Math 211

Final (Fall 2004)   Time 2hrs

Name : ____________________

ID # : ____________________

Circle your problem solving section number below :

Section 1   12:00 F   Section 2   1:00 F

Problem	Grade
1 (a, b, c) / 15	
1 (d, e) / 10	
1 (f, g) / 10	
2 / 15	
3 / 10	
4/ 14	
5, 6 / 16	
7 / 10	
TOTAL	/ 100

1- Answer appropriately the following:

(5 pts) a) Is the union of two transitive relations a transitive relation?

(5 pts) b) Is the function $f: IR \to IR$ defined by $f(x) = x^3 + 9$ a one to one function?

(5 pts) c) Is $[(p \wedge q) \vee (p \wedge \neg q)]$ logically equivalent to p ?

(5 pts) **d)** Let $f: X \to Y$ be a function. If $f^{-1}(B) = \{x \in X \text{ such that } f(x) \in B\}$, is $f^{-1}(f(A)) = A$ for any subset A of X?

(5 pts) **e)** Let A, B and C be sets. Is $(A - C) \cap (C - B) = \Phi$?

(5 pts) f) Let $f(n) = \left\lceil \frac{n}{2} \right\rceil - \left\lfloor \frac{n}{2} \right\rfloor$ for $n \in Z$, Is $\operatorname{Im} f = \{0,1\}$?

(5 pts) g) Is it true that "if 3 divides the product *m.n,* then 3 divides *m* or *n*" ?

(15 pts) 2- Let $\sum = \{0, 1\}$ and let s_n denote the number of strings of length n $(n \geq 1)$ with an even number of 0's.

a) Calculate s_1 , s_2 , s_3 and s_4 .

b) Let Σ_n be the set of strings of length n on Σ Give the recursive definition of Σ_n , then prove that $s_n = 2s_{n-1}$.

c) Solve the recurrence relation

(10 pts) 3- Let $s_n = (2n+1) + (2n+3) + (2n+5) + \ldots\ldots\ldots\ldots + (4n-1)$ for $n \geq 1$.

a) Give the expressions and values for s_n, $n = 1, 2, 3$

n	Expression of s_n	Value of s_n
1		
2		
3		

b) Use mathematical induction to prove that $s_n = 3n^2$

(8 pts) 4- Let $S = \{1, 2, 3, 4, \ldots\ldots, 2000\}$.

a) What is the number of integers in S that are divisible by 9, 11 or 13.?

(6 pts) **b)** What is the number of integers in S that are divisible by 9, 11or 13 but not by both 9 and 11 ?

(8 pts) 5- Nine students enter one elevator in the basement of College Hall. Each of these students will exit at either floor 1, 2, 3, 4, or 5. In how many ways can this happen?

(8 pts) 6- Show that if $s(n) = \theta(n^5)$ and $t(n) = \theta(n^2)$ then $\frac{s(n)}{t(n)} = \theta(n^3)$

(10 pts) 7- Define the relation $\leq$ on $IR \times IR$ by :

$$(x, y) \leq (z, w) \quad \text{if} \quad x^2 + y^2 < z^2 + w^2 \quad \text{or} \quad (x, y) = (z, w)$$

Show that $(IR \times IR, \leq)$ is a poset.