FINAL EXAM.; MATH 211

SPRING 2001: 8:00A.M.-10:00 A.M.

Name: Signature:

Student number:

Circle Your Section Number : 1 2 3 4

Instructions:

- Let **R**, **N**, **P**, and **Z** denote respectively the sets of reals, natural numbers, positive integers, and integers.
- The examination consists of 8 independent questions each of which consists of partial questions.
- You are expected to justify all your answers except in Question 8.

TOTAL GRADE/100:

1. (a) Use mathematical induction to prove that $1.2 + 2.3 + \cdots n(n+1)$ $= n(n+1)(n+2)/3$. (6 points)

(b) Show that $n^2 \not\equiv 2 \pmod 6$, $n \in \mathbf{Z}$. State whether your proof is direct or indirect? (6 points)

2. Let $A = \mathbf{R} \times (-\pi, \pi]$, $B = (\mathbf{R} \times \mathbf{R}) \setminus \{(0,0)\}$, and $f : A \to B$ be the function defined by $f(x, y) = (e^x \cos y, e^x \sin y)$.

(a) Graph the sets A and B in a cartesian system. (3 points)

(b) Show that f is a one-to-one correspondence. (6 points)

(c) Find the inverse function of f. (2 points)

3. (a) There are four large groups of people, each with 1000 members. Any two of these groups have 100 members in common. Any three of these groups have 10 members in common. And there is 1 person common to all four groups. All together, how many people are in these groups? (6 points)

(b) There are 10 questions in a Discrete Mathematics final examination. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points? (6 points)

(c) How many different words can be made from "SUCCESS" given that the first and last letters must both be "S"? (6 points)

4. (a) One hundred people are to be divided in ten discussion groups with ten people in each group? In how ways can this be done if the subject of discussion is the same to all? In how ways can this be done if each group has a different subject of discussion than the rest? Justify your answers.

(6 points)

(b) Twelve people join hands for a Dabke dance? In how many ways can they do this? (6 points)

5. Let $\Sigma = \{0, 1, 2\}$, and let s_n, $n \in \mathbf{N}$, be the number of words of length n not containing the string 11.

(a) Find a recursive definition of s_n. (6 points)

(b) Give an explicit formula for s_n dependent only on n. (6 points)

6. (a) Show by the binomial theorem that

$$2^n = \sum_{k=0}^{n} \binom{n}{k}.$$

Use a counting argument to conclude that 2^n is the number of all subsets of a set of n elements. Hint: count the sets of sizes $1, 2, \cdots, n$. (6 points)

(b) Suppose that the sequence s_n satisfies the recurrence relation $s_{2n} = 2s_n + n^3$, $n \in \mathbf{P}$. Give a formula for s_{2^m} and an argument that your formula is correct. Show that $s_n = O(n^3)$ for values $n = 2^m$. (6 points)

7. On the set $\mathbf{N} \times \mathbf{N}$ define $(m, n) \sim (k, l)$ if $m^2 - l^2 = k^2 - n^2$.

(a) Show that $\sim$ is an equivalence relation. (3 points)

(b) Describe the equivalence classes and represent their graphs in the cartesian product of $\mathbf{N} \times \mathbf{N}$. (4 points)

(c) Is the function $f : [\mathbf{N} \times \mathbf{N}]_{\sim} \to \mathbf{R}$ defined by $f([(m, n)] = (m^4 + 2m^2n^2 + n^4)/(m^2 + n^2 + 1)$ well-defined? Justify your answer. (4 points)

8. Answer by true (**T**) or false (**F**) the following statements: (2 points each)

(a) The coefficient of the $x^2y^3z^3$ in $(3 + x + y + z)^9$ is $9!/(2!2!2!)$.

(b) $1^n + 2^n + \cdots + n^n = O(n^{n+1})$.

(c) $1 + 1/2 + \cdots + 1/n = \Theta(\ln n)$.

(d) $[(p \vee q) \wedge (p - r) \wedge (q \to r)] \Leftrightarrow r$.

(e) If R_1 and R_2 are transitive relations on a set S, then $R_1 \cup R_2$ is transitive.

(f) The equation $3 *_7 x^2 - 6$ has exactly two solutions.