

AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Mathematics Department

MATH 211
MID-TERM EXAMINATION
Fall 2005-2006
Closed Book, 1H15MIN

WRITE YOUR ANSWERS ON THE QUESTION SHEET

STUDENT NAME	
ID NUMBER	

Problem	Out of	Grade
1	10	
2	10	
3	10	
4	10	
5	10	
TOTAL	50	

1. (10 points)

(a) (5 points) Prove or disprove :

$$(p \rightarrow (q \rightarrow r)) \equiv ((\neg p \rightarrow q) \wedge (p \vee r)).$$

(b) (5 points) Consider the following sets : $R = \{\{1\}, \{1,3\}, \{1,3,5\}\}$, $S = \{1,2,3\}$, $T = \{1,3\}$. What are :

i. (2 points) $R \cup 2^S$ (where 2^S is the power set of S)?

ii. (3 points) $(S \times T) \cup (T \times S)$?

2. (10 points) Solve the following :

(a) (5 points) $\forall n \in \mathbb{N}$ ($6|n^2$ if and only if $6|n$).

(b) (5 points) Prove that $\sqrt{6}$ is irrational.

3. (10 points) Given the universal set of the days of the week and the predicates :

$w(d)$: "I work on day d ", $s(d)$: "it is sunny on day d ", $p(d)$: "partly sunny on day d ".

write the following hypotheses in terms of logical connectives on r, f, s, d, t .

H_1 : "If I work on a day then it is either sunny or partly sunny".

H_2 : "I worked last Sunday or I worked last Friday".

H_3 : "It was not sunny on Tuesday". H_4 : "It was not partly sunny on Friday".

Draw your conclusion. Indicate the rules of inferences you are using.

4. (10 points)

(a) (5 points) State the necessary condition for an integer to be composite (not prime).

(b) (3 points) Check whether 143 and 1001 are prime integers?

(c) (2 points) Find the primes decomposition of 247.

5. (10 points) Let $f(n) = 2\lceil \frac{n}{3} \rceil - \lfloor \frac{2n}{3} \rfloor, n \in \mathbb{Z}$.

(a) (5 points) Using the Euclidean division theorem $n = 3 \times k + r$, find out the range (image) of the function f .

(b) (2 points) Find $f^{-1}(\{0\})$, the inverse image set of $\{0\}$.

(c) (3 points) Is $\lceil x \rceil + \lceil y \rceil - \lceil x + y \rceil = 0$ or $1 \forall x, y \in \mathbb{R}$.