AMERICAN UNIVERSITY OF BEIRUT

Math 211- Spring 2007 **Drop Quiz 11, section 2, May 24, 2007**

Name: ……………………………………… AUB ID number: ……………………

Section: …….

Question 1: (1.5 points)

Suppose that there are nine students in a discrete math class at a small college.
Show that the class must have at least three male students or at least seven female students.

Proof by contradiction: Assume that the class does not have at least 3 male students or at least 7 female students.
Then the class contains at most 2 male students and at most 6 female students
(De Morgan's law)

So there would be $2+6=8$ students at most, contradicting the assumption that there are 9 students in the class.

Question 2: (2 points)

A club has 25 members.

a) How many ways are there to choose four members of the club to serve on an executive committee?

There are as many ways as we can form a subset with 4 elements chosen from the 25 elements of a set.

$$\binom{25}{4} = \frac{25!}{4!\,21!} = \frac{25.24.23.22}{4.3.2.1} = 12\,650$$

b) How many ways are there to choose a president, vice-president, secretary, and treasurer of the club, where no person can hold more than one office?

There are as many ways as we can form 4-permutations of a set of 25 elements:

$$P(25,4) = \frac{25!}{21!} = 25.24.23.22 = 303\,600$$

Question 3: (2.5 point)

How many positive integers between 100 and 999 inclusive are divisible by 3 or 4?

Note first that there are exactly 900 integers in this range

* Number of integers divisible by 3:

Since the last integer (999) is divisible by 3, and since every third number is divisible by 3, so the answer is $\frac{1}{3} \cdot 900 = 300$

* Number of integers divisible by 4:

Since the first integer (100) is divisible by 4, and since every fourth integer is divisible by for, then the answer is $\frac{1}{4} \cdot 900 = 225$

* Number of integers divisible by 3 and 4 (i.e. by 12)

(Note that 100 and 999 are not divisible by 12, neither is the total number of integers, 900.)

Such integer can be written as a multiple of 12: $12k$ ($k \in \mathbb{N}$)

k must satisfy:
$$100 \le 12k \le 999$$
$$\frac{100}{12} \le k \le \frac{999}{12}$$
$$\left\lceil \frac{100}{12} \right\rceil \le k \le \left\lfloor \frac{999}{12} \right\rfloor$$
$$9 \le k \le 83$$

The number of such integers is then: $83-9+1 = 75$

* By the Inclusion-Exclusion Principle, the answer is:

$$300 + 225 - 75 = 450$$