AMERICAN UNIVERSITY OF BEIRUT
Math 211- Spring 2007 **Drop Quiz 9, section 3, May 17, 2007**

Name: .... AUB ID number: ....

Section: ....

Question 1: → BONUS / 0.5 point ~~(1 points)~~

How many strings of eight English letters are there:

a) If letters can be repeated? 26^8

b) That starts with X, if no letter can be repeated? $\underbrace{25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19}_{\text{For the 7 remaining letters}} = \frac{25!}{18!}$

c) That starts with X and ends with BO, if no letter can be repeated? $23 \times 22 \times 21 \times 20 \times 19 = \frac{23!}{18!}$

d) That contains exactly one X? 8×25^7

nb of places for the X

Question 2: → → → BONUS / 1 pt ~~(1.5 point)~~

a) How many different functions are there from a set of n elements to the set {1, 3, 5, 7, 9}? → 5^n

b) How many one-to-one functions are there from a set of n elements to the set {1, 3, 5, 7, 9}?

If $n=1$, there are 5 one-to-one functions
If $n=2$, there are 5×4 " "
If $n=3$, " " $5 \times 4 \times 3$ " "
If $n=4$, " " $5 \times 4 \times 3 \times 2$ " "
If $n=5$, " " $5 \times 4 \times 3 \times 2 \times 1$ " "
If $n>5$, there is no way to get a one-to-one function.
(cardinality of domain will be larger than cardinality of co-domain)

Question 3: ~~(1 point)~~ (2 points)

Give a recursive definition of the sequence $\{a_n\}$, $n = 0,1,2,...$ if:

a) $a_n = 4n - 2$.

$$\boxed{a_{n+1} = 4(n+1) - 2 = 4n + 2 = (4n-2) + 4 = a_n + 4}$$

$$\begin{cases} a_0 = -2 \\ a_{n+1} = a_n + 4 \quad \forall n \geq 1 \end{cases} \longrightarrow \text{(arithmetic sequence)}$$

b) $a_n = \frac{2}{3^n}(-1)^n = 2\left(-\frac{1}{3}\right)^n$

$$\boxed{a_{n+1} = \frac{2(-1)^{n+1}}{3^{n+1}} = \frac{2(-1)^n \times (-1)}{3^n \times 3} = -\frac{1}{3}a_n}$$

$$\begin{cases} a_0 = 2 \\ a_{n+1} = -\frac{1}{3}a_n \end{cases} \rightarrow \text{(geometric sequence)}$$

Question 4: ~~(2.5 points)~~ (4 pts)

a) Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7$, $a_1 = -4$ and $a_2 = 8$.
(use back of the page)

$$\boxed{a_n = 5 + 3(-2)^n - 3^n}$$

b) What is the general form of the particular solution of the linear nonhomogeneous recurrence relation $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3} + 3^n$? First guess for $a_n^{(P)}$: $a_n^{(P)} = D3^n$

But it duplicates a term in $a_n^{(H)}$ (see question a)
So the correct form is: $\boxed{a_n^{(P)} = Dn3^n}$

c) What is the general form of the particular solution of the linear nonhomogeneous recurrence relation $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3} + n^2$? First guess: $a_n^{(P)} = Dn^2 + En + F$

But it duplicates a term in $a_n^{(H)}$ (since 1 is a solution)
The corrected form is then: $\boxed{a_n^{(P)} = n(Dn^2 + En + F)}$

Question 4

a) $\begin{cases} a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3} \\ a_0 = 7, \quad a_1 = -4, \quad a_2 = 8 \end{cases}$ (1)

$a_n = r^n$ solution of (1) $\Longleftrightarrow$ r solution of $\underbrace{r^3 - 2r^2 - 5r + 6 = 0}_{\text{Characteristic equation}}$

$r = 1$ is a solution.

The equation can be factorized: $(r-1)(\underbrace{r^2 - r - 6}_{r=3 \text{ or } r=-2}) = 0$

So we have 3 distinct roots:

$r_1 = 1$, $r_2 = -2$ and $r_3 = 3$

General solution of (1) is:

$a_n = A + B(-2)^n + C\,3^n$

But $\begin{cases} a_0 = 7 \\ a_1 = -4 \\ a_2 = 8 \end{cases}$ so $\begin{cases} A + B + C = 7 & (a) \\ A - 2B + 3C = -4 & (b) \\ A + 4B + 9C = 8 & (c) \end{cases}$

(a) gives $C = 7 - A - B$

Replace in (b), (c): $\begin{cases} -2A - 5B = -25 \\ -8A - 5B = -55 \end{cases}$

$\ominus$ $6A = 30$

so $A = 5$, $B = 3$, $C = -1$

Therefore, the solution is:

$$\boxed{a_n = 5 + 3(-2)^n - 3^n}$$