Math 211

Quiz I (Fall 2004) **Time 75 min**

Name : ____________________

ID # : ____________________

Circle your problem solving section number below :

Section 1 12:00 F Section 2 1:00 F

Problem	Grade
1	/ 8
2 & 3	/ 9
4 & 5	/ 11
6	/ 7
7	/ 7
8	/ 9
9	/ 9
TOTAL	/ 60

I-LOGIC

(8 pts) **1.** Define the connective (↓) by $p \downarrow q$ is true when both p and q are false and it is false otherwise. Draw the truth table for this logical operation then prove or disprove the logical equivalence of $p \downarrow (q \wedge r)$ and $(p \downarrow q) \vee r$

p	q	p↓q
0	0	1
0	1	0
1	0	0
1	1	0

p	q	r	q∧r	p↓(q∧r)		p↓q	∨r
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	
0	1	0	0	1	0	0	
0	1	1	1	0	1	0	
1	0	0	0	0	0	0	
1	0	1	0	0	1	0	
1	1	0	0	0	0	0	
1	1	1	1	0	1	0	

(5 pts) **2.** Determine whether the following compound proposition is a tautology.
$[(p \vee \neg q) \wedge (p \rightarrow r) \wedge (\neg q \rightarrow r)] \rightarrow r$

p	q	r	$p \vee \neg q$	$p \rightarrow r$	$\neg q \rightarrow r$		
0	0	0	1	1	0	0	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	0	1
0	1	1	0	1	1	0	1
1	0	0	1	0	0	0	1
1	0	1	1	1	1	1	1
1	1	0	1	0	1	0	1
1	1	1	1	1	1	1	1

(4 pts) **3.** Translate:

a. The English statement "The product of two positive numbers is positive" into a logical expression , then write the corresponding negation.

b. Into English the logical expression: $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$

For every x, there exists y such that $x + y = 2$ and $2x - y = 1$.

II-PROOFS

(6 pts) **4.** Show that $\sqrt{5}$ is an irrational. Specify what method of proof you are using?

$5 = \frac{a}{b}$

(5 pts) **5.** Prove that the sum of a rational number and an irrational number is irrational. Name your method of proof.

III-FUNCTIONS

(7 pts) **6.** The function $f: IN \times IN \longrightarrow IN$ is defined by $f(x, y) = 2^x 6^y$.

a. Is f one to one? Justify your answer

$f(1,1) = 12$

$f(x_1, y_1) = f(x_2, y_2)$

$\Rightarrow$ $x_1 = x_2$
$y_1 = y_2$

$f(x_1, y_1) = 2^{x_1} 6^{y_1}$

$f(x_2, y_2) = 2^{x_2} 6^{y_2}$

$2^{x_1 - x_2} \cdot 6^{y_1 - y_2} = 1 \Rightarrow$

b. Is f onto? Justify your answer

c. Is f invertible? Justify your answer

(7 pts) **7.** Prove that: $\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil = n$ for all integers n.

IV-RELATIONS AND EQUIVALENCE RELATIONS

(9 pts) **8.** For m, n in IN define $m \approx n$ if $m^2 - n^2$ is a multiple of 3

a. Show that $\approx$ is an equivalence relation on IN.

b. Describe [0] and [1].

c. Show that [0] and [1] are the only equivalence classes for $\approx$? Justify your answer.

V-POSETS

(9 pts) **9.** Consider the set S={a,b,c}. Let ***P***(S) be the power set of S.

a. Describe the elements of ***P***(S).

b. Let ⊆ be the inclusion relation on ***P***(S). Show that (***P***(S),⊆) is a poset.

c. Draw the Hasse diagram of this poset.

d. Give the minimal and maximal elements of this poset.