Math 211 Final EXAMINATION
Time: 130 minutes

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Name ....

I.D ....

	GRADES	MAX
#1:		10
#2,3:		12
#4,5:		12
#6,7:		12
#8,9:		14
#10,11:		12
#12,13:		15
#14,15:		13
TOTAL:		

1. (10%) True –False
(**Penality: 2 wrong answers cancel 1 right answer**)

.... (a) $f^{\leftarrow}(A \cup B \cup C) = f^{\leftarrow}(A) \cup f^{\leftarrow}(B) \cup f^{\leftarrow}(C)$

.... (b) $f^{\leftarrow}(A \cap B) = f^{\leftarrow}(A) \cap f^{\leftarrow}(B)$

..............(c) $f(A \cap B) = f(A) \cap f(B)$

.... (d) $f(f^{\leftarrow}(B)) = B \cap \mathrm{Im}(f)$ (for every subset B of codomain(f))

.... (e) R is transitive $\Longleftrightarrow$ $R^{\leftarrow}$ is transitive

............(f) R_1 & R_2 are transitive $\Rightarrow$ $R_1 \cup R_2$ is transitive

.... (g) $g \circ f$ *is bijective* $\Longleftrightarrow$ *f is 1-1 and g is onto*

............(h) *f & g are both* 1-1 $\Longleftrightarrow$ $(f,g): X \times Y \to A \times B$ *is* 1−1
(where $(f,g)(x,y) = (f(x), g(y))$) and $f:X \to A$ & $g:Y \to B$ are functions)

..................(i) f o f o f has an inverse $\Longleftrightarrow$ f has an inverse. ($f: X \to X$ *is a function*)

..................(j) $f: X \to X$ *is onto* $\Longleftrightarrow$ f *is bijective* $\Longleftrightarrow$ *f has an inverse*

2. **(6 %)** Disprove **<u>any 2</u>** (false) parts of problem 1. (**<u>Specify ALL DETAILS</u>**: *What is f, A, B , ..etc ?*)

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3. **(6 %)** Prove **<u>any 2</u>** (true) parts of problem 1.

4. (6 %) Simplify $(B \cup (A \cap C)) \cap (D \cup C) \cap A$ (Hint: You may use Boolean Algebra)

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5. (6%) Find an explicit formula for s_n if $s_0 = 1$, $s_1 = 12$, **and** $s_n = 6s_{n-1} - 9s_{n-2}$ **for** $n \geq 2$

6. (6%) Recursively, define $a_0 = 1, a_1 = 5, a_2 = 10$ **and** $a_{n+1} = a_{n-1} + 75a_{n-2}$ **for** $n \geq 2$.

Find the best possible integer **r** such that $a_n \geq c\mathbf{r}^n$ **for** $n \in \mathbf{N}$ (Forget about c)

(Hint: Proceed as in Induction: Assume it is true for ……. , prove……..)

7. (6 %) Let $f: Z_{60} \rightarrow Z_5 \times Z_3 \times Z_4$ be given by $f[x]_{60} = ([11x]_5, [11x]_3, [11x]_4)$.

Prove that f is a **Well-defined** **Bijective** function

8. (8 %) In how many ways can we distribute **50 identical** scholarships over **4** universities $\{U_1,......,U_4\}$

(a) If U_1 gets at most **5** scholarships. (**Hint:** Take the complement, i.e, at least 6 ...)

(b) If each university gets at most (**max**) **20** scholarships. (**Hint:** Take the complement & use Inclusion- Exclusion.)

(BOX YOUR FINAL ANSWERS)

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9. (6%) How many words can be formed by permuting **SUCCESSLUNUS**

(a) with no (further) restrictions (You have 12 letters: 4S, 3U, 2C, 1E, 1L, 1N)

(b) such your word should NOT end with NUS.

10. (8 %) **True –False** **(Penality: 2 wrong answers will cancel 1 right answer)**

Reminder: A poset is called ***directed*** if each 2 elements have an upper bound and a lower bound.

