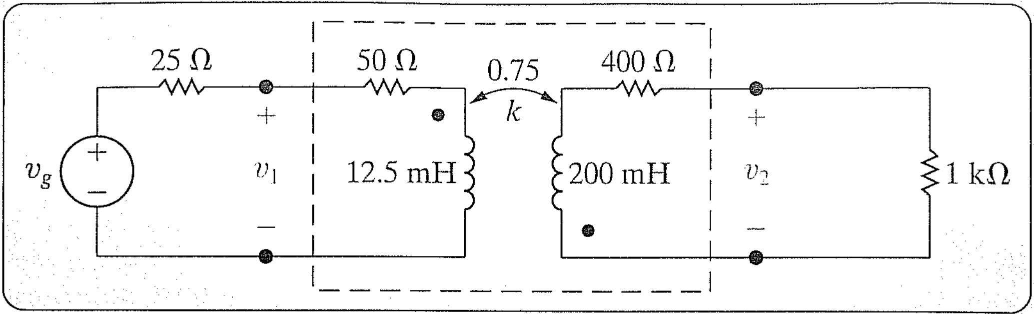
18.29 The linear transformer in the circuit shown in Fig. P18.29 has a coefficient of coupling of 0.75. The transformer is driven by a sinusoidal voltage source whose internal voltage is $v_g = 260 \cos 4000t$ V. The internal impedance of the source is $25 + j0\ \Omega$.

a) Find the frequency-domain a parameters of the linear transformer.

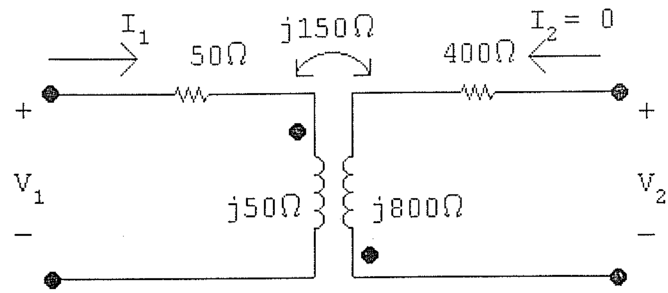
b) Use the a parameters to derive the Thévenin equivalent circuit with respect to the terminals of the load.

c) Derive the steady-state time-domain expression for v_2.

Figure P18.29



P 18.29 **[a]** For $I_2 = 0$:

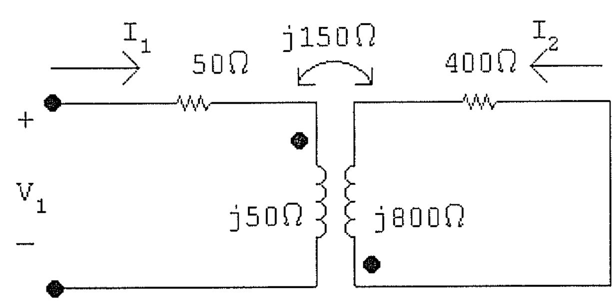


$$V_2 = -j150I_1 = -j150\frac{V_1}{50+j50} = \frac{-j3V_1}{1+j1}$$

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0} = \frac{1+j1}{-j3} = \frac{-1+j1}{3}$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} = \frac{1}{-j150} = \frac{j}{150}\text{ S}$$

For $V_2 = 0$:



$$V_1 = (50+j50)I_1 - j150I_2$$

$$0 = -j150I_1 + (400+j800)I_2$$

$$\Delta = \begin{vmatrix} 50+j50 & -j150 \\ -j150 & 400+j800 \end{vmatrix} = 2500(1+j24)$$

$$N_2 = \begin{vmatrix} 50+j50 & V_1 \\ -j150 & 0 \end{vmatrix} = j150V_1$$

$$I_2 = \frac{N_2}{\Delta} = \frac{j150V_1}{2500(1+j24)}$$

$$a_{12} = \frac{-V_1}{I_2}\bigg|_{V_2=0} = \frac{-50}{3}(24-j1)\ \Omega$$

$$j150I_1 = (400+j800)I_2$$

$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0} = -\frac{8}{3}(2 - j1)$$

