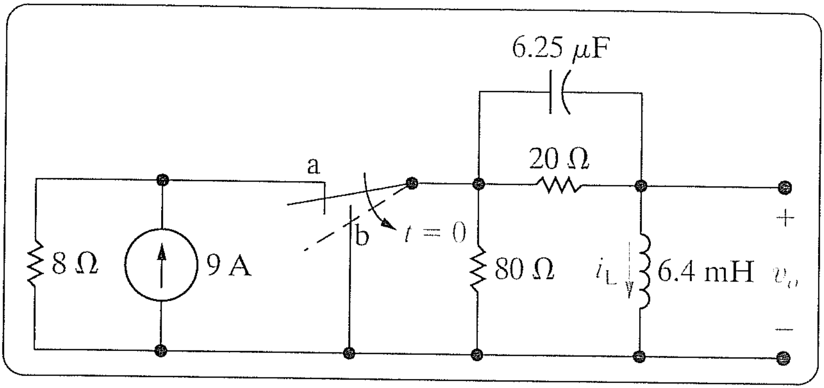
13.10 The switch in the circuit in Fig. P13.10 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b.

Figure P13.10



a) Construct the s-domain circuit for $t > 0$.

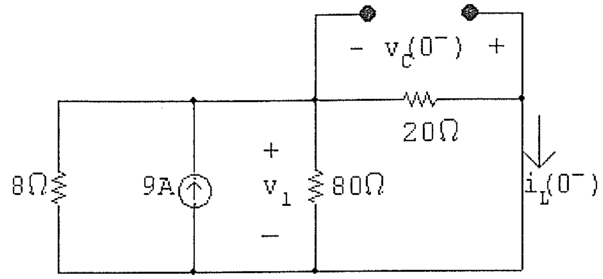
b) Find V_o.

c) Find I_L.

d) Find v_o for $t > 0$.

e) Find i_L for $t > 0$.

P 13.10 **[a]** For $t < 0$:



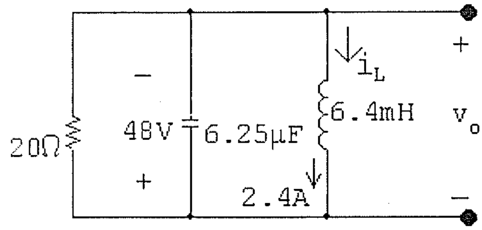
$$\frac{1}{R_e} = \frac{1}{8} + \frac{1}{80} + \frac{1}{20} = 0.1875; \qquad R_e = 5.33\,\Omega$$

$$v_1 = (9)(5.33) = 48\text{ V}$$

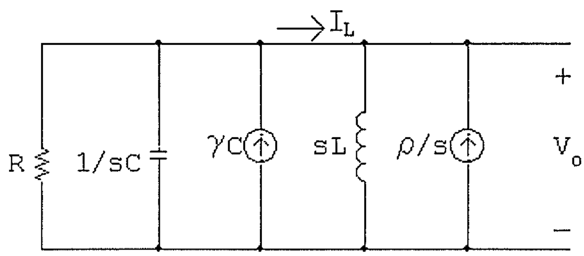
$$i_L(0^-) = \frac{48}{20} = 2.4\text{ A}$$

$$v_C(0^-) = -v_1 = -48\text{ V}$$

For $t = 0^+$:



s-domain circuit:



where

$$R = 20\,\Omega; \qquad C = 6.25\,\mu\text{F}; \qquad \gamma = -48\text{ V};$$

$$L = 6.4\text{ mH}; \qquad \text{and} \qquad \rho = -2.4\text{ A}$$

[b] $$\frac{V_o}{R} + V_o sC - \gamma C + \frac{V_o}{sL} - \frac{\rho}{s} = 0$$

$$\therefore \quad V_o = \frac{\gamma[s + (\rho/\gamma C)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{-2.4}{(-48)(6.25 \times 10^{-6})} = 8000$$

$$\frac{1}{RC} = \frac{1}{(20)(6.25 \times 10^{-6})} = 8000$$

$$\frac{1}{LC} = \frac{1}{(6.4 \times 10^{-3})(6.25 \times 10^{-6})} = 25 \times 10^6$$

$$V_o = \frac{-48(s + 8000)}{s^2 + 8000s + 25 \times 10^6}$$

[c] $$I_L = \frac{V_o}{sL} - \frac{\rho}{s} = \frac{V_o}{0.0064s} + \frac{2.4}{s}$$

$$= \frac{-7500(s + 8000)}{s(s^2 + 8000s + 25 \times 10^6)} + \frac{2.4}{s} = \frac{2.4(s + 4875)}{(s^2 + 8000s + 25 \times 10^6)}$$

