Math 211 — Fall 2006–07
Discrete Structures
Quiz 2, December 1 — Duration: 1 hour

GRADES (each problem is worth 14 or 15 points):

1	2	3	4	5	TOTAL/72

YOUR NAME:

YOUR AUB ID#:

PLEASE CIRCLE YOUR SECTION:

Section 1	Section 2	Section 3	Section 4
Recitation W 4	Recitation W 11	Recitation Th 12:30	Recitation Th 3:30
Professor Makdisi	Ms. Karam	Ms. Karam	Ms. Karam

INSTRUCTIONS:

1. **Write your NAME and AUB ID number, and circle your SECTION above.**
2. **Solve the problems inside the booklet. Explain your reasoning precisely and clearly to ensure full credit. Partial solutions will receive partial credit. Each problem is worth 14 or 15 points.**
3. **You may use the back of each page for scratchwork OR for solutions. There are two extra blank sheets at the end, for extra scratchwork or solutions. If you need to continue a solution on another page, INDICATE CLEARLY WHERE THE GRADER SHOULD CONTINUE READING.**
4. **Open book and notes. NO CALCULATORS ALLOWED. Turn OFF and put away any cell phones.**

NEW! 5. You do not need to simplify your answers to counting problems — for example, if the answer is $\binom{211}{30} + (12!)7 \cdot 6 \cdot 5$, just leave it that way.

GOOD LUCK!

An overview of the exam problems. Each problem is worth 14 or 15 points. Take a minute to look at all the questions, THEN solve each problem on its corresponding page INSIDE the booklet.

1. Note: the different parts are UNRELATED.

a) (4 pts) In how many ways can one rearrange the letters of `ALIBABA`?

b) (5 pts) A bakery sells three types of manaqish: zaatar, cheese, and kishk. In how many ways can one buy **no more than 12** manaqish? (It is possible to buy any number between zero and twelve manaqish.)

c) (5 pts) How many bitstrings of length 9 are there that have 000 at the beginning, the end, OR in the middle three positions? (In other words, the bitstring should have the form $000xxxxxx$ –**or**– $xxx000xxx$ –**or**– $xxxxxx000$.)

2. a) (6 pts) Show algebraically that $\binom{n}{k}\binom{n-k}{\ell} = \binom{n}{\ell}\binom{n-\ell}{k}$.

b) (8 pts) Show the above identity by a combinatorial argument.

3. a) (3 pts) Fill in the blanks for the definition of the function `implies`. Its input are boolean (i.e., logical) values `p, q` $\in$ {`true, false`} and its output is `true` or `false` depending on whether `p`$\rightarrow$`q` is true or not.

```
implies := function(p,q)
             if p then return(<fill in blank on page 3>);
                 else return(<fill in blank on page 3>);
             fi;
end;
```

b) (6 pts) Write a function function `sumofpositives` (in GAP or pseudocode) which takes as input a list `thelist`, and which outputs the sum of all the positive elements of `thelist`. For example, `sumofpositives([2,3,-4,5,-1]) = 10` because $2+3+5=10$.

c) (6 pts) Write a function `listofsquares` (in GAP or pseudocode) that takes as input a list `thelist`, and that outputs a list of the squares of the elements in `thelist`. For example, `listofsquares([-1,2,3,-1,1]) = [1,4,9,1,1]`. You may, if you wish, use the command `Add(ll,y)` which replaces the list `ll` by putting `y` in after the end of `ll`.

4. a) (7 pts) Show that

$$\forall x \geq 0, \quad x^4 \leq 5e^x.$$

Hint: study the function $h(x) = x^4e^{-x}$, and use the fact that $4^4e^{-4} \approx 4.689 < 5$.

b) (7 pts) Show from the definition of Θ that $e^x + 2x^4 + x^3$ is $\Theta(e^x)$. You may use the result of part (a) even if you were not able to solve it.

5. In this problem, we count functions $f : \{1,2,3,4,5,6\} \rightarrow \{10,20,30,40,50\}$ satisfying certain properties.

a) (3 pts) How many functions f are injective?

b) (6 pts) How many functions f satisfy $f^{-1}(\{10,20\}) = \{1,2,3,4\}$?

c) (6 pts) How many functions f satisfy $f(\{1,2,3,4\}) = \{10,20\}$? Hint: first count the number of surjective functions $g : \{1,2,3,4\} \rightarrow \{10,20\}$.