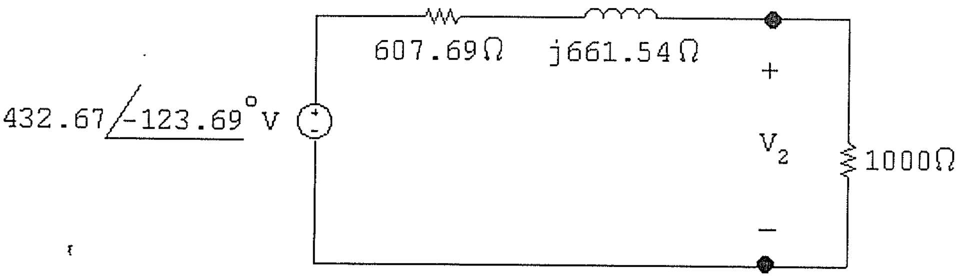
[b] $$V_{\text{Th}} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{260\underline{/0^\circ}}{(-1+j1)/3 + j25/150} = \frac{(260\underline{/0^\circ})6}{-2+j2+j1} = \frac{1560\underline{/0^\circ}}{-2+j3}$$

$$= 120(-2 - j3) = 432.67\underline{/-123.69^\circ}\text{ V}$$

$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{[-(50/3)(24-j1)] + [(-8/3)(2-j1)(25)]}{[(-1+j1)/3] + [(j/150)(25)]}$$

$$= \frac{-100(24-j1) - 16(2-j1)(25)}{-2+j2+j1} = \frac{-3200+j500}{-2+j3}$$

$$= 607.69 + j661.54\,\Omega$$



[c] $$V_2 = \frac{1000}{1607.69 + j661.54}(432.67\underline{/-123.69^\circ}) = 248.88\underline{/-146.06^\circ}$$

$$v_2(t) = 248.88\cos(4000t - 146.06^\circ)\text{ V}$$

18.31 The b parameters for the two-port circuit in Fig. P18.31 are

$$b_{11} = 1 + j\frac{1}{3}; \qquad b_{12} = -1 + j4\ \Omega;$$

$$b_{21} = \frac{1}{3}\ \text{S}; \qquad b_{22} = 1 + j1.$$

The load impedance Z_L is adjusted for maximum average power transfer to Z_L. The ideal voltage source is generating a sinusoidal voltage of

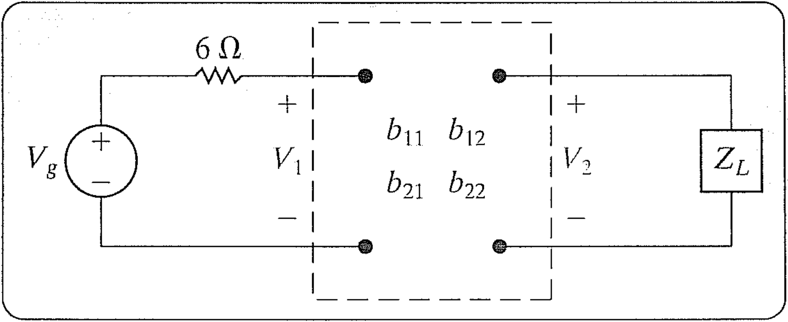
$$v_g = 90 \cos 8000t \text{ V}.$$

a) Find the rms value of V_2.

b) Find the average power delivered to Z_L.

c) What percentage of the average power developed by the ideal voltage source is delivered by Z_L?

Figure P18.31



P 18.31 **[a]** $Z_{\rm Th} = \dfrac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$

$$b_{11}Z_g = 6 + j2; \qquad b_{21}Z_g = 2$$

$$\therefore \qquad Z_{\rm Th} = \frac{6 + j2 - 1 + j4}{2 + 1 + j1} = \frac{5 + j6}{3 + j1} = 2.1 + j1.3\,\Omega$$

$$Z_{\rm L} = Z^*_{\rm Th} = 2.1 - j1.3\,\Omega$$

$$\frac{V_2}{V_g} = \frac{\Delta b Z_{\rm L}}{b_{12} + b_{11}Z_g + b_{22}Z_{\rm L} + b_{21}Z_gZ_{\rm L}}$$

$$\Delta b = \frac{3 + j1}{3}(1 + j1) - (-1 + j4)(1/3) = 1$$

$$b_{11}Z_g = 6 + j2$$

$$b_{22}Z_L = (1 + j1)(2.1 - j1.3) = 3.4 + j0.8$$

$$b_{21}Z_gZ_{\rm L} = 4.2 - j2.6$$

$$\frac{V_2}{V_g} = \frac{2.1 - j1.3}{-1 + j4 + 6 + j2 + 3.4 + j0.8 + 4.2 - j2.6} = \frac{2.1 - j13}{12.6 + j4.2}$$

