

AMERICAN UNIVERSITY OF BEIRUT

Math 211 - Summer 2010

CHECKPOINT -1-

Name: Solution



Check whether the following are logically equivalent. Justify your answer.

1. $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$.

2. $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$.

$$\begin{aligned} 1) \quad \neg p \rightarrow (q \rightarrow r) &\equiv \neg(\neg p) \vee (q \rightarrow r) \equiv p \vee (\neg q \vee r) \equiv p \vee \neg q \vee r \\ q \rightarrow (p \vee r) &\equiv \neg q \vee (p \vee r) \equiv \neg q \vee p \vee r \end{aligned}$$

Since $p \vee \neg q \vee r \equiv \neg q \vee p \vee r$, the above compound propositions are logically equivalent.

2) Take p as false, q as false, r as true and s as false

$$\begin{array}{l} p \rightarrow q \text{ is true} \\ r \rightarrow s \text{ is false} \end{array} \quad \text{so } (p \rightarrow q) \rightarrow (r \rightarrow s) \text{ is false}$$

$$\begin{array}{l} p \rightarrow r \text{ is true} \\ q \rightarrow s \text{ is true} \end{array} \quad \text{so } (p \rightarrow r) \rightarrow (q \rightarrow s) \text{ is true.}$$

So for the same truth values of p, q, r and s , the statements do not have the same truth value. They are not logically equivalent.