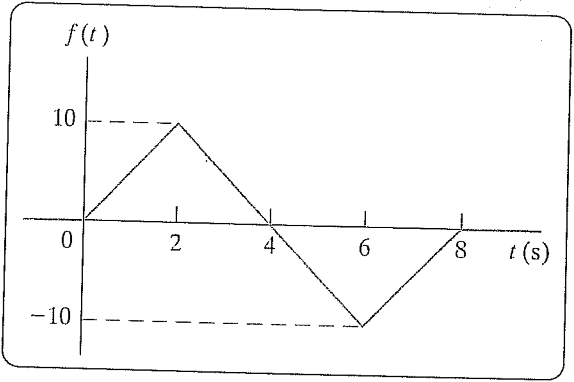
× **12.19** a) Find the Laplace transform of the function illustrated in Fig. P12.19.

b) Find the Laplace transform of the first derivative of the function illustrated in Fig. P12.19.

c) Find the Laplace transform of the second derivative of the function illustrated in Fig. P12.19.

Figure P12.19



P 12.19 **[a]** $f(t) = 5t[u(t) - u(t-2)]$

$$+(20-5t)[u(t-2) - u(t-6)]$$

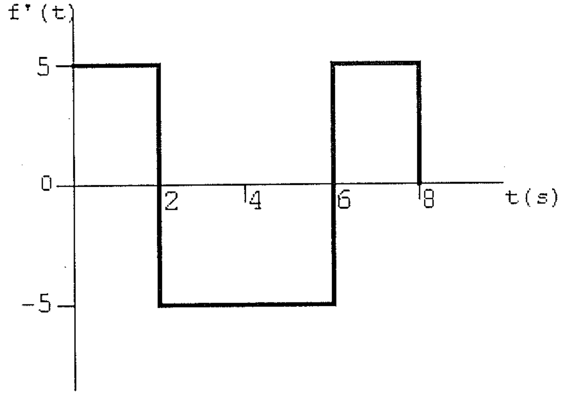
$$+(5t-40)[u(t-6) - u(t-8)]$$

$$= 5tu(t) - 10(t-2)u(t-2)$$

$$+10(t-6)u(t-6) - 5(t-8)u(t-8)$$

$$\therefore \quad F(s) = \frac{5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]}{s^2}$$

[b]



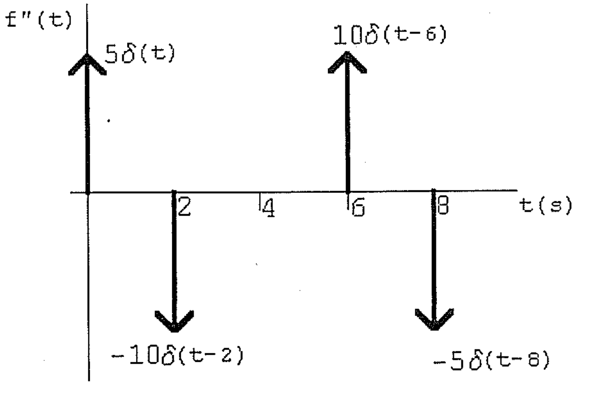
$$f'(t) = 5[u(t) - u(t-2)] - 5[u(t-2) - u(t-6)]$$

$$+5[u(t-6) - u(t-8)]$$

$$= 5u(t) - 10u(t-2) + 10u(t-6) - 5u(t-8)$$

$$\mathcal{L}\{f'(t)\} = \frac{5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]}{s}$$

[c]



$$f''(t) = 5\delta(t) - 10\delta(t-2) + 10\delta(t-6) - 5\delta(t-8)$$

$$\mathcal{L}\{f''(t)\} = 5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]$$

12.21 Find the Laplace transform of each of the following functions:

a) $f(t) = 40e^{-8(t-3)}u(t-3)$.

b) $f(t) = (5t - 10)[u(t-2) - u(t-4)] + (30 - 5t)[u(t-4) - u(t-8)] + (5t - 50)[u(t-8) - u(t-10)]$.

