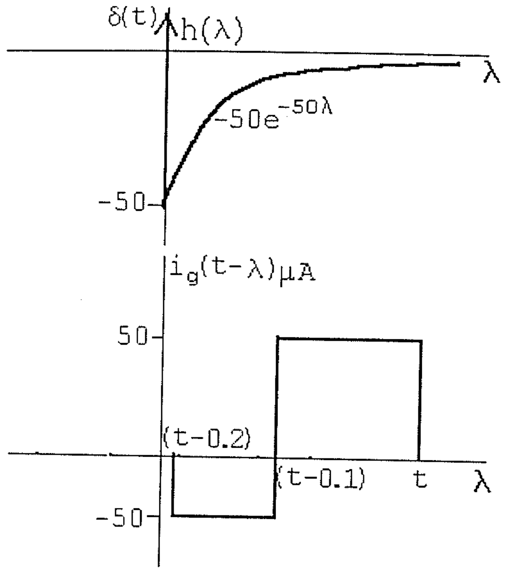
$$+ \int_{t-0.1}^{t} (50 \times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$= -50 \times 10^{-6} + 2500 \times 10^{-6} \frac{e^{-50\lambda}}{-50}\bigg|_0^{t-0.1} - 2500 \times 10^{-6} \frac{e^{-50\lambda}}{-50}\bigg|_{t-0.1}^{t}$$

$$= -50 \times 10^{-6} - 50 \times 10^{-6}[e^{-50(t-0.1)} - 1] + 50 \times 10^{-6}[e^{-50t} - e^{-50(t-0.1)}]$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)}\,\mu\text{A}$$

$0.2\,\text{s} < t < \infty$:



$$i_o = \int_{t-0.2}^{t-0.1} (-50\times10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$+\int_{t-0.1}^{t} (50\times10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)} + 50e^{-50(t-0.2)}\ \mu\text{A}$$

Summary:

$i_0 = 50e^{-50t}\ \mu\text{A} \qquad 0 \le t \le 0.1\,\text{s}$

$i_0 = 50e^{-50t} - 100e^{-50(t-0.1)}\ \mu\text{A} \qquad 0.1\,\text{s} \le t \le 0.2\,\text{s}$

$i_0 = 50e^{-50t} - 100e^{-50(t-0.1)} + 50e^{-50(t-0.2)}\ \mu\text{A} \qquad 0.2\,\text{s} \le t \le \infty$

[c] At $t = 0.1^-$:

$v_o = 5(1-e^{-5}) = 4.97\,\text{V}; \qquad i_{100\text{k}\Omega} = \dfrac{4.97}{0.1} = 49.66\,\mu\text{A}; \qquad i_g = 50\,\mu$

$\therefore\ i_o = 50 - 49.66 = 0.34\,\mu\text{A}$

From the solution for i_o we have $i_o(0.1^-) = 50e^{-5} = 0.34\,\mu\text{A}$ (Checks)
At $t = 0.1^+$:

$v_o(0.1^+) = v_o(0.1^-) = 4.97\,\mu\text{V}; \qquad i_{100\text{k}\Omega} = 49.66\,\mu\text{A}; \qquad i_g = -50\,\mu\text{A}$

$\therefore\ i_o(0.1^+) = -(50 + 49.66) = -99.66\,\mu\text{A}$

From the solution for i_o we have

$i_o(0.1^+) = 50e^{-5} - 100 = -99.66\,\mu\text{A}$ (Checks)

At $t = 0.2^-$:

$$v_o = 10e^{-5} - 5e^{-10} - 5 = -4.93\,\mu\text{V}$$

$$i_{100\text{k}\Omega} = -49.33\,\mu\text{A} \qquad i_g = -50\,\mu\text{A}$$

$$i_o = i_g - i_{100\text{k}\Omega} = -50 + 49.33 = -0.67\,\mu\text{A}$$

From the solution for i_o, $i_o(0.2^-) = 50e^{-10} - 100e^{-5} = -0.67\,\mu\text{A}$ (Checks)
At $t = 0.2^+$:

