FINAL EXAM.; MATH 211

FALL 2001, FEb. 3; 8:00 A.M.-10:00 A.M.

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Instructions:

- Make sure to write your NAME, ID NUMBER, and SECTION NUMBER (or CLASS MEETING TIME) on your Examination book.
- Write your answers on the colored EXAMINATION BOOKLET; reserve an independent page to each independent question 1, 2, $\cdots$, 9.
- The examination consists of 9 independent questions each of which consists of partial questions.
- The grade on each question is placed next to the question.
- The TOTAL GRADE is 100.
- **N**, **P**, and **Z** denote respectively the sets of natural numbers, positive integers, and integers.
- GOOD LUCK.

4. Let the function $f : \mathbf{R} \times \mathbf{R} \to \mathbf{R} \times \mathbf{R}$ be defined by

$$f(x, y) = (2x^3 + 3y^3, 3x^3 - 2y^3)$$

(a) Show that f is a one-to-one correspondence. (6 points)

(b) Find the inverse function of f. (2 points)

5. For $m, n \in \mathbf{N}$ define $m \sim n$ if $m^2 - n^2$ is a multiple of 5.

(a) Show that $\sim$ is an equivalence relation. (3 points)

(b) Describe the equivalence classes. How many equivalence classes are there? Justify your answer. (7 points)

6. (a) Let R_1 and R_2 be relations on a set S. Show that $(R_1 \cap R_2)^{\leftharpoondown} = R_1^{\leftharpoondown} \cap (R_2)^{\leftharpoondown}$. (5 points)

(b) Let R_1 and R_2 be relations on a set S. Must R_1 and R_2 be transitive if $R_1 \cup R_2$ is transitive? Justify your answer: If yes prove the statement, otherwise give a counter-example. (5 points)

7. (a) Let $f : S \to T$ and $g : T \to U$ be invertible functions. Show that $g \circ f$ is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. (5 points)

(b) Prove that the sequence $\{1, 5, \cdots, 5^n, \cdots\}$ has infinitely many terms such that the difference between any two of them is divisible by 11. Name your method of proof. (5 points)

8. (a) Show that $n^{1000} = O(n!)$. (5 points)

(b) Consider $\mathbf{N} \times \mathbf{N}$ and define the equivalence relation $(m, n) \sim (p, q)$ if $m = p$. Determine the partition and the natural function ν of $\mathbf{N} \times \mathbf{N}$ induced by $\sim$. (5 points)

9. Answer by true (T) or false (F) the following TEN questions.

(2pts. each)

(i) If R is a relation on a set S, then $R \cap R^{-}$ is symmetric.

(ii) $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$.

(iii) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$.

(iv) $\chi_E(n) = \lceil n/2 \rceil - \lfloor n/2 \rfloor$, $n \in \mathbf{Z}$, where E is the set of even integers.

(v) The equation $x +_5 3 = x *_5 2$ has two solutions.

(vi) There is only one equivalence class for the equivalence relation in $\mathbf{Z}$ defined by $m \equiv n \pmod 1$.

(vii) $\log_{10} n = \Theta(\log_2 n)$.

(viii) $(3n)! = O(2n!)$.

(ix) If $s_{2n} = 2s_n + 3 + 7n$, $n \in \mathbf{P}$, then $s_{2^m} = O(2^m)$.

(x) MATH 211 was FUN.