[d] $$V_o = \frac{-48(s + 8000)}{s^2 + 8000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 4000 - j3000} + \frac{K_1^*}{s + 4000 + j3000}$$

$$K_1 = \frac{-48(s + 8000)}{s + 4000 + j3000}\bigg|_{s=-4000+j3000} = 40\underline{/126.87^\circ}$$

$$v_o(t) = [80e^{-4000t}\cos(3000t + 126.87^\circ)]u(t)\text{ V}$$

[e] $$I_L = \frac{2.4(s + 4875)}{s^2 + 8000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 4000 - j3000} + \frac{K_1^*}{s + 4000 + j3000}$$

$$K_1 = \frac{2.4(s + 4875)}{s + 4000 + j3000}\bigg|_{s=-4000+j3000} = 1.25\underline{/-16.26^\circ}$$

$$i_L(t) = [2.5e^{-4000t}\cos(3000t - 16.26^\circ)]u(t)\text{ A}$$

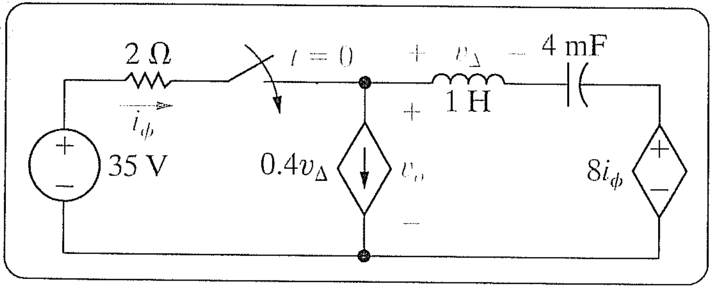
13.15 There is no energy stored in the circuit in Fig. P13.15 at the time the switch is closed.

P

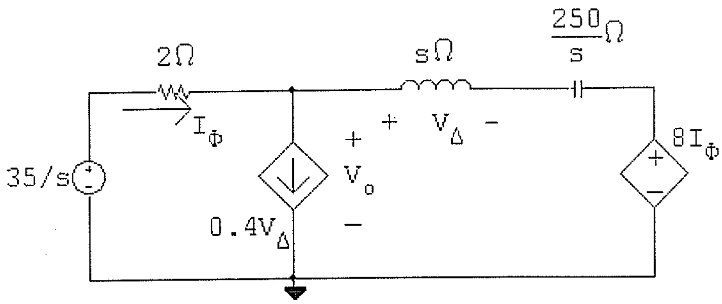
a) Find v_o for $t \geq 0$.

b) Does your solution make sense in terms of known circuit behavior? Explain.

Figure P13.15



P 13.15 **[a]**



$$\frac{V_o - 35/s}{2} + 0.4V_\Delta + \frac{V_o - 8I_\phi}{s + (250/s)} = 0$$

$$V_\Delta = \left[\frac{V_o - 8I_o}{s + (250/s)}\right] s; \qquad I_\phi = \frac{(35/s) - V_o}{2}$$

Solving for V_o yields:

$$V_o = \frac{29.4s^2 + 56s + 1750}{s(s^2 + 2s + 50)} = \frac{29.4s^2 + 56s + 1750}{s(s + 1 - j7)(s + 1 + j7)}$$

$$V_o = \frac{K_1}{s} + \frac{K_2}{s + 1 - j7} + \frac{K_2^*}{s + 1 + j7}$$

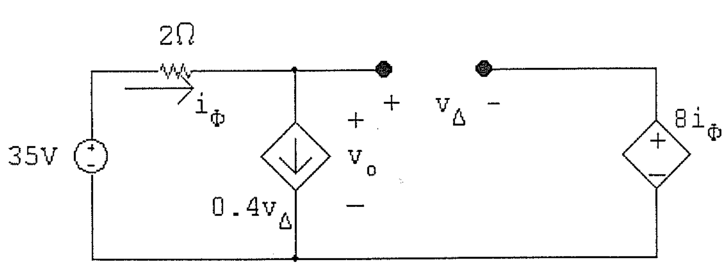
$$K_1 = \frac{29.4s^2 + 56s + 1750}{s^2 + 2s + 50}\bigg|_{s=0} = 35$$

$$K_2 = \frac{29.4s^2 + 56s + 1750}{s(s + 1 + j7)}\bigg|_{s=-1+j7}$$

$$= -2.8 + j0.6 = 2.86\underline{/167.91^\circ}$$

$$\therefore \quad v_o(t) = [35 + 5.73e^{-t}\cos(7t + 167.91^\circ)]u(t)\,\text{V}$$

