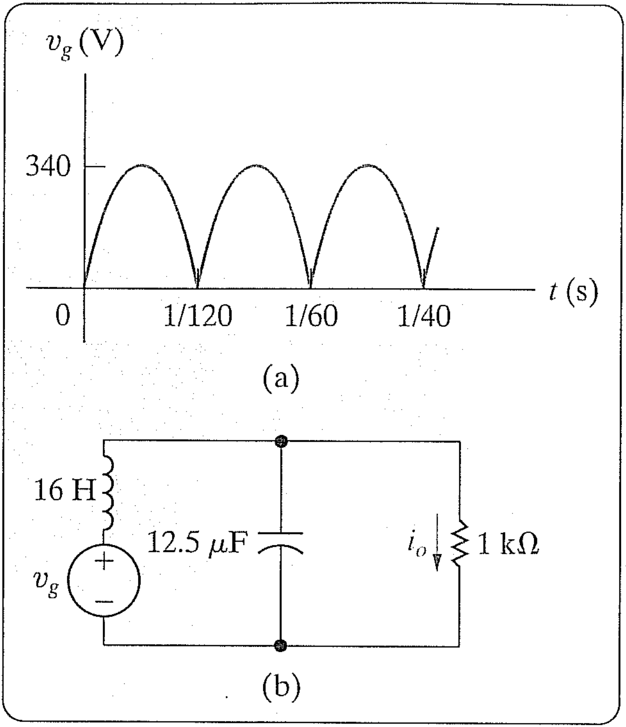
16.26 The full-wave rectified sine-wave voltage shown in Fig. P16.26(a) is applied to the circuit shown in Fig. P16.26(b).

Figure P16.26



a) Find the first four nonzero terms in the Fourier series representation of v_o.

b) Does your solution for v_o make sense? Explain.

P 16.26 **[a]** Note – find $i_o(t)$

$$\frac{V_0 - V_g}{16s} + V_0(12.5 \times 10^{-6}s) + \frac{V_0}{1000} = 0$$

$$V_0\left[\frac{1}{16s} + 12.5 \times 10^{-6}s + \frac{1}{1000}\right] = \frac{V_g}{16s}$$

$$V_0(1000 + 0.2s^2 + 16s) = 1000V_g$$

$$V_0 = \frac{5000V_g}{s^2 + 80s + 5000}$$

$$I_0 = \frac{V_0}{1000} = \frac{5V_g}{s^2 + 80s + 5000}$$

$$H(s) = \frac{I_0}{V_g} = \frac{5}{s^2 + 80s + 5000}$$

$$H(nj\omega_0) = \frac{5}{(5000 - n^2\omega_0^2) + j80n\omega_0}$$

$$\omega_0 = \frac{2\pi}{T} = 240\pi; \qquad \omega_0^2 = 57{,}600\pi^2; \qquad 80\omega_0 = 19{,}200\pi$$

$$H(jn\omega_0) = \frac{5}{(5000 - 57{,}600\pi^2 n^2) + j19{,}200\pi n}$$

$$H(0) = 10^{-3}$$

$$H(j\omega_0) = 8.82 \times 10^{-6}\underline{/-173.89^\circ}$$

$$H(j2\omega_0) = 2.20 \times 10^{-6}\underline{/-176.96^\circ}$$

$$H(j3\omega_0) = 9.78 \times 10^{-7}\underline{/-177.97^\circ}$$

$$H(j4\omega_0) = 5.5 \times 10^{-7}\underline{/-178.48^\circ}$$

$$v_g = \frac{680}{\pi} - \frac{1360}{\pi}\left[\frac{1}{3}\cos\omega_0 t + \frac{1}{15}\cos 2\omega_0 t + \frac{1}{35}\cos 3\omega_0 t + \frac{1}{63}\cos 4\omega_0 t + \ldots\right.$$

