14.5 A resistor denoted as R_L is connected in parallel with the capacitor in the circuit in Fig. 14.7. The loaded low-pass filter circuit is shown in Fig. P14.5.

a) Derive the expression for the voltage transfer function V_o/V_i.

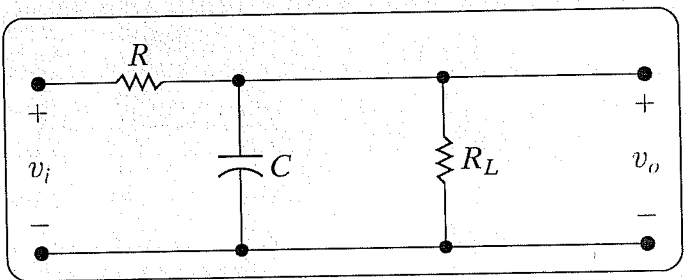
b) At what frequency will the magnitude of $H(j\omega)$ be maximum?

c) What is the maximum value of the magnitude of $H(j\omega)$?

d) At what frequency will the magnitude of $H(j\omega)$ equal its maximum value divided by $\sqrt{2}$?

e) Assume a resistance of 10 $k\Omega$ is added in parallel with the 100 nF capacitor in the circuit in Fig. P14.4. Find ω_c, $H(j0)$, $H(j\omega_c)$, $H(j0.1\omega_c)$, and $H(j10\omega_c)$.

Figure P14.5



P 14.5 **[a]** Let $Z = \dfrac{R_L(1/sC)}{R_L + 1/sC} = \dfrac{R_L}{R_LCs + 1}$

Then $H(s) = \dfrac{Z}{Z + R}$

$$= \frac{R_L}{RR_LCs + R + R_L}$$

$$= \frac{(1/RC)}{s + \left(\dfrac{R + R_L}{RR_LC}\right)}$$

[b] $|H(j\omega)| = \dfrac{(1/RC)}{\sqrt{\omega^2 + [(R + R_L)/RR_LC]^2}}$

$|H(j\omega)|$ is maximum at $\omega = 0$

[c] $|H(j\omega)|_{\text{max}} = \dfrac{R_L}{R + R_L}$

[d] $|H(j\omega_c)| = \dfrac{R_L}{\sqrt{2}(R + R_L)} = \dfrac{(1/RC)}{\sqrt{\omega_c^2 + [(R + R_L)/RR_LC]^2}}$

$\therefore \quad \omega_c = \dfrac{R + R_L}{RR_LC} = \dfrac{1}{RC}(1 + (R/R_L))$

[e] $\omega_c = \dfrac{1}{(10^3)(10^{-7})}[1 + (10^3/10^4)] = 10{,}000(1 + 0.1) = 11{,}000$ rad/s

$H(j0) = \dfrac{10{,}000}{11{,}000} = 0.9091\underline{/0^\circ}$

$$H(j\omega_c) = \frac{10{,}000}{11{,}000 + j11{,}000} = 0.6428\underline{/-45^\circ}$$

$$H(j0.1\omega_c) = \frac{10{,}000}{11{,}000 + j1100} = 0.9046\underline{/-5.71^\circ}$$

$$H(j10\omega_c) = \frac{10{,}000}{11{,}000 + j110{,}000} = 0.0905\underline{/-84.29^\circ}$$

14.6 Use a 0.5 μF capacitor to design a low-pass passive filter with a cutoff frequency of 50 krad/s.

P

a) Specify the cutoff frequency in hertz.

b) Specify the value of the filter resistor.

c) Assume the cutoff frequency cannot increase by more than 5%. What is the smallest value of load resistance that can be connected across the output terminals of the filter?

d) If the resistor found in (c) is connected across the output terminals, what is the magnitude of $H(j\omega)$ when $\omega = 0$?

P 14.6 [a] $f_c = \frac{\omega_c}{2\pi} = \frac{50,000}{2\pi} = \frac{50}{2\pi} \times 10^3 = 7957.75\,\text{Hz}$

[b] $\frac{1}{RC} = 50 \times 10^3$

$$R = \frac{1}{(50 \times 10^3)(0.5 \times 10^{-6})} = 40\,\Omega$$

[c] $\omega_c = \frac{1}{RC}\left(1 + \frac{R}{R_L}\right)$

$$\therefore \quad \frac{R}{R_L} = 0.05 \qquad \therefore \quad R_L = 20R = 800\,\Omega$$

[d] $H(j0) = \frac{R_L}{R + R_L} = \frac{800}{840} = 0.9524$

14.9 Using a 100 nF capacitor, design a high-pass passive filter with a cutoff frequency of 300 Hz.

P

a) Specify the value of R in kilohms.

b) A 47 kΩ resistor is connected across the output terminals of the filter. What is the cutoff frequency, in hertz, of the loaded filter?