$$\therefore \qquad V_2 = \frac{2.1 - j1.3}{12.6 + j4.2}(90\underline{/0^\circ}) = 10.71 - j12.86 = 16.74\underline{/-50.19^\circ}\,\rm V$$

$$V_2(\text{rms}) = \frac{16.74}{\sqrt{2}} = 11.84\,\rm V$$

[b] $I_2 = \dfrac{-(10.71 - j12.86)}{2.1 - j1.3} = 6.78\underline{/161.57^\circ}\,\rm A$

$$I_2(\text{rms}) = \frac{6.78}{\sqrt{2}} = 4.79\,\rm A$$

$$P = (4.79)^2(2.1) = 48.21\,\rm W$$

[c] $\dfrac{I_2}{I_1} = \dfrac{-\Delta b}{b_{11} + b_{21}Z_{\rm L}}$

$$\frac{-1}{(3 + j1)/3 + (2.1 - j1.3)/3} = \frac{-3}{5.1 - j0.3}$$

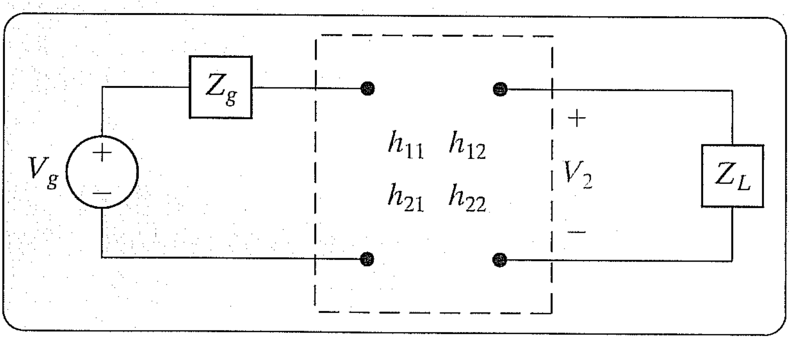
$$\therefore \qquad \frac{I_1}{I_2} = \frac{5.1 - j0.3}{-3} = -1.7 + j0.1$$

$$I_1 = (-1.7 + j0.1)(6.78\underline{/161.57^\circ}) = 11.54\underline{/-21.80^\circ}\ \text{A}$$

$$P_g\ (\text{dev}) = \frac{1}{2}(11.54)(90)\cos(21.80^\circ) = 482.15\ \text{W}$$

$$\% = \frac{48.21}{482.15}(100) \cong 10\%$$

Figure P18.32



18.33 For the terminated two-port amplifier circuit in Fig. P18.32, find

a) the value of Z_L for maximum average power transfer to Z_L

b) the maximum average power delivered to Z_L

c) the average power developed by the ideal voltage source when maximum power is delivered to Z_L.

P 18.33 **[a]** $Z_{\rm Th} = \dfrac{Z_g + h_{11}}{h_{22}Z_g + \Delta h}$

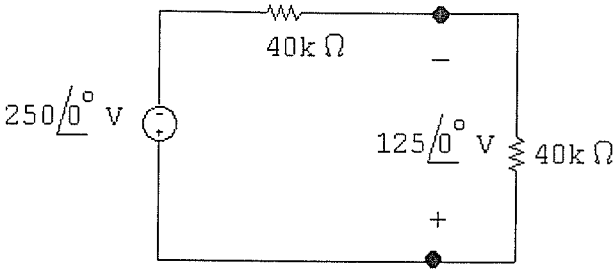
$$\Delta h = (500)(50 \times 10^{-6}) - 50 \times 10^{-3} = -25 \times 10^{-3}$$

$$h_{22}Z_g = 50 \times 10^{-6}(1500) = 75 \times 10^{-3}$$

$$Z_{\rm Th} = \frac{2000}{75 \times 10^{-3} - 25 \times 10^{-3}} = 40{,}000 + j0\,\Omega$$

$$Z_{\rm L} = Z_{\rm Th}^{*} = 40 + j0\,\text{k}\Omega$$

[b] $V_{\mathrm{Th}} = \dfrac{-h_{21}V_g}{50 \times 10^{-3}} = \dfrac{-50(250) \times 10^{-3}}{50 \times 10^{-3}} = -250\,\mathrm{V}$



$$P = \frac{1}{2}\frac{(125)^2}{40{,}000} = 196.3125\,\mathrm{mW}$$

[c] $I_2 = \dfrac{125}{40{,}000} = 3.125\,\mathrm{mA}$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_{\mathrm{L}}} = \frac{50}{1 + (50 \times 10^{-6})(40{,}000)} = \frac{50}{3}$$

$$\frac{I_1}{I_2} = \frac{3}{50} = 0.06$$

$$I_1 = 0.06I_2 = 187.5\,\mu\mathrm{A}$$

$$P_g\ (\mathrm{dev}) = \frac{1}{2}(250)(187.5) \times 10^{-6} = 23.4375\,\mathrm{mW}$$