[b] At $t = 0^+$ $\qquad v_o = 35 + 5.73\cos(167.91^\circ) = 29.4\,\text{V}$

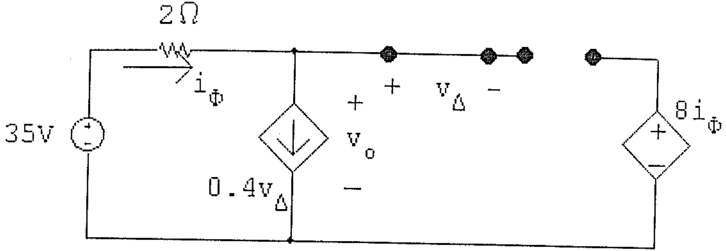


$$\frac{v_o - 35}{2} + 0.4v_\Delta = 0; \qquad v_o - 35 + 0.8v_\Delta = 0$$

$$v_o = v_\Delta + 8i_\phi = v_\Delta + 8(0.4v_\Delta) = 4.2v_\Delta$$

$$v_o + (0.8)\frac{v_o}{4.2} = 35; \qquad \therefore \quad v_o(0^+) = 29.4\,\text{V}(\text{Checks})$$

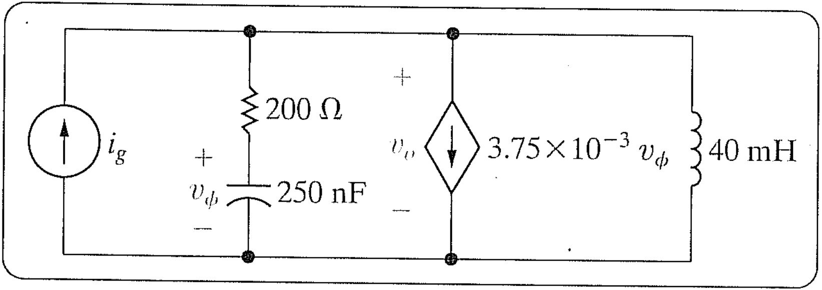
At $t = \infty$, the circuit is



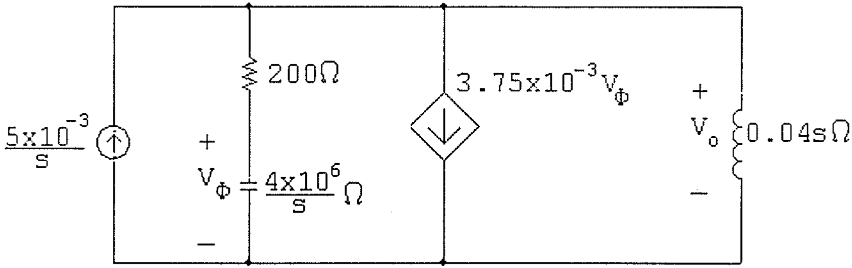
$v_\Delta = 0. \quad i_\phi = 0 \qquad \therefore \quad v_o = 35\text{ V}(\text{Checks})$

13.17 Find v_o in the circuit shown in Fig. P13.17 if $i_g = 5u(t)$ mA. There is no energy stored in the circuit at $t = 0$.

Figure P13.17



P 13.17



$$\frac{5\times10^{-3}}{s} = \frac{V_o}{200 + 4\times10^6/s} + 3.75\times10^{-3}V_\phi + \frac{V_o}{0.04s}$$

$$V_\phi = \frac{4\times10^6/s}{200+4\times10^6/s}V_o = \frac{4\times10^6 V_o}{200s+4\times10^6}$$

$$\therefore \quad \frac{5\times10^{-3}}{s} = \frac{V_o s}{200s+4\times10^6} + \frac{15{,}000V_o}{200s+4\times10^6} + \frac{25V_o}{s}$$

$$\therefore \quad V_o = \frac{s+20{,}000}{s^2+20{,}000s+10^8} = \frac{K_1}{(s+10{,}000)^2} + \frac{K_2}{s+10{,}000}$$

$$K_1 = 10{,}000; \qquad K_2 = 1$$

$$V_o = \frac{10{,}000}{(s+10{,}000)^2} + \frac{1}{s+10{,}000}$$

$$v_o(t) = [10{,}000te^{-10{,}000t} + e^{-10{,}000t}]u(t)\ \mathrm{V}$$

13.38 P

The magnetically coupled coils in the circuit seen in Fig. P13.38 carry initial currents of 15 and 10 A, as shown.

a) Find the initial energy stored in the circuit.