$$\begin{aligned} i_0 = {} & \frac{680}{\pi} \times 10^{-3} - \frac{1360}{3\pi}(8.82 \times 10^{-6})\cos(\omega_0 t - 173.89^\circ) \\ & - \frac{1360}{15\pi}(2.20 \times 10^{-6})\cos(2\omega_0 t - 176.96^\circ) \\ & - \frac{1360}{35\pi}(9.78 \times 10^{-7})\cos(3\omega_0 t - 177.97^\circ) \\ & - \frac{1360}{63\pi}(5.5 \times 10^{-7})\cos(4\omega_0 t - 178.48^\circ) - \ldots \\ & = 216.45 \times 10^{-3} - 1.27 \times 10^{-3}\cos(\omega_0 t - 173.89^\circ) \\ & - 6.35 \times 10^{-5}\cos(2\omega_0 t - 176.96^\circ) \\ & - 1.21 \times 10^{-5}\cos(3\omega_0 t - 177.97^\circ) \\ & - 3.8 \times 10^{-6}\cos(4\omega_0 t - 178.48^\circ) - \ldots \end{aligned}$$

$$i_0 \cong 216.45 - 1.27\cos(\omega_0 t - 173.89^\circ)\ \text{mA}$$

Note that the sinusoidal component is very small compared to the dc component, so

$$i_0 \cong 216.45\ \text{mA} \quad \text{(a dc current)}$$

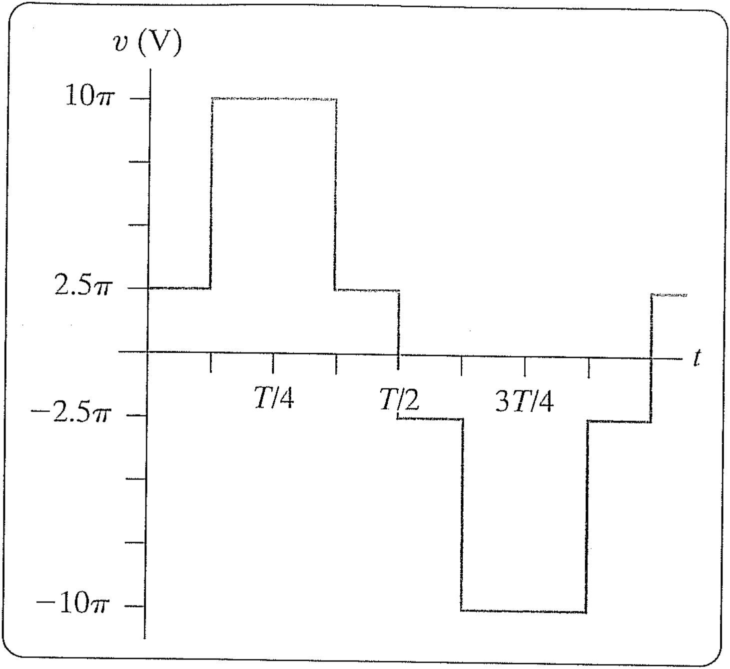
[b] Yes, the solution makes sense. The circuit is a low-pass filter which nearly eliminates all but the dc component.

16.37 a) Use the first four nonzero terms in the Fourier series approximation of the periodic voltage shown in Fig. P16.37 to estimate its rms value.

b) Calculate the true rms value of the voltage.

c) Calculate the percentage of error in the estimated value.

Figure P16.37



P 16.37 [a] v has half-wave symmetry, quarter-wave symmetry, and is odd

$$\therefore \quad a_v = 0, \; a_k = 0 \text{ all } k. \; b_k = 0 \; k\text{-even}$$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\,dt, \qquad k\text{-odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8}\frac{V_m}{4}\sin k\omega_o t\,dt + \int_{T/8}^{T/4} V_m \sin k\omega_o t\,dt\right\}$$

$$= \frac{8V_m}{4T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_0^{T/8}\right] + \frac{8V_m}{T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4}\right]$$