P 12.21 **[a]** $\mathcal{L}\{40e^{-8(t-3)}u(t-3)\} = \dfrac{40e^{-3s}}{(s+8)}$

[b] First rewrite $f(t)$ as

$$f(t) = (5t-10)u(t-2) + (40-10t)u(t-4) + (10t-80)u(t-8) + (50-5t)u(t-10)$$
$$= 5(t-2)u(t-2) - 10(t-4)u(t-4) + 10(t-8)u(t-8) - 5(t-10)u(t-10)$$

$$\therefore \quad F(s) = \frac{5[e^{-2s} - 2e^{-4s} + 2e^{-8s} - e^{-10s}]}{s^2}$$

12.26 In the circuit shown in Fig. 12.16, the dc current source is replaced with a sinusoidal source that delivers a current of $1.2 \cos t$ A. The circuit components are $R = 1\ \Omega$, $C = 625$ mF, and $L = 1.6$ H. Find the numerical expression for $V(s)$.

P 12.26 $I_g(s) = \frac{1.2s}{s^2+1}; \qquad \frac{1}{RC} = 1.6; \qquad \frac{1}{LC} = 1; \qquad \frac{1}{C} = 1.6$

$$\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C[sV(s) - v(0^-)] = I_g(s)$$

$$V(s)\left[\frac{1}{R} + \frac{1}{Ls} + sC\right] = I_g(s)$$

$$V(s) = \frac{I_g(s)}{\frac{1}{R} + \frac{1}{Ls} + sC} = \frac{LsI_g(s)}{\frac{L}{R}s + 1 + s^2LC} = \frac{\frac{1}{C}sI_g(s)}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$= \frac{(1.6)(1.2)s^2}{(s^2+1.6s+1)(s^2+1)} = \frac{1.92s^2}{(s^2+1.6s+1)(s^2+1)}$$

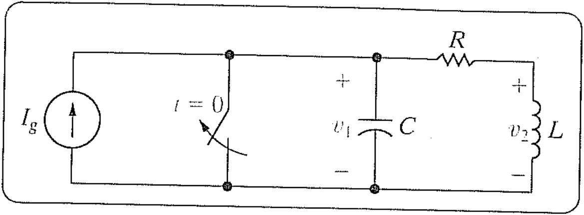
X **12.31** There is no energy stored in the circuit shown in Fig. P12.31 at the time the switch is opened.

a) Derive the integrodifferential equations that govern the behavior of the node voltages v_1 and v_2.

b) Show that

$$V_2(s) = \frac{s I_g(s)}{C[s^2 + (R/L)s + (1/LC)]}.$$

Figure P12.31



P 12.31 **[a]** $C\frac{dv_1}{dt} + \frac{v_1 - v_2}{R} = i_g$

$$\frac{1}{L}\int_0^t v_2\,d\tau + \frac{v_2 - v_1}{R} = 0$$

or

$$C\frac{dv_1}{dt} + \frac{v_1}{R} - \frac{v_2}{R} = i_g$$

$$-\frac{v_1}{R} + \frac{v_2}{R} + \frac{1}{L}\int_0^t v_2\,d\tau = 0$$

[b] $CsV_1(s) + \frac{V_1(s)}{R} - \frac{V_2(s)}{R} = I_g(s)$

$$-\frac{V_1(s)}{R} + \frac{V_2(s)}{R} + \frac{V_2(s)}{sL} = 0$$

or

$$(RCs + 1)V_1(s) - V_2(s) = RI_g(s)$$

$$-sLV_1(s) + (R + sL)V_2(s) = 0$$

Solving,

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}$$

X **12.32** The circuit parameters in the circuit in Fig. P12.31 are $R = 1600\ \Omega$; $L = 200$ mH; and $C = 0.2\ \mu$F. If $I_g(t) = 6$ mA, find $v_2(t)$.

P

P 12.32 $\frac{1}{C} = 5 \times 10^6; \qquad \frac{1}{LC} = 25 \times 10^6; \qquad \frac{R}{L} = 8000$

$$V_2(s) = \frac{(6 \times 10^{-3})(5 \times 10^6)}{s^2 + 8000s + 25 \times 10^6}$$

$$s_{1,2} = -4000 \pm j3000$$

$$V_2(s) = \frac{30{,}000}{(s + 4000 - j3000)(s + 4000 + j3000)}$$

$$= \frac{K_1}{s + 4000 - j3000} + \frac{K_1^*}{s + 4000 + j3000}$$

$$K_1 = \frac{30.000}{j6000} = -j5 = 5\underline{/-90^\circ}$$

$$v_2(t) = 10e^{-4000t}\cos(3000t - 90^\circ)$$

$$= [10e^{-4000t}\sin 3000t]u(t)\ \text{V}$$