AMERICAN UNIVERSITY OF BEIRUT
Math 211- Spring 2007 **Drop Quiz 10, section 1, May 17, 2007**

Name: ……………………………………… AUB ID number: ……………………

Section: …….

Question 1: → BONUS / 0.5 point ~~(1 point)~~

How many subsets of a set of n elements contain more than one element?

Number of subsets containing 0 elements: 1

" " " " 1 element: n

Total number of subsets 2^n

Number of subsets containing more than one element: $2^n - n - 1$

Question 2: → BONUS / 1 point ~~(1.5 points)~~

How many positive integers between 100 and 999 inclusive: → There are 900 nbs in this range

a) Have distinct digits? $9 \times 9 \times 8$

b) Do not contain the same digit three times? $9 \times 10^2 - 9 = 891$

c) Begin with an odd digit? 5×10^2

d) Are divisible by 9? $100 \leq 9k \leq 999$

$11,1\ldots = \frac{100}{9} \leq k \leq 111$

$11 < k \leq 111$ → There are 100 such numbers.

Question 3: 2 points ~~(1 points)~~

Let $a_n = 2(-4)^n + 3$, for $n = 0, 1, 2, \ldots$

Is the sequence $\{a_n\}$ solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$?

$$
\begin{aligned}
-3a_{n-1} + 4a_{n-2} &= -3\left[2(-4)^{n-1} + 3\right] + 4\left[2(-4)^{n-2} + 3\right] \\
&= -6(-4)^{n-1} - 9 + 8(-4)^{n-2} + 12 \\
&= -6(-4)^{n-1} + 3 - 2(-4)^{n-1} \\
&= (-4)^{n-1}\left[-6 - 2\right] + 3 \\
&= 2(-4)^n + 3 \\
&= a_n
\end{aligned}
$$

$\therefore$ a_n is a solution.

Question 4: 4 points ~~(2.5 points)~~

a) Find the solution to $a_n = a_{n-4}$ with $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_3 = 4$.
(use back of the page)

$$a_n = \frac{5}{2} - \frac{1}{2}(-1)^n - \cos\left(n\frac{\pi}{2}\right) - \sin\left(n\frac{\pi}{2}\right)$$

b) What is the general form of the particular solution of the linear nonhomogeneous recurrence relation $a_n = a_{n-4} + n(3)^n$?

First guess $a_n^{(P)} = (En + F)3^n$ (no duplication)

accepted

c) What is the general form of the particular solution of the linear nonhomogeneous recurrence relation $a_n = a_{n-4} + n^2(-1)^n$?

First guess: $a_n^{(P)} = (En^2 + Fn + G)(-1)^n$

It duplicates one term in $a_n^{(H)}$ (see question a)

Corrected guess: $a_n^{(P)} = n(En^2 + Fn + G)(-1)^n$

Question 4

a) $\begin{cases} a_n = a_{n-4} & (1) \\ a_0 = 1, \quad a_1 = 2, \quad a_2 = 3, \quad a_3 = 4 \end{cases}$

$a_n = r^n$ solution of (1) $\iff$ r solution of $\underbrace{r^4 - 1 = 0}_{\text{characteristic equation}}$

$(r^2-1)(r^2+1) = 0$

$(r-1)(r+1)(r^2+1) = 0$

$r_1 = 1$, $r_2 = -1$, $r_3 = i$ $\quad r_4 = -i$

form $r_3 = \alpha + i\beta$ where $\alpha = 0$, $\beta = 1$

so $\rho = \sqrt{\alpha^2+\beta^2} = 1$

and $\theta = \frac{\pi}{2}$

General solution of (1):

$$a_n = A + B(-1)^n + C\cos\left(n\frac{\pi}{2}\right) + D\sin\left(n\frac{\pi}{2}\right)$$

But $\begin{cases} a_0 = 1 \\ a_1 = 2 \\ a_2 = 3 \\ a_3 = 4 \end{cases}$ so $\begin{cases} A + B + C = 1 & (a) \\ A - B + D = 2 & (b) \\ A + B - C = 3 & (c) \\ A - B - D = 4 & (d) \end{cases}$

(a) − (c) gives $2C = -2$ $\quad \underline{C = -1}$

(b) − (d) " $2D = -2$ $\quad \underline{D = -1}$

Substitute in (a) $A + B = 2$

(b) $A - B = 3$

$2A = 5$ $\quad \underline{A = 5/2}$, $\underline{B = -1/2}$

Solution: $\boxed{a_n = \frac{5}{2} - \frac{1}{2}(-1)^n - \cos\left(n\frac{\pi}{2}\right) - \sin\left(n\frac{\pi}{2}\right)}$