AMERICAN UNIVERSITY OF BEIRUT
Math 211- Spring 2007

Drop Quiz 8, section 4, April 26, 2007

Name: ....

AUB ID number: ....

Section: ....

Question 1: (1.5 points)

Explain which rules of inference are used for each step of the following argument:
"Logic is difficult or not many students like logic.
If mathematics is easy, then logic is not difficult.
Therefore: mathematics is not easy, if many students like logic."

d : " logic is difficult
s : " not many students like logic"
e : " mathematics is easy

Given that: $d \vee s$ (1)
and $e \to \neg d$. (2)

Let us assume $\neg s$ (3)
$\therefore$ d (4) (disjunctive syllogism using (1))
but $d \to \neg e$ (5) (contrapositive of (2))
$\therefore$ $\neg e$ (6) (modus ponens using (4) and (5))

$\therefore$ $\neg s \Rightarrow \neg e$ i.e: if many students like logic, then mathematics is not easy

Question 2: (2.5 points)

What relevant conclusions can be drawn for the following sets of premises? *(Explain the rules of inference used to obtain each conclusion)*

a) "If I eat onions at dinner, then I have nightmares". "If there is thunder while I sleep, then I have nightmares". "I am tired the next day, whenever I have nightmares". "Today, I am not tired".

Let : o: "I eat onions at dinner"
n: "I have nightmares
th: " there is a thunder while I sleep
t: "I am tired"
(we could have used predicates where the variable is the day)

Givens
$o \to n$ (1)
$th \to n$ (2)
$n \to t$ (3)
Today : $\neg t$ (4)

$\therefore$ $\neg n$ (5) modus tollens using (3) and (4)
$\therefore$ $\neg th$ (6) " " " (5) and (2)
and $\neg o$ (7) " " " (5) and (1)

Relevant conclusions: I did not eat onions yesterday at dinner
There was no thunder last night and I had no nightmares.

b) "If I go to the beach, then I will see your brother". "I am depressed, whenever I see your brother". "I am depressed".

b : "I go to the beach"
s : "I see your brother"
d : "I am depressed".

Givens :
$b \to s$ (1)
$s \to d$ (2)
d (3)

No relevant conclusions .

Question 3: (2 points)

Prove, using mathematical induction; one of the following statements .

a) $n! < n^n$, for all n greater then 1.

b) $1.2.3+2.3.4+...+n(n+1)(n+2)=\dfrac{n(n+1)(n+2)(n+3)}{4}$, for every positive integer n.

a) $\forall n > 1$, $n! < n^n$ $(P(n))$

• Basis step: $P(2)$ true, since $2! < 2^2$

• Inductive step:
Assume $P(n)$ true : $n! < n^n$ [we must show: $P(n+1)$ true i.e: $(n+1)! < (n+1)^{n+1}$]

But $(n+1)! = n!\,(n+1)$
$< n^n.(n+1)$ using the inductive hypothesis
$< (n+1)^n\,(n+1)$ since $n < n+1$ and $n \geq 2$ (power fct is increasing)

$\therefore$ $(n+1)! < (n+1)^{n+1}$

• Conclusion : It follows, by the principle of mathematical induction that $\forall n \geq 2$, $n! < n^n$

$\rightarrow$

b] $\forall n \geq 1$, $\underbrace{\overbrace{1.2.3 + 2.3.4 + \cdots + n(n+1)(n+2)}^{S_n} = \frac{n(n+1)(n+2)(n+3)}{4}}_{P(n)}$

- Basis step: $P(1)$ true, since $S_1 = 1.2.3 = \frac{1.(1+1)(1+2)(1+3)}{4}$
- Inductive step

Assume $P(n)$ true: $S_n = \frac{n(n+1)(n+2)(n+3)}{4}$

we must show that $P(n+1)$ is true: $S_{n+1} = \frac{(n+1)(n+2)(n+3)(n+4)}{4}$

But
$$\begin{aligned}
S_{n+1} &= \underbrace{1.2.3 + 2.3.4 + \cdots + n(n+1)(n+2)} + (n+1)(n+2)(n+3) \\
&= S_n + (n+1)(n+2)(n+3) \\
&= \frac{n(n+1)(n+2)(n+3)}{4} + (n+1)(n+2)(n+3) \\
&= (n+1)(n+2)(n+3)\left[\frac{n}{4} + 1\right] \\
&= \frac{(n+1)(n+2)(n+3)(n+4)}{4}
\end{aligned}$$

- Conclusion: $\forall n \geq 1$, $S_n = \frac{n(n+1)(n+2)(n+3)}{4}$

(by the principle of mathematical induction)