Time: 2 hours	MATHEMATICS 211	Name ----------------------
January 23, 2003	First Semester 2003-04	ID # ------------------
Prof. H. Abu-Khuzam	Final Examination	
Ms. N. Fleihan		

Circle your section number below:

#1. (12:00 F) #2 (8:00 F) #3 (1:00 F)

#4 (10:00 F) #5 (9:00 F) #6 (2:00 F)

PROBLEM	GRADE
PART I	
1	------- / 7
2	------- / 7
3	------- / 7
4.	------- / 6
5	------- / 7.
6	------- / 8
7	------- / 10
8	------- / 24
PART II	
9-16	------- / 24
TOTAL	------- / 100

Throughout, N denotes the set of natural numbers, P denotes the set of positive integers, Z denotes the set of integers, and Q denotes the set of rational numbers.

PART I. Solve each of the following problems (Problems 1, 2, 3 , 4, 5, 6,7, and 8) in the space provided for each problem.[76 points]

1. Use induction to show that

   $5^n \equiv 4n+1 \pmod 8$ *for every* $n \in P$

   [7 points]

2. Verify the following logical implication using a truth table

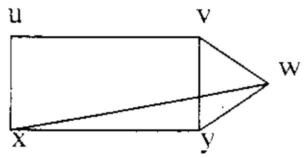
   $[(p \to q) \wedge \neg q] \Rightarrow \neg p$

   [7 points]

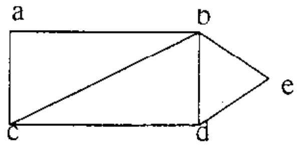
3. How many ways are possible to place 10 identical letters into 4 mailboxes such that the first box remains empty and each of the other mailboxes must receive at least 2 letters?

[7 points]

4. Find the degree sequence of each of the following graphs. Are these graphs isomorphic?



Graph G



Graph H

[6 points]

5. Define the relation ~ on $Z\setminus\{0\}$ by

$$m \sim n \;\; if \; mn > 0$$

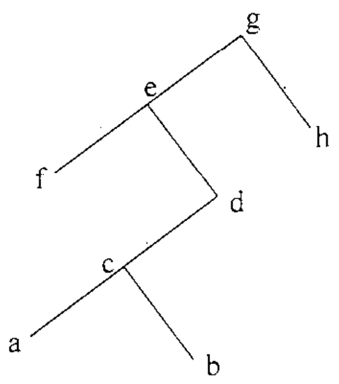
(a) Show that ~ is an equivalence relation

[5 points]

(b) Find the equivalence classes of this relation.

[2 points]

6. Let S={a,b,c,d,e,f,g,h}. (S, $\leq$) is a poset with the following Hasse diagram:



(i) What are the maximal and minimal elements of S?

[3 points]

(ii) Let T={a,b,d,f} be a subset of S. Does each of the following exist? If yes, find it
max(T), lub(T), and glb(T)

[3 points]

(iii) Is S a lattice? Explain.

[2 points]

7. (a) Let $f: R \times R \rightarrow R \times R$ be the function defined by $f(x, y) = (x + y, x - y)$. This function is invertible (Do not prove). Show that the inverse function is given by

$$f^{-1}(a,b) = (\frac{a+b}{2}, \frac{a-b}{2})$$

[5 points]

(b) Show that $3n^2 + 14n + n\log_2 n = \Theta(n^2)$

[5 points]

8. **Prove** or **Disprove** the following:

(a) If $f: X \rightarrow Y$ is a function and A, B are subsets of X, then $f(A \setminus B) = f(A) \setminus f(B)$

[6 points]

(b) If $f: S \rightarrow S$ and $g: S \rightarrow S$ are functions such that the composition $g \circ f$ is one-to-one, then g is one-to-one.

[6 points]

(c) If A is a countable subset of the set of real numbers R, then R\A is uncountable
[6 points]

(d) The set $S = \{\sqrt{n} + 5 : \; n \in P\}$ is countable
[6 points]

PART II. Circle the correct answer in each of the following problems (Problems 9,10,11,12,13,14,15,and 16). [3 points for each correct answer, and –1/2 point penalty for each wrong answer]. [24 points]

9. Let S={A,B,C,D} and T={1,2,3,4,5}. The number of license plates consisting of three letters from S followed by two numbers from T (repetition is allowed) such that the letter A appears at least once is:

(a) 480
(b) 1600
(c) 925
(d) 700
(e) none of the above

[3 points]

10. One card is to be drawn at random from a deck of 52 cards. Find the probability of selecting a queen or a heart.

(a) $^{16}/_{52}$
(b) $^{18}/_{52}$
(c) $^{12}/_{52}$
(d) $^{17}/_{52}$
(e) none of the above

[3 points]

11. Six airline companies have submitted applications for operating over a new international route. Only two of the companies will be awarded permits to operate over the route. The number of different sets of airlines that could be selected is equal to:

(a) 20
(b) 15
(c) 10
(d) 30
(e) none of the above

[3 points]

12. An urn contains 3 red balls and 4 black balls. A set of three balls is removed at random from the urn without replacement. What is the probability that the selected balls are one red and two black?

(a) $^{3}/_{35}$
(b) $^{12}/_{35}$
(c) $^{18}/_{35}$
(d) $^{6}/_{35}$
(e) none of the above

[3 points]

13. Which one of the following statements is FALSE?

(a) $Z \times Z$ has the same size as $N \times Q$
(b) The set of irrational numbers is uncountable
(c) If $\Sigma = \{a, b\}$, then the set Σ^* of all words using letters from Σ is countable
(d) The set of points in the plane with integer coordinates is countable
(e) An infinite subset of an uncountable set is uncountable

[3 points]

14. Which one of the following statements is FALSE?

(a) N with the usual order is well ordered
(b) A chain cannot have more than one minimal element
(c) If S and T are two chains, Then S×T with the filing order is a chain
(d) N×N with the product order is a chain
(e) The interval (0,1) in (R, $\leq$) has no maximum.

[3 points]

15. Let S={1,2,3,4}, T={a,b,c,d,e}.Then the number of functions $f : S \to T$ such that $f(1) = a$ *and* $f(3) = b$ is equal to:

(a) 625
(b) 125
(c) 25
(d) 20
(e) none of the above

[3 points]

16. The sequence s_n is defined recursively by
$s_0=0$, $s_1=3$ and $s_n = -s_{n-1} + 2s_{n-2}$ *for* $n \geq 2$. Then

(a) $s_{20} = 2^{20} + 1$
(b) $s_{20} = 2^{20} - 1$
(c) $s_{20} = -2^{20} + 1$
(d) $s_{20} = -2^{20} - 1$
(e) none of the above

[3 points]