-------(a) Every chain is a directed poset

-------(b) Every directed poset is a chain

-------(c) Every minimal element of a directed poset is a minimum element.

------- (d) The poset $([50, \infty), \leq)$ is well-ordered .

------- (e) The set of irrational numbers **R – Q** can be well-ordered.

--------(f) The filing product of normal posets is a normal poset.

------- (f) The cartesian product of chains is a chain.

------- (g) If a finite poset has a unique minimal element M, then M is a minimum .

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11. (4 %) Disprove (by a counter example) **Any 2** (false) parts of problem 10 above.

12. (7 %) Suppose $s(n) = n^6 + 3n^4 + O(n^3)$ and $t(n) = n^8 + O(n^3)$,

Find the smallest possible k such that $s(n)\,t(n) = n^{14} + 3n^{12} + O(n^k)$.

(**Be Careful**: When you expand, you must have **six terms**)

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13. (8 %) **True- False**

(**Penality**: **2 wrong answers cancel 1 right answer**)

Reminder: Infinite countable sets are bijective

.... (a) $\mathbf{Q^3}$ **and** $(\mathbf{Q^3 - Z^3})$ **are bijective sets**

.... (b) Any 100 uncountable sets are bijective

.... (c) **INF-SEQ{0,1} is an uncountable set** (See Notation below)

.... (d) **FIN-SEQ(Z) is an uncountable set**

.... (e) $\mathbf{R^3}$ **is bijective to** $\wp(\mathbf{R^3})$

.... (f) **R is NOT bijective to** $\wp(\mathbf{N})$

............(g) **R is bijective to** $\Omega(\mathbf{R})$ **= All finite subsets of R**

----------- (h) the set of all polynomials with coefficients in Q is a countable set.

Notation: N = The set of natural numbers; Z= The set of integers

Q = The set of rational numbers; R= The set of real numbers

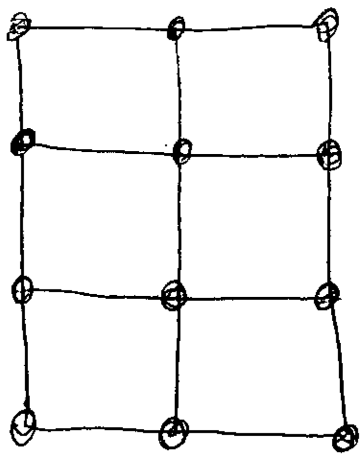
$M_{n\times n}(S)$ = ***The set of*** $n\times n$ ***matrices with entries in S***

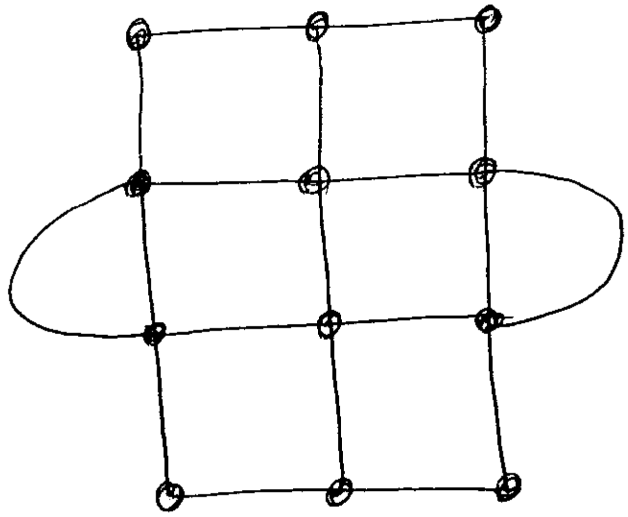
$\wp(S)$ = **The power set of S**

INF-SEQ (S) = The set of all infinite sequences with elements in S

FIN-SEQ (S) = The set of all finite sequences with elements in S

14. (5%) Find (if possible) an **Euler path** of the following graphs.
(To show your path, **put a, b ,c, ...** on the edges of your path)





15. (8%) Find 2 non-isomorphic **minimal spanning trees** of the following weighted graph.
Draw the trees separately & **clearly** (after ....). (Use any method)