American University of Beirut

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NAME:-- ID.----------------------------------

I. 1. Evaluate the sum: $\binom{n}{0}+3\binom{n}{1}+3^2\binom{n}{2}+3^3\binom{n}{3}+\dots\dots\dots+3^n\binom{n}{n}$.

2.Find the coefficient of $x^3y^2z^6$ in the multinomial expansion : $(x+2y-3z^2)^8$.

II. 1.How many arrangements of the letters of the word MISSISSIPPI exist in which no two I's are next to each other?

Hint:First find the number of arrangements of the letters MSSSSPP.*Then distribute the four* I's *as required?*

2. M.S. , the formula one driver has to complete 30 laps in a particular race , during which he will make exactly four pit stops. A stop should happen only at a complete lap , and there must be at least one lap just before and just after each pit stop.In how many ways can M.S. choose his stops?

III. True or false? In each case give a reason.

1. $10^n = O(5^n)$.

2. $(\frac{n\pi}{2})^2 = \Theta(n^2)$.

IV.1. In how many ways can 9 prisoners be placed in three distinct cells A , B and C if cell A must take 2 , cell B must take 3 , and cell C must take the others?

2. It was observed at a conference that :

100 persons can speak English ,
90 persons can speak French,
60 persons can speak Russian,
30 persons can speak English and French,
20 persons can speak English and Russian,
10 persons can speak French and Russian,
5 persons can speak all the three languages.

How many persons were there at the conference?

V. Let

$f: P \times P \to P$ *be defined by*: $f(m,n) = mn \; ; \forall\, m, n \in P.$

1. *Find* $f^{\leftarrow}(2), f^{\leftarrow}(2^2)$, *and* $f^{\leftarrow}(2^3)$.
2. *Find* $f^{\leftarrow}(p^r)$, *where* $r \in P$ *and* p *is prime.*
3. *Prove or disprove*: i) f *is* 1 − 1.
   ii) f *is onto.*

VI.

Let $\Sigma = \{a, b\}$ *be an alphabet. Answer the following questions*:

a. *What is the number of elements of the set* Σ^6 *of all six letter words of* Σ?

b. *What is the number of elements of the set* $\Sigma^{\oplus}$ *of all words of at most six letters of* Σ?

c. *Define on* Σ^6 *the relation* R *by*: wRw' *iff* w *and* w' *have the same first letter but different last letter. Is R reflexive? Is it antireflexive? Is it symmetric? Is it transitive? Give reasons.*

d. *Show that the set of all strings of a's and b's is uncountable.*

e. *Show that the set of all words having exactly one a is countable.*

VII. 1. Give an explicit formula for s_n if: $s_0 = 5, s_1 = 8$ *and* $s_n = 4s_{n-1} - 4s_{n-2}\; ; n \geq 2.$

2. Give a recursive relation for the sequence:

$$(3, 3^2, 3^{3^2-1}, 3^{(3^{3^2-1})-1}, \ldots\ldots\ldots\ldots)$$

VIII. Use mathematical induction to prove that :

1. $15 - 4n + 5^n$ *is divisible by* 16; $\forall\, n \in P$.

2. $$\frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots\ldots + \frac{1}{2n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots + \frac{1}{2n-1} - \frac{1}{2n}\; ; \forall\, n \in P.$$

IX. 1. Find a 1-1 correspondence between the intervals of real numbers (0,1) and (5,9) .

2. Given the set D={1,2,3,….,9}.
   a) How many four digit numbers are possible using four distinct digits of D?
   b) How many four digit numbers are possible,if repetition is allowed?
   c) How many four digit numbers are possible if two distinct even and two distinct odd digits of D are used ?

GRADE DISTRIBUTION: **20 points for question VI. 10 points for each of the other eight questions.**