American University of Beirut

Faculty of Engineering and Architecture

Spring 2014

Quiz 2

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Test ID 1001-1002

Section:

EECE 380- Engineering Electromagnetics

CLOSED BOOK Exam:

- Programmable Calculators are not allowed
- Provide your answers to the multiple choice questions on the computer card only
- Provide your answers to the comprehensive problems on the exam sheet

Section I: Multiple Choice Questions (40 pts- 8 pts each)

MCQ 1:

At Cartesian point (-3, 4,-1) which of these is incorrect:

a. $\rho = -5$
b. $r = \sqrt{26}$
c. $\theta = \tan^{-1}(\frac{5}{-1})$
d. $\phi = \tan^{-1}(\frac{4}{-3})$

Complete Solution

a.

MCQ 2:

If $\vec{E} = 4\hat{a}_\rho - 3\hat{a}_\phi + 5\hat{a}_z$ at (1, π/2, 0), the component of $\vec{E}$ parallel to surface ρ=1 is:

a. $4\hat{a}_\rho$
b. $5\hat{a}_z$
c. $-3\hat{a}_\phi$
d. $-3\hat{a}_\phi + 5\hat{a}_z$
e. $5\hat{a}_\phi + 3\hat{a}_z$

Complete Solution

d.

MCQ 3:

Point charges Q_1= 1 nC and Q_2= 2 nC are at a distance apart. Which of the following statements is incorrect?

a. The force on Q_1 is repulsive
b. The force on Q_2 is the same in magnitude as that on Q_1
c. As the distance between them decreases, the force on Q_1 increases linearly
d. The force on Q_2 is along the line joining them
e. None of the above

Complete Solution

C

MCQ 4:

Point charges Q_1 and Q_2 are, respectively, located at (4, 0, -3) and (2, 0, 1). If Q_2= 4 nC, find Q_1 such that the electric field at (5, 0, 6) has no Z component.

a. Q_1= -5.4 nC
b. Q_1= -3.33 nC
c. Q_1= -8.323 nC
d. Q_1= -2.53 nC
e. Q_1= -10.34 nC

Complete Solution

$$E(5,0,6) = \frac{Q_1}{4\pi\varepsilon_0}\frac{[(5,0,6)-(4,0,-3)]}{|(5,0,6)-(4,0,-3)|^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{[(5,0,6)-(2,0,1)]}{|(5,0,6)-(2,0,1)|^3}$$

$$= \frac{Q_1}{4\pi\varepsilon_0}\frac{(1,0,9)}{(\sqrt{82})^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{(3,0,5)}{(34)^{3/2}}$$

If $E_z = 0$, then

$$\frac{9Q_1}{4\pi\varepsilon_0}\frac{1}{(82)^{3/2}} + \frac{5Q_2}{4\pi\varepsilon_0}\frac{1}{(34)^{3/2}} = 0$$

$$Q_1 = -\frac{5}{9}Q_2\left(\frac{82}{34}\right)^{3/2} = -\frac{5}{9}4\left(\frac{82}{34}\right)^{3/2} \text{ nC}$$

$$= \underline{\underline{-8.3232 \text{ nC}}}$$

MCQ 5:

A can of tomatoes is in the form of a wedge that is defined by 0<ρ<2, 0<Φ<π/4, 0<z<1. Given that the wedge has a volume charge distribution $\rho_v = 4\rho^2 z\cos\phi$ nC/m^3, find the total charge contained in the wedge.

a. Q= 6.754 nC
b. Q= 5.657 nC
c. Q= 7.856 nC
d. Q= 9.324 nC
e. None of the above

Complete Solution

$$Q = \int_v \rho_v dv = \iint\int 4\rho^2 z\cos\phi \rho d\rho d\phi dz \qquad \text{nC}$$

$$= 4\int_0^2 \rho^3 d\rho \int_0^1 zdz \int_0^{\pi/4} \cos\phi d\phi = \rho^4 \Big|_0^2 \frac{z^2}{2}\Big|_0^1 (\sin\phi)\Big|_0^{\pi/4}$$

$$= (16)(0.5)(\sin\pi/4) = \underline{\underline{5.657 \text{ nC}}}$$

MCQ 7:

Find the electric field at point P(0, 0, 8 m) resulting from a surface charge density $\rho_s = 5nC/m^2$ existing on the z=0 plane from ρ=2 m to ρ=6 m.

a. $\vec{E} = 24\hat{a}_z V/m$
b. $\vec{E} = 12\hat{a}_z V/m$
c. $\vec{E} = 16\hat{a}_z V/m$
d. $\vec{E} = 8\hat{a}_z V/m$
e. $\vec{E} = 48\hat{a}_z V/m$

Complete Solution

Find an expression for potential and then evaluate the gradient at the point.

$$V = \int \frac{dQ}{4\pi\varepsilon_o R}, \quad R = \sqrt{\rho^2 + h^2}, dQ = \rho_S \rho d\rho d\phi, \text{ so } V = \int \frac{\rho_S \rho d\rho d\phi}{4\pi\varepsilon_o\sqrt{\rho^2 + h^2}}$$

$$V = \frac{\rho_S}{2\varepsilon_o}\int_a^b \frac{\rho d\rho}{\sqrt{\rho^2 + h^2}} = \frac{\rho_S}{2\varepsilon_o}\sqrt{\rho^2 + h^2}\Big|_a^b = \frac{\rho_S}{2\varepsilon_o}\left[\sqrt{b^2 + h^2} - \sqrt{a^2 + h^2}\right].$$

Now we let h = z and $\mathbf{E} = -\nabla V$;

$$\mathbf{E} = -\frac{\rho_S}{2\varepsilon_o}\left[\frac{\partial}{\partial z}\left(b^2 + z^2\right)^{1/2} - \frac{\partial}{\partial z}\left(a^2 + z^2\right)^{1/2}\right]\mathbf{a}_z$$

$$= -\frac{\rho_S}{2\varepsilon_o}\left[\frac{1}{2}\left(b^2 + z^2\right)^{-1/2} 2z - \frac{1}{2}\left(a^2 + z^2\right)^{-1/2} 2z\right]\mathbf{a}_z = -\frac{\rho_S}{2\varepsilon_o}\left[\frac{z}{\sqrt{b^2 + z^2}} - \frac{z}{\sqrt{a^2 + z^2}}\right]\mathbf{a}_z$$

Plugging in the values we find **E** = 48 V/m $\mathbf{a}_z$.

MCQ 8:

Two point charges Q=2 nC and Q= - 4 nC are located at (1, 0, 3) and (-2, 1, 5), respectively. Determine the potential at P(1, -2, 3)

a. V=1.235 V
b. V=1.325 V
c. V=3.125 V
d. V=5.312 V
e. V=2.315 V

Complete Solution

$$V_P = \frac{Q_1}{4\pi\varepsilon_o r_1} + \frac{Q_2}{4\pi\varepsilon_o r_2} = \frac{10^{-9}}{4\pi \times \frac{10^{-9}}{36\pi}} \left[\frac{2}{|(1,-2,3)-(1,0,3)|} - \frac{4}{|(1,-2,3)-(-2,1,5)|} \right]$$

$$= 9\left[\frac{2}{2} - \frac{4}{\sqrt{9+9+4}}\right] = 1.325 \text{ V}$$

Section II: Comprehensive Problems

Problem 1 (20 pts):

In a region in free space, the electric flux density is found to be:

$$D = \begin{cases} \rho_0 (Z+d)\hat{a}_z \; C/m^2 & (-2d \le z < 0) \\ -\rho_0 (Z-2d)\hat{a}_z \; C/m^2 & (0 \le z \le 2d) \end{cases}$$

Everywhere else D=0.

a) Find the volume charge density ρ_v as a function of position everywhere (5 pts)

