AMERICAN UNIVERSITY OF BEIRUT

Math 211- Spring 2007 — Drop Quiz 12, section 4, May 24, 2007

Name: ……………………………………… AUB ID number: ……………………

Section: …….

**Question 1:** (1.5 point)

In how many ways can a photographer at a wedding arrange six people in a row from a group of 10 people, where the bride is among these 10 people and must be in the picture?

We first place the bride in any of the 6 positions.
Then, from left to right in the remaining positions, we choose the other five people to be in the picture. This can be done in $9.8.7.6.5 = 15120$ ways $(= P(9,5))$
Therefore, the answer is: $6 \times 15120 = 90720$

**Question 2:** (2 points)

A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

a) How many socks must he take out to be sure that he has at least two socks of the same colour?

There are 2 colours (these are the pigeonhole, $k=2$)
We want to know the least number $N$ of socks (pigeons) that must be taken out to be sure that at least 2 socks are of the same colour.
By the pigeonhole principle: $N = k+1 = 2+1 = 3$

b) How many socks must he take out to be sure that he has at least two black socks?

In the worst case, he can select 12 brown socks before selecting any black sock. The next 2 socks will be black.
Therefore, the answer is: $12+2 = 14$

**Question 3:** (2.5 points)

Suppose that a department contains 10 men and 15 women.

a) How many ways are there to form a committee with six members if it must have the same number of men and women? i.e. comittee must contain 3 men and 3 women.

There are $\binom{10}{3} = \frac{10!}{3!\,7!} = \frac{10.9.8}{3.2} = 120$ ways to choose 3 men out of 10 men

and $\binom{15}{3} = \frac{15!}{3!\,12!} = \frac{15.14.13}{3.2} = 455$ ways to choose 3 women out of 10 women.

Therefor, the answer is: $120 \times 455 = 54\,600$

b) How many ways are there to form a committee with six members if it must have more women than men?

Case 1: The committee is composed only of women:
There are $\binom{15}{6} = 5005$ such committees.

Case 2: The committee contains 5 women and 1 man:
There are $\binom{15}{5}\cdot\binom{10}{1} = 30\,030$ such committees.

Case 3: The committee contains 4 women and 2 men:
there are $\binom{15}{4}\cdot\binom{10}{2} = 61\,425$ such committees.

Therefore, the answer is:
$5005 + 30030 + 61425 = 96\,460$