P 14.9 **[a]** $\omega_c = \frac{1}{RC} = 2\pi(300) = 600\pi$ rad/s

$$\therefore \quad R = \frac{1}{\omega_c C} = \frac{1}{(600\pi)(100 \times 10^{-9})} = 5305.16\,\Omega$$

[b] $R_e = 5305.16 \| 47{,}000 = 4767.08\,\Omega$

$$\omega_c = \frac{1}{R_e C} = \frac{1}{(4767.08)(100 \times 10^{-9})} = 2097.7 \text{ rad/s}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{2097.7}{2\pi} = 333.86\,\text{Hz}$$

14.14 Use a 5 nF capacitor to design a series *RLC* bandpass filter, as shown at the top of Fig. 14.27. The center frequency of the filter is 8 kHz, and the quality factor is 2.

a) Specify the values of *R* and *L*.

b) What is the lower cutoff frequency in kilohertz?

c) What is the upper cutoff frequency in kilohertz?

d) What is the bandwidth of the filter in kilohertz?

P 14.14 **[a]** $\omega_o^2 = \frac{1}{LC}$ so $L = \frac{1}{[8000(2\pi)]^2(5 \times 10^{-9})} = 79.16\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{8000(2\pi)(79.16 \times 10^{-3})}{2} = 1.99\,\text{k}\Omega$$

[b] $f_{c1} = 8000\left[-\frac{1}{4} + \sqrt{1 + \frac{1}{16}}\right] = 6.25\,\text{kHz}$

[c] $f_{c2} = 8000\left[\frac{1}{4} + \sqrt{1 + \frac{1}{16}}\right] = 10.25\,\text{kHz}$

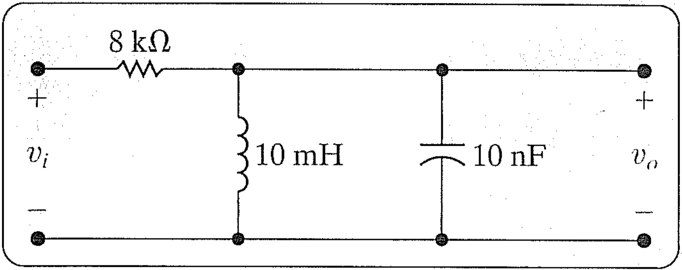
[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{8000}{2} = 4\,\text{kHz}$$

14.15 For the bandpass filter shown in Fig. P14.15, find (a) ω_o, (b) f_o, (c) Q, (d) ω_{c1}, (e) f_{c1}, (f) ω_{c2}, (g) f_{c2}, and (h) β.

Figure P14.15



P 14.15 **[a]** $\omega_o^2 = \frac{1}{LC} = \frac{1}{(10\times 10^{-3})(10\times 10^{-9})} = 10^{10}$

$$\omega_o = 10^5\text{ rad/s} = 100\,\text{krad/s}$$

[b] $f_o = \frac{\omega_o}{2\pi} = \frac{10^5}{2\pi} = 15.92\,\text{kHz}$

[c] $Q = \omega_o RC = (100\times 10^3)(8000)(10\times 10^{-9}) = 8$

[d] $\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1+\left(\frac{1}{2Q}\right)^2}\right] = 10^5\left[-\frac{1}{16} + \sqrt{1+\frac{1}{256}}\right] = 93.95\,\text{krad/s}$

[e] $\therefore \quad f_{c1} = \frac{\omega_{c1}}{2\pi} = 14.96\,\text{kHz}$

[f] $\omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 10^5 \left[\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 106.45\,\text{krad/s}$

[g] $\therefore \quad f_{c2} = \frac{\omega_{c2}}{2\pi} = 16.94\,\text{kHz}$

[h] $\beta = \frac{\omega_o}{Q} = \frac{10^5}{8} = 12.5\,\text{krad/s or } 1.99\,\text{kHz}$