$$= \frac{8V_m}{4k\omega_o T}\left[1 - \cos\frac{k\pi}{4}\right] + \frac{8V_m}{Tk\omega_o}\left[\cos\frac{k\pi}{4} - 0\right]$$

$$= \frac{8V_m}{k\omega_o T}\left\{\frac{1}{4} - \frac{1}{4}\cos\frac{k\pi}{4} + \cos\frac{k\pi}{4}\right\}$$

$$= \frac{4V_m}{\pi k}\left\{\frac{1}{4} + 0.75\cos\frac{k\pi}{4}\right\} = \frac{1}{k}[10 + 30\cos(k\pi/4)]$$

$$b_1 = 10 + 30\cos(\pi/4) = 31.21$$

$$b_3 = \frac{1}{3}[10 + 30\cos(3\pi/4)] = -3.74$$

$$b_5 = \frac{1}{5}[10 + 30\cos(5\pi/4)] = -2.24$$

$$b_7 = \frac{1}{7}[10 + 30\cos(7\pi/4)] = 4.46$$

$$V(\text{rms}) \approx V_m\sqrt{\frac{31.21^2 + 3.74^2 + 2.24^2 + 4.46^2}{2}} = 22.51$$

[b] Area under $v^2 = 2\left[2(2.5\pi)^2\left(\frac{T}{8}\right) + 100\pi^2\left(\frac{T}{4}\right)\right] = 53.125\pi^2 T$

$$V(\text{rms}) = \sqrt{\frac{1}{T}(53.125\pi^2)T} = \sqrt{53.125}\pi = 22.90$$

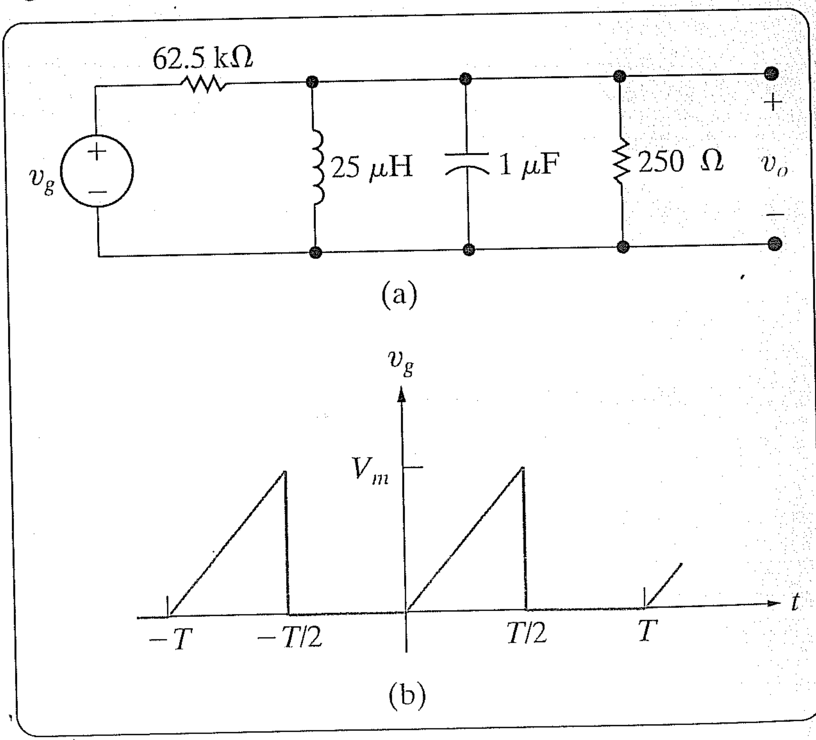
[c] % Error $= \left(\frac{22.51}{22.90} - 1\right)(100) = -1.7\%$

16.43 The periodic voltage source in the circuit shown in Fig. P16.43(a) has the waveform shown in Fig. P16.43(b).

a) Derive the expression for C_n.

b) Find the values of the complex coefficients $C_o, C_{-1}, C_1, C_{-2}, C_2, C_{-3}, C_3, C_{-4}$, and C_4 for the input voltage v_g if $V_m = 54$ V and $T = 10\pi\,\mu$s.

