AMERICAN UNIVERSITY OF BEIRUT
Math 211- Spring 2007

Drop Quiz 3, section 2, March 15, 2007

/6 pts

Name: ....

AUB ID number: ....

Section: ....

Question 1: Express the following statement using predicates, quantifiers and logical connectives, if necessary:

"The e-mail address of every user can be retrieved whenever the archive contains at least one message sent by every user on the system"

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U: all users
M: all messages
A(m): archive contains message m
S(u, m): user u sent message m

R(u): e-mail address of u can be retrieved

$$\left[\forall u\ \exists m\ (A(m) \wedge S(u,m))\right] \Longrightarrow \forall u,\ R(u)$$

Question 2: Negate the following statement so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives):

1

$$\neg\left(\forall x(\exists y \forall z P(x,y,z) \wedge \exists z \forall y P(x,y,z))\right)$$

$$\equiv \exists x\left[\neg(\exists y \forall z P(x,y,z)) \vee \neg(\exists z \forall y P(x,y,z))\right]$$

$$\equiv \exists x\left[\forall y \exists z \neg P(x,y,z) \vee \forall z \exists y \neg P(x,y,z)\right]$$

Question 3: Let A, B and C be sets.

Explain why $(A\times B)\times(C\times D)$ and $A\times(B\times C)\times D$ are not the same,

1

Elements of $(A\times B)\times(C\times D)$ are ordered pairs of the form $((a,b),(c,d))$
Elements of $A\times(B\times C)\times D$ are 3-tuples of the form $(a,(b,c),d)$
So $(A\times B)\times(C\times D) \neq A\times(B\times C)\times D$

Question 4: Let A, B and C be sets.

Show that: $A\oplus C = B\oplus C \Rightarrow A = B$?

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It is equivalent to show: $A \neq B \Longrightarrow A\oplus C \neq B\oplus C$ (contrapositive)

Assumption $A\neq B$
it means: $(\exists x\in A \mid x\notin B) \vee (\exists x\in B \mid x\notin A)$
let us assume $\exists x\in A \mid x\notin B$ (without loss of generality)
(we have to show that $A\oplus C \neq B\oplus C$)

Case 1: $x\in C$ but $x\in A$ } so $x\notin A\oplus C$
and $x\in C$, $x\notin B$ } so $x\in B\oplus C$
therefore $A\oplus C \neq B\oplus C$

Case 2: $x\notin C$ but $x\in A$ } so $x\in A\oplus C$
and $x\notin B$, $x\notin C$ } so $x\notin B\oplus C$
Therefore $A\oplus C \neq B\oplus C$

In both cases $A\oplus C \neq B\oplus C$
It follows: $A\neq B \Longrightarrow A\oplus C \neq B\oplus C$
similar to $A\oplus C = B\oplus C \Longrightarrow A = B$

Q4 Direct proof: $A \oplus C = B \oplus C \implies A = B$

Assumption: $A \oplus C = B \oplus C$

Step 1: $\boxed{A \subseteq B}$

Sub-assumption: x arbitrary, $x \in A$

Case 1 $x \in C$

$x \notin A \oplus C$

$\left.\begin{array}{l} x \notin B \oplus C \\ \text{but } x \in C \end{array}\right\}$ so $x \in B$.

Case 2: $x \notin C$

$x \in A \oplus C$

$\left.\begin{array}{l} x \in B \oplus C \\ x \notin C \end{array}\right\}$ $x \in B$

In both cases $x \in B$

so $[\forall x, \ x \in A \implies x \in B] \equiv \underline{A \subseteq B}$

Step 2: $\boxed{B \subseteq A}$

Sub-assumption: x arbitrary in B

Case 1: $x \in C$

$x \notin B \oplus C$

$\left.\begin{array}{l} x \notin A \oplus C \\ x \in C \end{array}\right\}$ $x \in A$

Case 2: $x \notin C$

$x \in B \oplus C$

$\left.\begin{array}{l} x \in A \oplus C \\ x \notin C \end{array}\right\}$ $x \in A$

Both cases: $x \in A$

so $[\forall x, \ x \in B \implies x \in A] \equiv \underline{B \subseteq A}$

Conclusion: $\left.\begin{array}{l} A \subseteq B \\ B \subseteq A \end{array}\right\} \iff \boxed{A = B}$