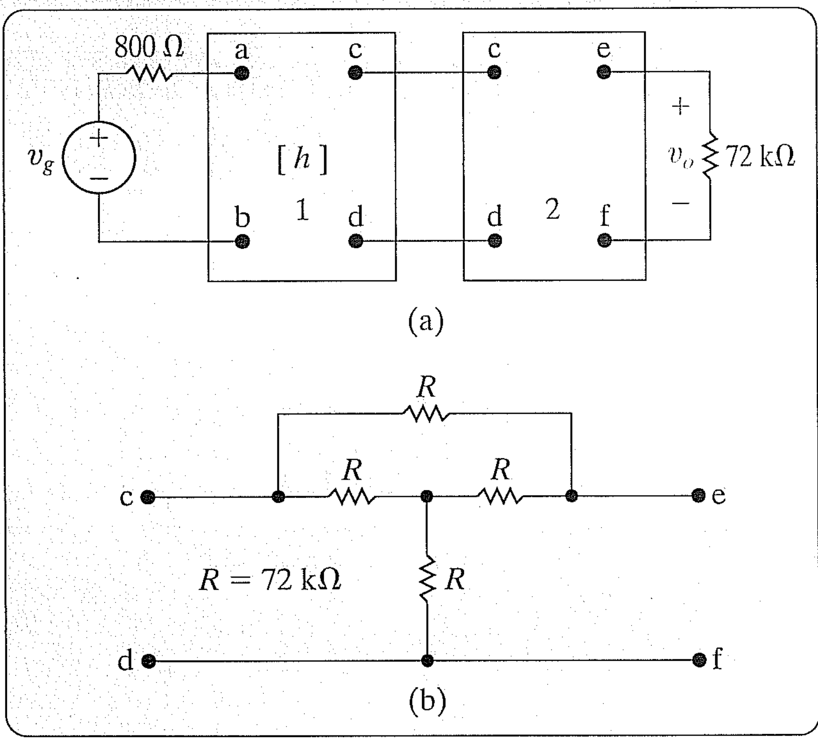
18.39 The h parameters of the first two-port circuit in Fig. P18.39(a) are

$$h_{11} = 1000\ \Omega; \qquad h_{12} = 5 \times 10^{-4};$$

$$h_{21} = 40; \qquad h_{22} = 25\ \mu\text{S}.$$

The circuit in the second two-port circuit is shown in Fig. P18.39(b), where $R = 72\ \text{k}\Omega$. Find v_o if $v_g = 9$ mV dc.

Figure P18.39



P 18.39 The a parameters of the first two-port are

$$a'_{11} = \frac{-\Delta h}{h_{21}} = \frac{-5 \times 10^{-3}}{40} = -125 \times 10^{-6}$$

$$a'_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25\,\Omega$$

$$a'_{21} = \frac{-h_{22}}{h_{21}} = \frac{-25}{40} \times 10^{-6} = -625 \times 10^{-9}\,\text{S}$$

$$a'_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -25 \times 10^{-3}$$

The a parameters of the second two-port are

$$a''_{11} = \frac{5}{4}; \quad a''_{12} = \frac{3R}{4}; \quad a''_{21} = \frac{3}{4R}; \quad a''_{22} = \frac{5}{4}$$

or $a''_{11} = 1.25$; $a''_{12} = 54\,\text{k}\Omega$; $a''_{21} = \dfrac{1}{96}\,\text{mS}$; $a''_{22} = 1.25$

The a parameters of the cascade connection are

$$a_{11} = -125 \times 10^{-6}(1.25) + (-25)(10^{-3}/96) = \frac{-10^{-2}}{24}$$

$$a_{12} = -125 \times 10^{-6}(54 \times 10^3) + (-25)(1.25) = -38\,\Omega$$

$$a_{21} = -625 \times 10^{-9}(1.25) + (-25 \times 10^{-3})(10^{-3}/96) = \frac{-10^{-4}}{96}\,\mathrm{S}$$

$$a_{22} = -625 \times 10^{-9}(54 \times 10^3) + (-25 \times 10^{-3})(1.25) = -65 \times 10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = \frac{-10^{-4}}{96}(800) = \frac{-10^{-2}}{12}$$