Figure P16.43



c) Repeat (b) for v_o.

d) Use the complex coefficients found in (c) to estimate the average power delivered to the 250 Ω resistor.

P 16.43 **[a]** $C_o = a_v = \frac{(1/2)(T/2)V_m}{T} = \frac{V_m}{4}$

$$C_n = \frac{1}{T}\int_0^{T/2} \frac{2V_m}{T} t e^{-jn\omega_o t}\, dt$$

$$= \frac{2V_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\right]_0^{T/2}$$

$$= \frac{V_m}{2n^2\pi^2}[e^{-jn\pi}(jn\pi + 1) - 1]$$

Since $e^{-jn\pi} = \cos n\pi$ we can write

$$C_n = \frac{V_m}{2\pi^2 n^2}(\cos n\pi - 1) + j\frac{V_m}{2n\pi}\cos n\pi$$

[b] $C_o = \frac{54}{4} = 13.5\text{ V}$

$$C_{-1} = \frac{-54}{\pi^2} + j\frac{27}{\pi} = 10.19\underline{/122.48^\circ}\text{ V}$$

$$C_1 = 10.19\underline{/-122.48^\circ}\text{ V}$$

$$C_{-2} = -j\frac{13.5}{\pi} = 4.30\underline{/-90^\circ}\text{ V}$$

$$C_2 = 4.30\underline{/90^\circ}\text{ V}$$

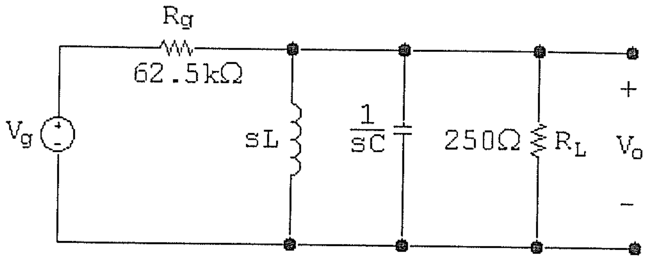
$$C_{-3} = \frac{-6}{\pi^2} + j\frac{9}{\pi} = 2.93\underline{/101.98^\circ}\text{ V}$$

$$C_3 = 2.93\underline{/-101.98^\circ}\text{ V}$$

$$C_{-4} = -j\frac{6.75}{\pi} = 2.15\angle{-90^\circ}\text{ V}$$

$$C_4 = 2.15\angle{90^\circ}\text{ V}$$

[c]



$$\frac{V_o}{250} + \frac{V_o}{sL} + V_o sC + \frac{V_o - V_g}{62.5 \times 10^3} = 0$$

$$\therefore \qquad (250LCs^2 + 1.004sL + 250)V_o = 0.004sLV_g$$

$$\frac{V_o}{V_g} = H(s) = \frac{(1/62,500C)s}{s^2 + 1/249C + 1/LC}$$

$$H(s) = \frac{16s}{s^2 + 1/249Cs + 4 \times 10^{10}}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 2 \times 10^5 \text{ rad/s}$$

$$H(j0) = 0$$

$$H(j2 \times 10^5 k) = \frac{jk}{12,500(1 - k^2) + j251k}$$

Therefore,

$H_{-1} = 0.0398\angle{0^\circ}$; $\qquad H_1 = 0.0398\angle{0^\circ}$

$H_{-2} = \dfrac{-j2}{-37,500 - j20} = 5.33 \times 10^{-5}\angle{86.23^\circ}$; $\qquad H_2 = 5.33 \times 10^{-5}\angle{-89.2}$

$H_{-3} = \dfrac{-3j}{-10^{-5} - j753} = 3.00 \times 10^{-5}\angle{89.57^\circ}$; $\qquad H_2 = 3.00 \times 10^{-5}\angle{-89.57}$

