AMERICAN UNIVERSITY OF BEIRUT
Math 211- Spring 2007

Drop Quiz 5, section 4, March 29, 2007

Name: ………………………………………

AUB ID number: ……………………

Section: …….

3 pts **Question 1:** Let $f: \mathbf{R} \to \mathbf{R}$ and $f(x) = \dfrac{x+1}{x+2}$.

3/4 a) Show that f is not a bijection from **R** to **R**.

$f(-2)$ is not defined.
So f is not one-to-one from $\mathbb{R}$ to $\mathbb{R}$.
Therefore f is not a bijection from $\mathbb{R}$ to $\mathbb{R}$

3/4 b) Find $f^{-1}(\{0\})$, $f^{-1}(\{2,3\})$ and $f^{-1}(\{1\})$. $\frac{x+1}{x+2} = 0 \Rightarrow x+1=0 \Leftrightarrow x=-1$

$\frac{x+1}{x+2} = 2 \Rightarrow x+1 = 2x+4 \Rightarrow x = -3$

$\frac{x+1}{x+2} = 3 \Rightarrow x+1 = 3x+6 \Rightarrow x = -5/2$

$\frac{x+1}{x+2} = 1 \Rightarrow x+1 = x+2$ Impossible

Therefore: $\boxed{f^{-1}(\{0\}) = \{-1\}}$

$\boxed{f^{-1}(\{2,3\}) = \{-3, -\frac{5}{2}\}}$ $\boxed{f^{-1}(\{1\}) = \phi}$

1 1/2 c) Change the domain and codomain of f in a way that makes f bijective, then give its inverse function. What is $f^{-1}(0)$? * The domain must not contain (-2).

* Let $y = \frac{x+1}{x+2}$ $(x \neq -2)$ $\Rightarrow yx + 2y = x+1 \Rightarrow x(y-1) = 1-2y$

1/2 $\boxed{x = \frac{1-2y}{y-1}}$ with $y \neq 1$

this last identity defines a unique pre-image $\forall y \in \mathbb{R}$, excep for $y = 1$ that doesn't have any pre-image.
Therefore, the co-domain must not contain 1.

* Answer: If the domain is $\boxed{\mathbb{R} - \{-2\}}$ 1/4
and the codomain is $\boxed{\mathbb{R} - \{1\}}$
then f is bijective 1/2

$* f^{-1}(0) = -1$ 1/4

4 pts **Question 2:** R is a relation on the set $A = \{1,2,3,4\}$ given by: $R = \{(1,2),(1,3),(2,4),(3,2),(4,4)\}$.

1 a) Is R reflexive? Irreflexive? Symmetric? Antisymmetric? Asymmetric? Transitive?

R is not reflexive, since $(1,1) \notin R$ for example
R is not irreflexive, since $(4,4) \in R$
R is not symmetric, since $(1,2) \in R$ and $(2,1) \notin R$
R is antisymmetric, since we have no pairs (a,b) and (b,a) with a≠
R is not asymetric, since $(4,4) \in R$

1/2 b) Find $R \cap R^{-1}$. $R^{-1} = \{(2,1), (3,1), (4,2), (2,3), (4,4)\}$

So $R \cap R^{-1} = \{(4,4)\}$

1/2 c) Find $R^2 = R \circ R = \{(1,4), (1,2), (2,4), (3,4), (4,4)\}$

1 pt **Question 3:** Let R be a reflexive relation on a set A. Show that $R \subseteq R^2$.

Assumption: R reflexive
Let x, y be arbitrary in A.
Subassumption: $(x,y) \in R$
but $(y,y) \in R$ since R is reflexive
$(x,y) \in R \circ R = R^2$
Conclusion: $\forall x \forall y \; (x,y) \in R \Rightarrow (x,y) \in R^2$
which means $R \subseteq R^2$

Conclusion: $R \subseteq R^2$