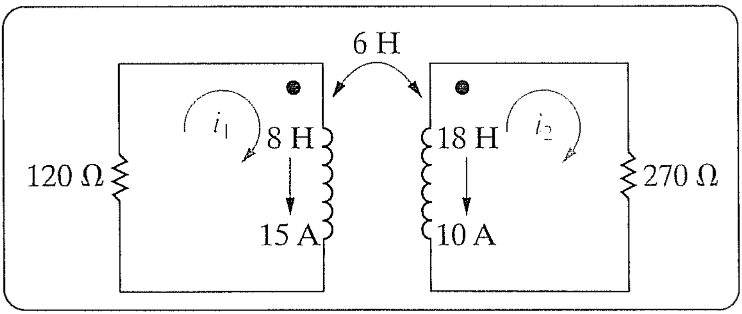
b) Find I_1 and I_2.

c) Find i_1 and i_2.

d) Find the total energy dissipated in the 120 and 270 Ω resistors.

e) Repeat (a)–(d), with the dot on the 18 H inductor at the lower terminal.

Figure P13.38



P 13.38 **[a]** $W = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2$

$$W = 4(15)^2 + 9(100) + 150(6) = 2700\,\text{J}$$

[b] $120i_1 + 8\frac{di_1}{dt} - 6\frac{di_2}{dt} = 0$

$$270i_2 + 18\frac{di_2}{dt} - 6\frac{di_1}{dt} = 0$$

Laplace transform the equations to get

$$120I_1 + 8(sI_1 - 15) - 6(sI_2 + 10) = 0$$

$$270I_2 + 18(sI_2 + 10) - 6(sI_1 - 15) = 0$$

In standard form,

$$(8s + 120)I_1 - 6sI_2 = 180$$

$$-6sI_1 + (18s + 270)I_2 = -270$$

$$\Delta = \begin{vmatrix} 8s + 120 & -6s \\ -6s & 18s + 270 \end{vmatrix} = 108(s + 10)(s + 30)$$

$$N_1 = \begin{vmatrix} 180 & -6s \\ -270 & 18s + 270 \end{vmatrix} = 1620(s + 30)$$

$$N_2 = \begin{vmatrix} 8s + 120 & 180 \\ -6s & -270 \end{vmatrix} = -1080(s + 30)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{1620(s+30)}{108(s+10)(s+30)} = \frac{15}{s+10}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-1080(s+30)}{108(s+10)(s+30)} = \frac{-10}{s+10}$$

[c] $i_1(t) = 15e^{-10t}u(t)$ A; $\qquad i_2(t) = -10e^{-10t}u(t)$ A

[d] $W_{120\Omega} = \int_0^\infty (225e^{-20t})(120)\,dt = 27{,}000\frac{e^{-20t}}{-20}\Big|_0^\infty = 1350\,\text{J}$

$$W_{270\Omega} = \int_0^\infty (100e^{-20t})(270)\,dt = 27{,}000\frac{e^{-20t}}{-20}\Big|_0^\infty = 1350\,\text{J}$$

$W_{120\Omega} + W_{270\Omega} = 2700\,\text{J}$ (Checks)

[e] $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = 900 + 900 - 900 = 900\,\text{J}$

With the dot reversed the s-domain equations are

$$(8s+120)I_1 + 6sI_2 = 60$$

$$6sI_1 + (18s+270)I_2 = -90$$

As before, $\Delta = 108(s+10)(s+30)$. Now,

$$N_1 = \begin{vmatrix} 60 & 6s \\ -90 & 18s+270 \end{vmatrix} = 1620(s+10)$$

$$N_2 = \begin{vmatrix} 8s+120 & 60 \\ 6s & -90 \end{vmatrix} = -1080(s+10)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{15}{s+30}; \qquad I_2 = \frac{N_2}{\Delta} = \frac{-10}{s+30}$$