Using $\nabla \cdot \mathbf{D} = \rho_v$, find the volume charge density as a function of position everywhere: Use

$$\rho_v = \nabla \cdot \mathbf{D} = \frac{dD_z}{dz} = \begin{cases} \rho_0 & (-2d \le z \le 0) \\ -\rho_0 & (0 \le z \le 2d) \end{cases}$$

b) Determine the electric flux that passes through the surface defined by Z=0, $-a \le x \le a$, $-b \le y \le b$ (5 pts)

determine the electric flux that passes through the surface defined by $z = 0$, $-a \le x \le a$, $-b \le y \le b$: In the x-y plane, $\mathbf{D}$ evaluates as the constant $\mathbf{D}(0) = 2d\rho_0\,\mathbf{a}_z$. Therefore the flux passing through the given area will be

$$\Phi = \int_{-a}^{a}\int_{-b}^{b} 2d\rho_0\,dx dy = \underline{8abd\,\rho_0}\ \text{C}$$

c) Determine the total charge contained within the region $-a \le x \le a$, $-b \le y \le b$, $-d \le z \le d$ (5 pts)

determine the total charge contained within the region $-a \le x \le a$, $-b \le y \le b$, $-d \le z \le d$: From part a, we have equal and opposite charge densities above and below the x-y plane. This means that within a region having equal volumes above and below the plane, the net charge is zero.

d) Determine the total charge contained within the region $-a \le x \le a$, $-b \le y \le b$, $0 \le z \le 2d$ (5 pts)

determine the total charge contained within the region $-a \le x \le a$, $-b \le y \le b$, $0 \le z \le 2d$. In this case,

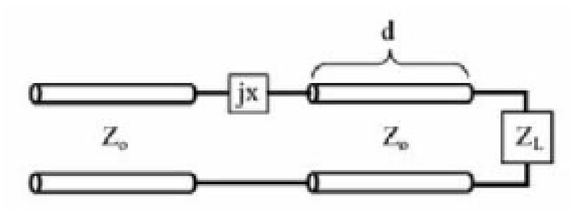
$$Q = -\rho_0\,(2a)\,(2b)\,(2d) = \underline{-8abd\,\rho_0}\ \text{C}$$

This is equivalent to the net *inward* flux of $\mathbf{D}$ into the volume, as was found in part b.

Problem 2 (20 pts):

A matching network, using a reactive element in series with a length d of a transmission line, is to be used to match 35-j50 Ω load to a 100 Ω transmission line. Find the through line length d and the value of the reactive element for F=1 GHz if:

a) a series capacitor is used. (10 pts)



First we normalize the load and locate it on the Smith Chart (point a, at z_L = 0.35-j0.5, WTG = 0.419λ).

(a) need to move to point b, at z = 1+j1.4, so that a capacitive element of value jx = -j1.4 can be added to provide an impedance match. Moving to this point b gives d = 0.500λ+0.173 λ -0.419 λ = 0.254 λ. The capacitance is