$$a_{11} + a_{21}Z_g = \frac{-10^{-2}}{24} + \frac{-10^{-2}}{12} = \frac{-10^{-2}}{8}$$

$$(a_{11} + a_{21}Z_g)Z_L = \frac{-10^{-2}}{8}(72{,}000) = -90$$

$$a_{22}Z_g = -65 \times 10^{-3}(800) = -52$$

$$\frac{V_o}{V_g} = \frac{72{,}000}{-90 - 38 - 52} = -400$$

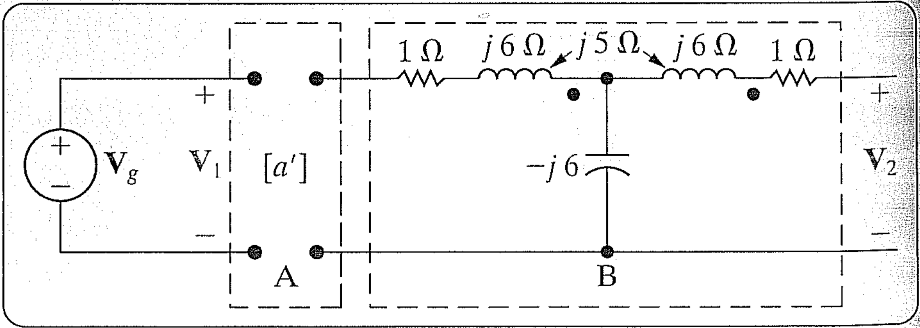
$$v_o = V_o = -400V_g = -3.6\,\mathrm{V}$$

18.40 The networks A and B in the circuit in Fig. P18.40 are reciprocal and symmetric. For network A, it is known that $a'_{11} = 2$ and $a'_{12} = 1\ \Omega$.

a) Find the a parameters of network B.

b) Find $\mathbf{V}_2$ when $\mathbf{V}_g = 75\ \angle 0°$ V, $Z_g = 1\ \angle 0°\ \Omega$, and $Z_L = 14\ \angle 0°\ \Omega$.

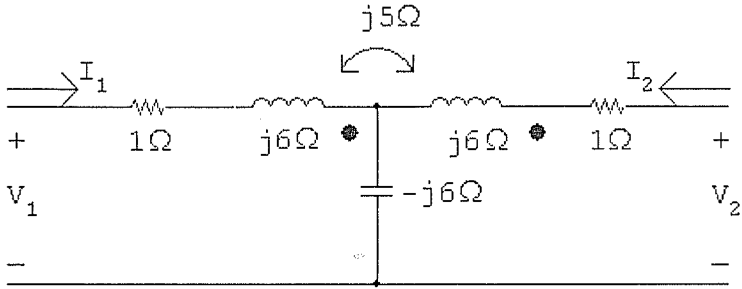
Figure P18.40



P 18.40 **[a]** From reciprocity and symmetry

$$a'_{11} = a'_{22}, \quad \Delta a' = 1; \qquad \therefore \quad 4 - a'_{21} = 1, \quad a'_{21} = 3\ \text{S}$$

For network B



$$a''_{11} = \frac{\mathbf{V}_1}{\mathbf{V}_2}\bigg|_{I_2=0}$$

$$\mathbf{V}_1 = (1 + j6 - j6)\mathbf{I}_1 = \mathbf{I}_1$$

$$\mathbf{V}_2 = (-j5 - j6)\mathbf{I}_1 = -j11\mathbf{I}_1$$

$$a''_{11} = \frac{1}{-j11} = \frac{j}{11}$$

$$a''_{21} = \frac{\mathbf{I}_1}{\mathbf{V}_2}\bigg|_{I_2=0} = \frac{1}{-j11} = \frac{j}{11}\ \text{S}$$

$$a''_{22} = a''_{11} = \frac{j}{11}$$

$$\Delta a'' = 1 = (j/11)(j/11) - (j/11)a''_{12}$$

$$\therefore \quad a''_{12} = \frac{122}{11}j$$

[b] $a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = 2(j/11) + 1(j/11) = j3/11$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = 2(j122/11) + 1(j/11) = j245/11$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = 3(j/11) + 2(j/11) = j5/11$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = 3(j122/11) + 2(j/11) = j368/11$$

$$I_2 = \frac{-V_g}{a_{11}Z_{\text{L}} + a_{12} + a_{21}Z_gZ_{\text{L}} + a_{22}Z_g} = j1.14$$

$$V_2 = -14I_2 = -j15.9\ \text{V}$$