Math 211

Quiz I (Spring 2005) **Time: 75 min.**

Name : ____________________

ID # : ______________________

Circle your problem solving section number below :

Section 1 11:00 F Section 2 1:00 F Section3: 1:00F

Problem	Grade
1, 2	/ 9
3, 4, 5	/ 9
6, 7	/ 12
8, 9	/ 12
10	/ 8
TOTAL	/ 50

(5 pts) **1.** Determine whether the following compound proposition is a tautology.
$\{(p \to q) \wedge (q \wedge \neg r)\} \to [\neg p \vee (q \to r)]$

(4 pts) **2.** Consider the statement p:="Flowers do not grow if you do not water them".

a. Give the contra-positive form of p.

b. Give the converse form of p.

(5 pts) **3.** Use De Morgan's law to prove :

$\left(\overline{\overline{A} \cup \overline{B}}\right) \cup C = \overline{\left(A \cap \overline{C}\right) \cup \left(B \cap \overline{C}\right)}$ for all subsets A, B and C of some universal set U.

$\bar{A} \wedge \bar{B} \wedge \bar{C}$

(2 pts) **4.** Show that $\chi_{\overline{A}} = 1 - \chi_A$ for all sets A where χ_E is the characteristic function of a set E.

$1 - \chi_A = \phi$

$\chi_{\bar{A}} = 1, x \notin A.$
$= 0, x \in A.$

(2 pts) **5.** Show that $member(A - B) = member(A) \wedge not\ member(B)$ for all subsets A and B of a universal set U.

(6 pts) **6.** Show that $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$, for all subsets S and T of the co-domain of f.

$x \in f^{-1}(S \cup T)$

$\Rightarrow x \in [f^{-1}(S) \vee f^{-1}(T)]$

$x \in f^{-1}(S) \vee x \in f^{-1}(T)$

$x \in [f^{-1}(S) \vee f^{-1}(T)]$

(6 pts) **7.** Show that $\left\lceil \frac{3n}{4} \right\rceil + \left\lfloor \frac{n}{4} \right\rfloor = n$ for all integers n. Name whether your proof is direct or indirect.

$= 4K$ 4K + 4

$4K + 1$

$4K + 2$

$4K + 3$

$4K$

$n = 4K + 1$

$4K + 2$

$4K + 3$

$3K + K =$

$4K = n$

$\frac{12K + 3}{4} = 3K + \frac{3}{4}$

$\frac{4K + 1}{4}$

$K + \frac{1}{4}$

K

$3K + 1 + K$

$= 4K$

(6 pts) **8.** Use mathematical induction to show that :

$$\underbrace{(4n+1)+(4n+3)+(4n+5)+\ldots\ldots+(8n-1)}=12n^2, \text{for all } n \geq 1$$

$(4m+2)+(4m+4)+(4m+6)+\ldots+(8m)=12(m+1)^2$

$4(n+1)+2$

$\boxed{=12m^2+24m+12}$

$(4m+n)(4m+7)(4m+9)\ldots(8m+7)$ $n=\log n^2$

$=12m^2-[(4m+1)+(4m+3)]+(8m+1)+(8m+3)(8m+5)+(8m+7)$

$n+1=\log(n+1)^2$

$=$

(6 pts) **9.** Show that $\sqrt[3]{2}$ is an irrational. Specify what method of proof you are using and if any state preliminary results (lemmas)?

$(4m+5)+(4m+7)+(4m+9)+\ldots+(8m+7)=$

$=12m^2$.

(8 pts) **10.** Consider the function $f : IR \times IR \longrightarrow IR \times IR$ defined by $f(x, y) = (x\text{-}2y\,,\, x + 2y)$.

a. Is f one to one? Justify your answer

$(1,1)$

$(3,3)\ (-4, 8)$

$\cancel{f(x_1, y_1) = x_1 2y}$

$f(x_1, y_1) = (x_1 2y_1, x_1 + 2y_1)$

$\Leftrightarrow$ $f(x_2, y_2) = (x_2 - 2y_2, x_2 + 2y_2)$

$(x_1 - 2y_1, x_1 + 2y_1) = (x_2 - 2y_2, x_2 + 2y_2)$

$$\begin{cases} x_1 - 2y_1 = x_2 - 2y_2 \\ x_1 + 2y_1 = x_2 + 2y_2 \end{cases}$$

b. Is f onto? Justify your answer

c. Is f invertible? Justify your answer