$i_1(t) = 15e^{-30t}u(t)$ A; $\qquad i_2(t) = -10e^{-30t}u(t)$ A

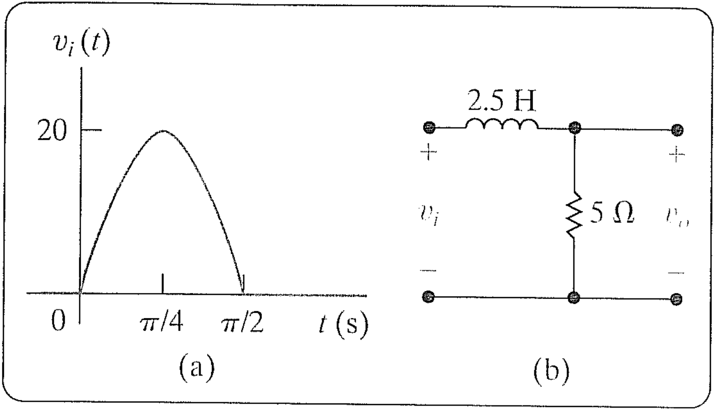
$$W_{270\Omega} = \int_0^\infty (100e^{-60t})(270)\,dt = 450\,\text{J}$$

$$W_{120\Omega} = \int_0^\infty (225e^{-60t})(120)\,dt = 450\,\text{J}$$

$W_{120\Omega} + W_{270\Omega} = 900\,\text{J}$ (Checks)

13.67 The sinusoidal voltage pulse shown in Fig. P13.67(a) is applied to the circuit shown in Fig. P13.67(b). Use the convolution integral to find the value of v_o at $t = 2.2$ s.

Figure P13.67



P 13.67 $H(s) = \frac{V_o}{V_i} = \frac{5}{5+2.5s} = \frac{2}{s+2}$

$$h(\lambda) = 2e^{-2\lambda}; \qquad h(t-\lambda) = 2e^{-2(t-\lambda)} = 2e^{-2t}e^{2\lambda}$$

$$\frac{T}{2} = \frac{\pi}{2}; \qquad T = \pi\,\text{s}; \qquad f = \frac{1}{\pi}\,\text{Hz}$$

$$v_i(\lambda) = (20\sin 2\lambda)[u(\lambda) - u(\lambda - \pi/2)]$$

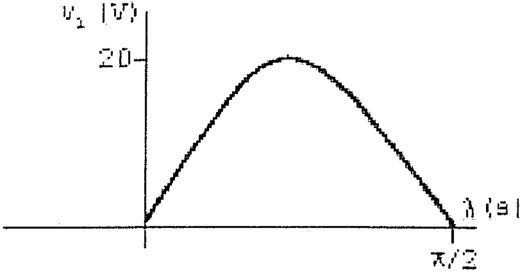
$(\pi/2)\,\text{s} \leq t \leq \infty$:

$$v_o = \int_0^{\pi/2} (2e^{-2t}e^{2\lambda})(20\sin 2\lambda)\,d\lambda = 40e^{-2t}\int_0^{\pi/2} e^{2\lambda}\sin 2\lambda\,d\lambda$$

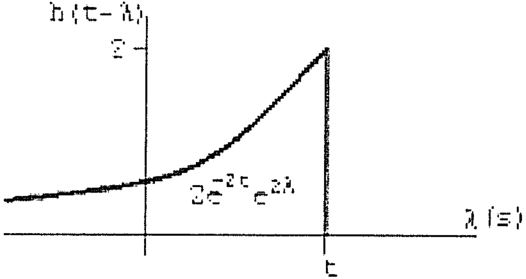
$$= 40e^{-2t}\left[\frac{e^{2\lambda}}{8}(2\sin 2\lambda - 2\cos 2\lambda)\right]_0^{\pi/2} = 10e^{-2t}[e^{\pi}(\sin\pi - \cos\pi) - 1(0-1)$$

$$= 10e^{-2t}(e^{\pi}+1) = 10(e^{\pi}+1)e^{-2t}\,\text{V}$$

$$v_o(2.2) = 241.41e^{-4.4} = 2.96\,\text{V}$$



v_1 (V)
20
λ (s)
π/2



h(t−λ)
2
$2e^{-2t}e^{2\lambda}$
λ (s)
t