

AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Mathematics Department



MATH 211
FINAL EXAMINATION
SUMMER 2005
Closed Book, 2 HOURS



WRITE YOUR ANSWERS ON THE QUESTION SHEET

STUDENT NAME	
ID NUMBER	

Problem	Out of	Grade
1	10	
2	10	
3	10	
4	10	
5	20	
TOTAL	60	





1. (10 points)

(a) (2 points) Prove without using truth tables that the implication

$$p \wedge q \rightarrow p,$$

is a tautology.

(b) (3 points) Prove without using truth tables that the compound proposition

$$\neg p \wedge (p \vee q) \rightarrow q,$$

is a tautology.

(c) (2 points) Let $P(x)$ and $Q(x)$ be predicates on a universal set. Prove the equivalence :

$$(\forall x (P(x) \wedge Q(x))) \equiv (\forall x P(x)) \wedge (\forall x Q(x)).$$

(d) (3 points) Let $P(x)$ be a predicate on a universal set and R a proposition. Prove the equivalence :

$$(\forall x P(x)) \vee R \equiv (\forall x (P(x) \vee R)).$$

2. (10 points) Consider the function $f : A = \{-2, -1, 0, 1\} \rightarrow B = \{-1, 0, 1, 2, 3\}$, given by $f(x) = x^2 + 2x$.

(a) (3 points) Give the range (or image) of A under f . Is f on-to? One-to-one?

(b) (4 points) Consider the subset S of $B \times A$ defined by :

$$S = \{(b, a) | b = f(a)\}.$$

Give the elements of (the relation) S . Give your answer in the form of a table (with B horizontal and A vertical).

(c) (3 points) Is the relation S a function? Justify your answer.

