Math 211

Final (Spring 2005) Time 2hrs

Name : ____________________ ID # : ______________________

Circle your problem solving section number below :

Miss Jaafar Section 1 11:00 F Section 2 1:00 F

Miss Fuleihan Section 3 1:00 F

Problem	Grade
1 / 16	
2 & 3 / 12	
4 / 12	
5 / 8	
6 & 7 / 16	
8/ 24	
9 / 12	
TOTAL	/ 100

(16 pts)

I- Consider the set $S = \{2, 3, 4, 6, 8, 24\}$. Define on S the relation " a | b if a divides b".

a. Show that (S , |) is a poset.

b. Draw the Hasse diagram of (S , |)

c. Find the maximal and the minimal elements of (S , |) if they exist.

d. Is (S , |) a lattice?

(4 pts) 2- Show that $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology

(8 pts) 3- Use mathematical induction to show that

$$1^2 + 3^2 + 5^2 + \dots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

(12 pts)

4- Consider the function $f : IR \times IR \longrightarrow IR \times IR$ defined by
$f(x,y) = (x + 2y\,,\,2x - y)$.

a. Show that f is one to one.

b. Show that f is onto

c. Determine the inverse function of f.

(8 pts)

5- On the set $IN \times IN$ define the relation $(m, n) \approx (p, q)$ if $m - 3q = p - 3n$

a. Show that $\approx$ is an equivalence relation

b. Describe the equivalence classes

(12 pts) 6- How many integers from 1 through 1000 are

a. divisible by 3 or divisible by 7?

b. divisible by 3 or divisible by 7 but not by both?

c. not divisible by 3 or divisible by 7?

(4 pts) 7- Show that if R is a relation on a set S, then $R \cap R^{-1}$ is a symmetric relation on S.

(24 pts)

8- Prove or disprove

a. $A \cap (B \cup C) = (A \cap B) \cup C$

b. $n^3 + 2n^2 + 3n + 1 = \theta(n^3)$

c. $f(f^{-1}(B)) = B$

Prove or disprove

d. $f(S \cup T) = f(S) \cup f(T)$

e. $(2^{2n} - 1)$ is divisible by 3 for all $n \in IN$.

f. $n^3 \equiv 2 \pmod 4$ $n \in Z$

(12 pts) 9- Prove that if n is an integer and $n^3 + 5$ is odd, then n is even using

a. a direct proof

b. an indirect proof

c. a proof by contradiction