AMERICAN UNIVERSITY OF BEIRUT
Math 211- Spring 2007 **Drop Quiz 7, section 2, April 19, 2007**

Name: …………………………………… AUB ID number: ……………………

Section: …….

Question 1: (2 points)

1/2 1. Evaluate: (- 90) mod 17 $= 12$ $\left[\text{since } (-90) = (-6).17 + 12\right]$

1/2 2. Is it true that: $-18 \equiv 2 \pmod 5$? *Justify your answer.*

$5 \mid (-18-2 = -20)$ so $-18 \equiv 2 \pmod 5$

1 3. Let $a = 4455$ and $b = 6534$. Find $gcg(a,b)$ and $lcm(a,b)$.

$a = 3^4.5.11$
$b = 2.3^3.11^2$

$\gcd(a,b) = 3^3.11$
$\mathrm{lcm}(a,b) = 2.3^4.5^3.11^2$

6534	2
3267	3
1089	3
363	3
121	11
11	11
1	

4455	3
1485	3
495	3
165	3
55	5
11	11
1	

Question 2: Determine whether each of these arguments is valid. (1 point)
If it is, what rule of inference has been used? If it is not, what logical error occurs?

1/2 a) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \le 9$. Then $n \le 3$.

$[n > 3 \rightarrow n^2 > 9]$ (hyp)
$n^2 \le 9 \equiv \neg(n^2 > 9)$ (hyp)
so by modus tollens: $\neg(n > 3) \equiv n \le 3$ (Valid argumen

1/2 b) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n \le 3$. Then $n^2 \le 9$.

$[n > 3 \rightarrow n^2 > 9]$ (hyp)
$n \le 3 \equiv \neg(n > 3)$ (hyp)
It is incorrect to conclude $\neg(n^2 > 9)$ since $p \rightarrow q \not\equiv \neg p \rightarrow \neg q$

Question 3: (2 points)

1 a) Explain which rules of inference are used for each step of the following argument: "All songs composed by Mr.Smith become hits. Mr. Smith composed a love song. Therefore, there is a hit which is a love song."

$S(x)$: "song x is composed by M. Smith"
$H(x)$: "song x is a hit"
$L(x)$: "song x is a love song"

(1) $\forall x \; S(x) \rightarrow H(x)$ (Hyp)
(2) $\exists x \; S(x) \wedge L(x)$ (Hyp)
(3) $S(a) \wedge L(a)$ for some song a (existential instantiati
(4) $S(a)$ (simplification
(5) $S(a) \rightarrow H(a)$ (universal instantiation)
(6) $H(a)$ (modus ponens using (4) and (5))
(7) $L(a)$ simplification of (3)
(8) $H(a) \wedge L(a)$ from (6) &
(9) $\exists x \, [H(x) \wedge L(x)]$ (existential generalizati

1 b) What relevant conclusions can be drawn for the following set of premises: "I am either clever or lucky". "If I am lucky and I am inspired, then I will win the lottery". "I am not clever". "I did not win the lottery".

c: "I am clever"
l: "I am lucky"
i: "I am inspired"
w: "I win the lottery"

(1) $c \vee l$
(2) $(l \wedge i) \rightarrow w$ premises.
(3) $\neg c$
(4) $\neg w$

(5) $\neg(l \wedge i)$ modus tollens using (2) and (4)
(6) $\neg l \vee \neg i$
(7) l disjunctive syllogism using (1) and (3)
(8) $\neg i$ disj. syll. using (6) and (7)

Relevant Conclusion: I am luck but I am not inspi

Question 4: Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod 8$. (1 point)

Assumption: n is an odd positive integer

$\therefore \exists k \in \mathbb{Z}^+, \; n = 2k+1$
$\therefore n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4(k^2+k) + 1$
$\therefore n^2 = 4\,\underbrace{k(k+1)}\, + 1$

product of 2 consecutive integers: one of them must be even so the product is even

$\therefore \exists k' \in \mathbb{Z}^+, \; k(k+1) = 2k'$
$\therefore n^2 = 4.2k' + 1$
$\therefore n^2 = 8k' + 1$

Conclusion: $n \equiv 1 \pmod 8$