AMERICAN UNIVERSITY OF BEIRUT
Math 211- Spring 2007 **Drop Quiz 4, section 1, March 22, 2007**

Name: ..... AUB ID number: .....

Section: .....

Question 1: let $f: R \to R$ and $f(x) = \frac{x^2+1}{x^2+2}$.

a) Is f one-to-one? No since $-1 \neq 1$ and $f(-1) = f(1) = \frac{2}{3}$

b) Is f onto? Let $y \in \mathbb{R}$.
If $\exists x \in \mathbb{R} \mid f(x) = y$, then $\frac{x^2+1}{x^2+2} = y \iff x^2+1 = yx^2+2y$
$x^2(1-y) = 2y-1$
$x^2 = \frac{2y-1}{1-y}$ $x = \pm\sqrt{\frac{2y-1}{1-y}}$

Therefore, $y=1$ has no preimage so f is not onto.

c) Is f bijective?
f is not bijective (not even one-to-one or onto)

Question 2: Let f be the function from **R** to **R** defined by: $f(x) = x^2$.

a) Find $f^{-1}(\{1\})$ and $f^{-1}(\{x \mid 4 < x < 9\})$. $f^{-1}(\{1\}) = \{x \in \mathbb{R} \mid x^2 = 1\} = \{-1, 1\}$

$f^{-1}(\{x \mid 4 < x < 9\}) = \{x \in \mathbb{R} \mid 4 < x^2 < 9\} = \{x \in \mathbb{R} \mid (2 < x < 3) \vee (-3 < x < -2)\}$
$= (-3, -2) \cup (2, 3)$

b) Is f invertible? f is not one-to-one, since for example $f(-2) = f(2) = 4$
Therefore f is not bijective and not invertible.

Question 3: Give an example of an invertible function (justify your choice) and give an explicit formula for its inverse function.

$f(x) = 2x - 3$ f is one-to-one since $\forall x_1, x_2 \in \mathbb{R}$, $x_1 \neq x_2 \Rightarrow 2x_1 \neq 2x_2$
$2x_1 - 3 \neq 2x_2 - 3$
$f(x_1) \neq f(x_2)$

f is onto since $\forall y \in \mathbb{R}$, $\exists x \in \mathbb{R} \mid f(x) = y \iff 2x - 3 = y$
We let $x = \frac{y+3}{2}$

Therefore f is bijective and invertible, with $\boxed{f^{-1}(x) = \frac{x+3}{2}}$

Question 4: We consider the relation R on the set of people where $(a,b) \in R$ if and only if a is a cousin of b. Is R reflexive? Symmetric? Antisymmetric?

* I am not the cousin of myself, so R is not reflexive.
* If any person a is a cousin of a person b, then b is a cousin of a.
Therefore R is symmetric.
* Me and my cousin are cousins, but we are not equal!
Therefore, R is not Antisymmetric.

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