$H_{-4} = \dfrac{-4j}{-187,500 - j1004} = 2.13 \times 10^{-5}\angle{89.69^\circ}$; $\qquad H_2 = 2.13 \times 10^{-5}\angle{-8}$

The output voltage coefficients:

$$C_0 = 0$$

$$C_{-1} = (10.19\angle{122.48^\circ})(0.00398\angle{0^\circ}) = 0.0406\angle{122.48^\circ}\text{ V}$$

$$C_1 = 0.0406\underline{/-122.48^\circ}\ \text{V}$$

$$C_{-2} = (4.30\underline{/-90^\circ})(5.33\times 10^{-5}\underline{/86.23^\circ}) = 2.29\times 10^{-4}\underline{/-3.77^\circ}\ \text{V}$$

$$C_2 = 2.29\times 10^{-4}\underline{/3.77^\circ}\ \text{V}$$

$$C_{-3} = (2.93\underline{/101.98^\circ})(3.00\times 10^{-5}\underline{/89.57^\circ}) = 8.79\times 10^{-5}\underline{/191.55^\circ}\ \text{V}$$

$$C_3 = 8.79\times 10^{-5}\underline{/-191.55^\circ}\ \text{V}$$

$$C_{-4} = (2.15\underline{/-90^\circ})(2.13\times 10^{-5}\underline{/89.69^\circ}) = 4.58\times 10^{-5}\underline{/-0.31^\circ}\ \text{V}$$

$$C_4 = 4.58\times 10^{-5}\underline{/0.31^\circ}\ \text{V}$$

[d] $V_{\text{rms}} \cong \sqrt{C_o^2 + 2\sum_{n=1}^{4}|C_n|^2} \cong \sqrt{2\sum_{n=1}^{4}|C_n|^2}$

$$\cong \sqrt{2(0.0406^2 + (2.29\times 10^{-4})^2 + (8.79\times 10^{-5})^2 + (4.58\times 10^{-5})^2} \cong$$

$$P = \frac{(0.0574)^2}{250} = 13.2\,\mu\text{W}$$

16.47 A periodic voltage is represented by a truncated Fourier series. The amplitude and phase spectra are shown in Fig. P16.47(a) and (b), respectively. (See page 802.)

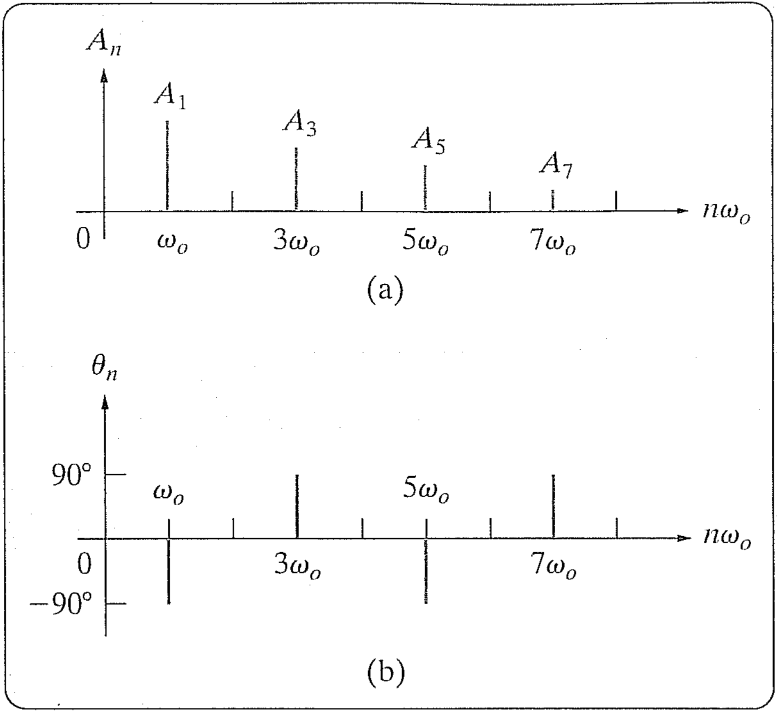
a) Write an expression for the periodic voltage using the form given by Eq. 16.38.

b) Is the voltage an even or odd function of t?

c) Does the voltage have half-wave symmetry?

d) Does the voltage have quarter-wave symmetry?