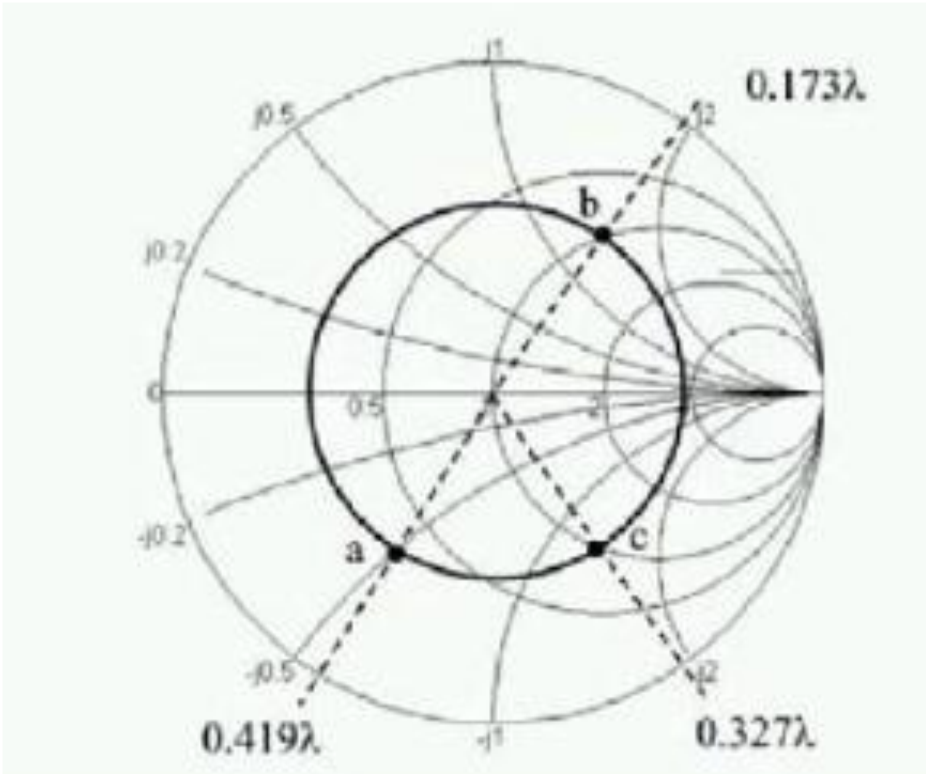
$$\frac{-j}{\omega C Z_o} = -j1.4,$$

$$C = \frac{1}{2\pi\left(1x10^9\right)(100)(1.4)} = 1.14pF$$

b) If an inductor is used. (10 pts)

(b) Now we need to move to point c, at z = 1-j1.4, so that an inductive element of value jx = +j1.4 can be added. Moving to this point c gives d = 0.500 λ + 0.327 λ − 0.419 λ = 0.408 λ. The inductance is

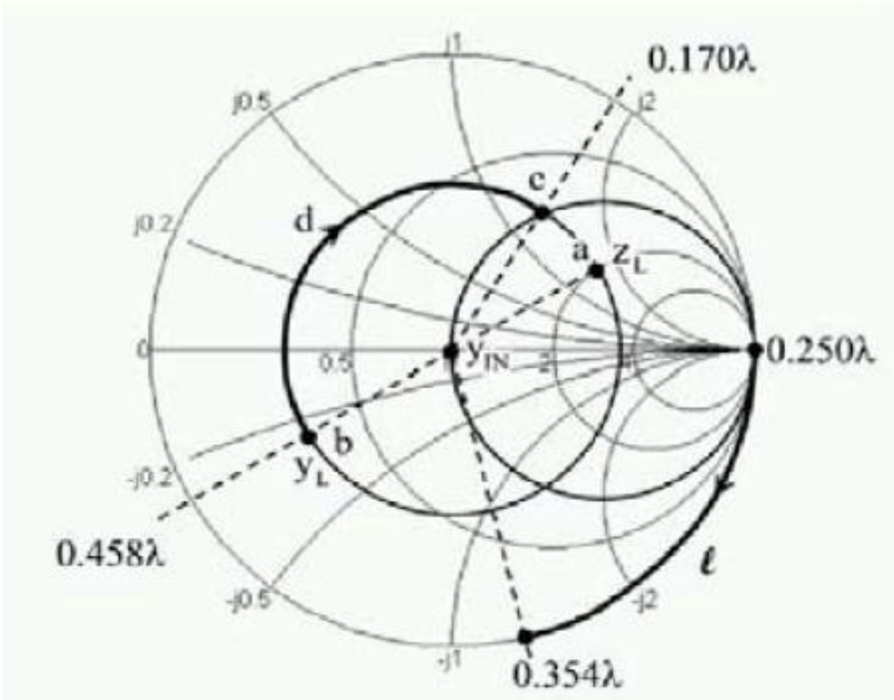
$$\frac{j\omega L}{Z_o} = j1.4, \quad L = \frac{(1.4)(100)}{2\pi\left(1x10^9\right)} = \text{ 22.3 nH}$$



<u>Problem 3 (20 pts):</u>

A load impedance Z_L=200+j160 Ω is to be matched to a 100 Ω line using a shorted shunt stub tuner. Find the solution that minimizes the length of the shorted stub.

Refer to Fig. P2.37a for the shunt stub circuit.
(1)Normalize the load (point a, z_L = 2.0 + j1.6).
(2) locate the normalized load admittance: y_L (point b)
(3) move from point b to point c, at the y=1+jb circle(0.500λ + 0.170λ -0.458λ = 0.212λ)
(4) move from the shorted end of the stub (normalized admittance point) to the point y = 0 – jb. (ℓ = 0.354 λ – 0.250 λ = 0.104 λ.)



0.170λ
c
d
a
z_L
y_{IN}
0.250λ
b
y_L
0.458λ
ℓ
0.354λ