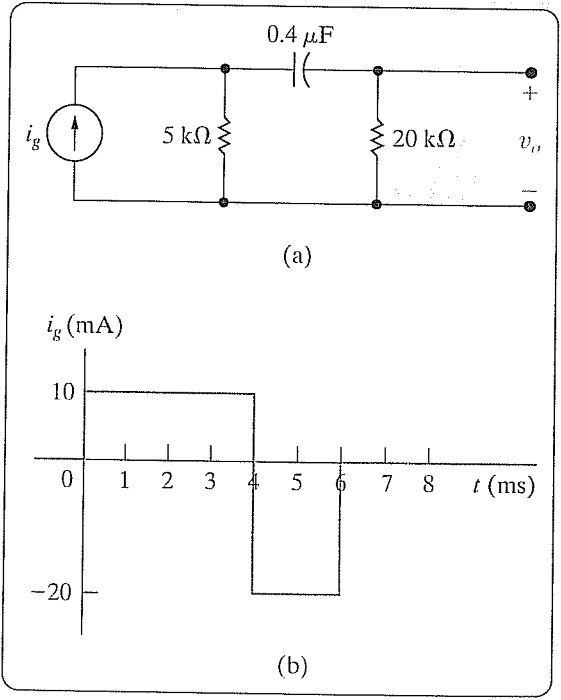
$$v_o(0.2^+) = i_o(0.2^-) = -4.93\,\text{V}; \qquad i_{100\text{k}\Omega} = -49.33\,\mu\text{A}: \qquad i_g = 0$$

$$i_o = i_g - i_{100\text{k}\Omega} = 49.33\,\mu\text{A}$$

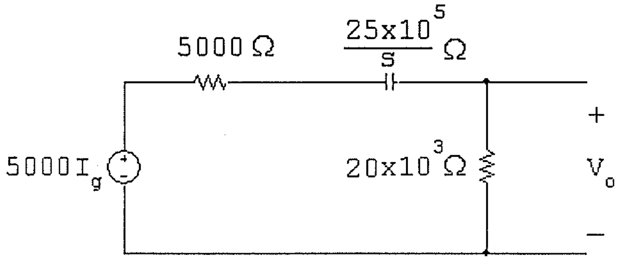
From the solution for i_o,
$i_o(0.2^+) = 50e^{-10} - 100e^{-5} + 50 = 49.33\,\mu\text{A}$(Checks)

×**13.70** The current source in the circuit shown in Fig. P13.70(a) is generating the waveform shown in Fig. P13.70(b). Use the convolution integral to find v_o at $t = 5$ ms.

Figure P13.70



P 13.70



$$V_o = \frac{20 \times 10^3}{5000 + 25 \times 10^5/s + 20 \times 10^3}(5000 I_g)$$

$$\frac{V_o}{I_g} = H(s) = \frac{4000s}{s + 100}$$

$$H(s) = 4000\left[1 - \frac{100}{s + 100}\right] = 4000 - \frac{4 \times 10^5}{s + 100}$$

$$h(\lambda) = 4000\delta(\lambda) - 400{,}000 e^{-100\lambda} u(\lambda)$$

$$v_o = \int_0^{10^{-3}} (-20 \times 10^{-3})[4000\delta(\lambda) - 400{,}000e^{-100\lambda}]\,d\lambda$$

$$+\int_{10^{-3}}^{5\times 10^{-3}} (10 \times 10^{-3})[-400{,}000e^{-100\lambda}]\,d\lambda$$

$$= -80 + 8000\int_0^{10^{-3}} e^{-100\lambda}\,d\lambda - \int_{10^{-3}}^{5\times 10^{-3}} 4000e^{-100\lambda}\,d\lambda$$

$$= -80 - 80(e^{-0.1} - 1) + 40(e^{-0.5} - e^{-0.1})$$

$$v_o(5 \times 10^{-3}) = 40e^{-0.5} - 120e^{-0.1} = 24.26 - 108.58 = -84.32\,\text{V}$$