Figure P16.47



P 16.47 [a] $v = A_1 \cos(\omega_o t + 90^\circ) + A_3 \cos(3\omega_o t - 90^\circ)$

$$+A_5 \cos(5\omega_o t + 90^\circ) + A_7 \cos(7\omega_o t - 90^\circ)$$

$$v = -A_1 \sin \omega_o t + A_3 \sin 3\omega_o t - A_5 \sin 5\omega_o t + A_7 \sin 7\omega_o t$$

[b] $v(-t) = A_1 \sin \omega_o t - A_3 \sin 3\omega_o t + A_5 \sin 5\omega_o t - A_7 \sin 7\omega_o t$

$\therefore \quad v(-t) = -v(t);$ odd function

[c] $v(t - T/2) = -A_1 \sin(\omega_o t - \pi) + A_3 \sin(3\omega_o t - 3\pi)$

$$-A_5 \sin(5\omega_o t - 5\pi) + A_7 \sin(7\omega_o t - 7\pi)$$

$$= A_1 \sin \omega_o t - A_3 \sin 3\omega_o t + A_5 \sin 5\omega_o t - A_7 \sin 7\omega_o t$$

$\therefore \quad v(t - T/2) = -v(t)$, yes, the function has half-wave symmetry

[d] Since the function is odd, with hws, we test to see if

$$f(T/2-t)=f(t)$$

$$\begin{aligned} f(T/2-t) &= -A_1\sin(\pi-\omega_o t)+A_3\sin(3\pi-3\omega_o t) \\ &\quad A_5\sin(5\pi-5\omega_o t)+A_7\sin(7\pi-7\omega_o t) \\ &= -A_1\sin\omega_o t+A_3\sin 3\omega_o t-A_5\sin 5\omega_o t+A_7\sin 7\omega_o t \end{aligned}$$

$\therefore\quad f(T/2-t)=f(t)$ and the voltage has quarter-wave symmetry

16.49 The input signal to a third-order low-pass Butterworth filter is a half-wave rectified sinusoidal voltage. The corner frequency of the filter is 100 rad/s. The amplitude of the sinusoidal voltage is 54π V and its period is 5π ms. Write the first three terms of the Fourier series that represents the steady-state output voltage of the filter.

P 16.49 From Table 15.1 we have

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

After scaling we get

$$H'(s) = \frac{10^6}{(s+100)(s^2+100s+10^4)}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{5\pi} \times 10^3 = 400 \text{ rad/s}$$

$$\therefore \quad H'(jn\omega_o) = \frac{1}{(1+j4n)[(1-16n^2)+j4n]}$$

It follows that

$$H(j0) = 1\underline{/0^\circ}$$

$$H(j\omega_o) = \frac{1}{(1+j4)(-15+j4)} = 0.0156\underline{/-241.03^\circ}$$

$$H(j2\omega_o) = \frac{1}{(1+j8)(-63+j8)} = 0.00195\underline{/-255.64^\circ}$$

$$v_g(t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega_o t - \frac{2A}{\pi}\sum_{n=2,4,6.}^{\infty}\frac{\cos n\omega_o t}{n^2-1}$$

$$= 54 + 27\pi\sin\omega_o t - 36\cos 2\omega_o t - \cdots \text{ V}$$

$$\therefore \quad v_o = 54 + 1.33\sin(400t - 241.03^\circ) - 0.07\cos(800t - 255.64^\circ) - \cdots \text{ V}$$