AMERICAN UNIVERSITY OF BEIRUT
Math 211- Spring 2007 | **Drop Quiz 6, section 3, April 12, 2007**

Name: .... AUB ID number: ....

Section: ....

1 pt **Question 1:**

1/2 1. Evaluate: a) (- 97) mod 11 = 2 since $-97 = (-9).11 + 2$
b) (- 1001) div 13 = -77 since $-1001 = (-77).13 + 0$

1/2 2. Decide whether each of these integers is congruent to 4 modulo 12:
a) -4 : $-4 = (-1).12 + 8$ so $-4 \not\equiv 4 \pmod{12}$
b) 1204 : $1204 = (100).12 + 4$ so $1204 \equiv 4 \pmod{12}$
c) -32 : $-32 = (-3).12 + 4$ so $-32 \equiv 4 \pmod{12}$

(in another way: 12 divides $(1204-4)$ and $(-32-4)$ but 12 doesn't divide $(-4-4)$)

1 pt **Question 2:**

1/2 1. Are 29 and 203 prime numbers?
29 is prime, $203 = 7.29$ is not prime
Are they relatively prime?
No, since $\gcd(29,203) = 29 \neq 1$

1/2 2. Let $a = 3^7.5^3.7^3$ and $b = 2^{11}.3^5.5^9$.
Find: $gcg(a,b) = 3^5.5^3$
$lcm(a,b) = 2^{11}.3^7.5^9.7^3$

3 pts **Question 3**: R_1 is the "divides" relation on the set of all positive integers: $R_1 = \{(a,b) \mid a \text{ divides } b\}$.

1 1/4 1. Is R_1 reflexive? Symmetric? Antisymmetric? Asymmetric? Transitive?
R_1 is reflexive: $\forall x \in \mathbb{Z}^+$, x divides x. So: $(x,x) \in R_1$.
R_1 is not symmetric: counterexample: 2 divides 8, but 8 doesn't divide 2. So $(2,8) \in R_1$ and $(8,2) \notin R_1$
R_1 is antisymmetric: $\forall x, \forall y, [(x \text{ divides } y) \text{ and } (y \text{ divides } x)] \Rightarrow x = y$
i.e.: $\forall x, \forall y, [(x R_1 y) \wedge (y R_1 x)] \Rightarrow x = y$
R_1 is not asymmetric, since R_1 is reflexive
R_1 is transitive: $\forall x, \forall y, \forall z$ [(x divides y) and (y divides z)] $\Rightarrow$ (x divides z)

1 1/4 2. R_2 is the "is a multiple of" relation on the set of all positive integers: $R_2 = \{(a,b) \mid a \text{ is a multiple of } b\}$.
Find: a) $R_1 \cap R_2 = \{(a,b) \mid (a|b) \wedge (b|a)\} = \{(a,b) \mid a = \pm b\}$
b) $R_1 \cup R_2 = \{(a,b) \mid (a|b) \vee (b|a)\}$ (No easier way to state this)
c) $R_1 - R_2 = \{(a,b) \mid (a|b) \wedge (b \nmid a)\} = \{(a,b) \mid (a|b) \wedge (a \neq \pm b)\}$
d) $R_2 - R_1 = \{(a,b) \mid (b|a) \wedge (a \nmid b)\} = \{(a,b) \mid (b|a) \wedge (a \neq \pm b)\}$
e) $R_1 \oplus R_2 = \{(a,b) \mid [(a|b) \vee (b|a)] \wedge (a \neq \pm b)\}$

(Note that: $R_2 = R_1^{-1}$, since $(a,b) \in R_2 \iff (b,a) \in R_1$
a is a multiple of $b \iff b$ divides a)

1/2 3. Find $R_1 \circ R_1$
$R_1 \circ R_1 = \{(a,b)$ for which $\exists c \mid [(a|c) \wedge (c|b)]\}$
Such c exists whenever a divides b: we let for example $c=a$ or $c=$
Therefore: $R_1 \circ R_1 = R_1$

1 pt **Question 4**: n and m are two positive integers greater than 1, and a and b are two integers.
Show that if $n \mid m$ and $a \equiv b \pmod{m}$, then $a \equiv b \pmod{n}$.

Assumption: $n|m$ and $a \equiv b \pmod{m}$
So $\exists k_1 \in \mathbb{Z}$, $m = k_1 n$
and $\exists k_2 \in \mathbb{Z}$, $a - b = k_2 m$
It follows: $a - b = k_2(k_1 n)$
$a - b = (k_2 k_1) n$ we let $= k_1 k_2$
Therefore: $\exists k \in \mathbb{Z}$, $a - b = k n$
which is equivalent to saying: $a \equiv b \pmod{n}$