American University of Beirut

Faculty of Engineering and Architecture

Spring 2014

Profs:

Joseph Costantine

Shahwan Khoury

Student Name:

ID number:

Test ID 1001-1002

EECE 380- Engineering Electromagnetics

CLOSED BOOK Exam:

- Programmable Calculators are not allowed
- Provide your answers to the multiple choice questions on the computer card only
- Provide your answers to the comprehensive problems on the exam sheet

Section I: Multiple Choice Questions (40 pts- 8 pts each)

MCQ 1:

The electric field component of a wave in free space is given by $\vec{E} = 10\cos(10^7 t + kz)\hat{a}_y$ V/m, it can be inferred that:

a. The wave propagates along $\hat{a}_y$

b. The wavelength is approximately λ=188.5 m

c. The phase constant is β=0.33 rad/m

d. The wave attenuates as it travels

e. The attenuation constant is $10\sqrt{2}$ Np/m

Complete Solution

b.

Version 1:

$$w = 2\pi f = 10^7 \Rightarrow f = \frac{10^7}{2\pi} \Rightarrow \lambda = c/f = \frac{3\times10^8 \times 2\times\pi}{10^7} = 188.5$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{188.4} = 0.033 rad/m$$

Version 2:

$$\vec{E} = 10\cos(2\times10^7 t + kz)\hat{a}_y$$

$$w = 2\pi f = 2\times10^7 \Rightarrow f = \frac{2\times10^7}{2\pi} \Rightarrow \lambda = c/f = \frac{3\times10^8 \times 2\times\pi}{2\times10^7} = 94.25$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{188.4} = 0.033 rad/m$$

MCQ 2:

A wave traveling along a string is given by: $y(x,t) = 2\sin(4\pi t + 10\pi x)$, identify the wrong statement below

a. The wave propagates in the –x direction
b. The frequency of operation is 2 Hz
c. The phase velocity is 0.4 m/s
d. The amplitude of the wave is 2
e. The reference phase Φ_0 =0°

Complete Solution

Solution:

(a) We start by converting the given expression into a cosine function of the form given by (1.17):

$$y(x,t) = 2\cos\left(4\pi t + 10\pi x - \frac{\pi}{2}\right) \quad \text{(cm)}.$$

Since the coefficients of t and x both have the same sign, the wave is traveling in the negative x-direction.

(b) From the cosine expression, $\phi_0 = -\pi/2$.

(c) $\omega = 2\pi f = 4\pi$,

$$f = 4\pi/2\pi = 2 \text{ Hz}.$$

(d) $2\pi/\lambda = 10\pi$,

$$\lambda = 2\pi/10\pi = 0.2 \text{ m}.$$

(e) $u_p = f\lambda = 2 \times 0.2 = 0.4$ (m/s).

Version 2:

Y(x,t)=2 sin(4πt+10πx+30°)

Answer is e. reference phase= - 60°

MCQ 3:

The impedance and propagation constant at 100 MHz for a transmission line are determined to be $Z_0 = 18.6 - j0.253\Omega$ and $\gamma = 0.0638 + j4.68 m^{-1}$. Calculate the approximated distributed parameters

a. $R' = 2.37\frac{\Omega}{m}, L' = 138.5\frac{nH}{m}, G' = 7.629\frac{\mu S}{m}, C' = 400.43\frac{pF}{m}$

b. $R' = 3.27\frac{\Omega}{m}, L' = 319.5\frac{nH}{m}, G' = 3.7629\frac{\mu S}{m}, C' = 400.43\frac{pF}{m}$

c. $R' = 3.27\frac{\Omega}{mm}, L' = 319.5\frac{nH}{mm}, G' = 3.7629\frac{\mu S}{mm}, C' = 400.43\frac{pF}{mm}$

d. $R' = 1.27\frac{\Omega}{m}, L' = 250.5\frac{nH}{m}, G' = 6.5\frac{\mu S}{m}, C' = 451\frac{pF}{m}$

e. None of the above

Complete Solution

$$Z_o = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}, \quad \gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$Z_o \gamma = R' + j\omega L' = 2.37 + j87.0$$

$$\therefore R' = 2.37 \frac{\Omega}{m}, \quad \omega L' = 87.0 \text{ so } L' = 139 \frac{nH}{m}$$

$$\frac{\gamma}{Z_o} = G' + j\omega C' = 7.63x10^{-6} + 0.252,$$

$$G' = 7.629 \frac{\mu S}{m} \text{ and } C' = 400.43 \frac{pF}{m}$$

Version 2:

F=500 MHz

$$L' = 27.7 \frac{nH}{m}$$

$$C' = 80 \; pF / m$$

MCQ 4:

The reflection coefficient at the load for a 50Ω line is measured as $\Gamma_L = 0.516 e^{j8.2^\circ}$ at f=1 GHz. Find the equivalent circuit for Z_L in its approximated values.

a. R=120Ω, L=5.2 nH in series
b. R=125Ω, L=2.5 nH in parallel
c. R=150Ω, L=4.8 nH in series
d. R=510Ω, L=8.4 nH in parallel
e. None of the above

Complete Solution

Rearranging $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$, we find $Z_L = Z_o \frac{1+\Gamma_L}{1-\Gamma_L} = 150 + j30\ \Omega$.

This is a resistor in series with an inductor. The inductor is found by considering

$j\omega L = j30$, or $L = \frac{30}{2\pi\left(1x10^9\right)} = 4.8nH$,

So the load is a 150 Ω resistor in series with a 4.8 nH inductor.

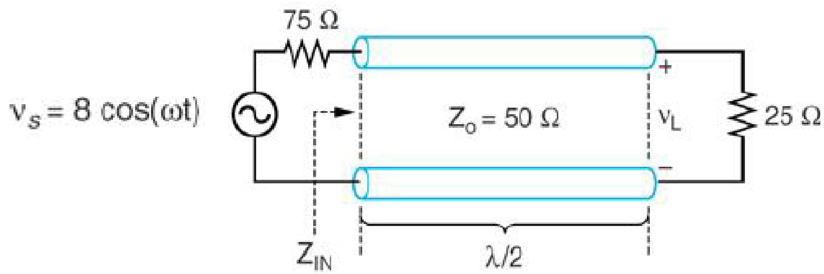
Version 2:

The transmission line has a characteristic impedance of 75Ω.

$$Z_L = 225 + j45$$
$$R = 225\Omega$$
$$L = \frac{45}{2\pi \times 10^9} = 7.16nH$$

MCQ 5:

For the lossless transmission line circuit shown below, determine the input impedance Z_{in} and V_L, the instantaneous voltage at the load end.



a. Z_{in}=30Ω, V_L= 2cos(wt+60°) v
b. Z_{in}=35Ω, V_L= 2cos(wt+90°) v
c. Z_{in}=50Ω, V_L= 2cos(wt+120°) v
d. Z_{in}=25Ω, V_L= 2cos(wt+180°) v
e. None of the above

Complete Solution

$$\Gamma_L = \frac{25-50}{25+50} = -\frac{1}{3}, \quad \beta\ell = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi, \ \tan\pi = 0$$

$$Z_{in} = Z_o \frac{Z_L + 0}{Z_L + 0} = Z_L = 25\Omega$$

$$V_{in} = \frac{25}{25+75} 8V = 2V = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$$

$$2 = V_o^+ \left(e^{j\beta\ell} + \Gamma_L e^{-j\beta\ell}\right)$$

$$e^{j\pi} = \cos\pi + j\sin\pi = -1, \quad e^{-j\pi} = -1,$$

$$V_o^+ \left(-1 - \frac{1}{3}(-1)\right) = \frac{-2}{3} V_o^+ = 2; \quad V_o^+ = -3V$$

$$V_L = V_o^+ (1 + \Gamma_L) = -3\left(1 - \frac{1}{3}\right) = -2V$$

$$v_L = 2\cos\left(\omega t + 180^\circ\right) V$$

Version 2:

R_g=25Ω

Z_{in}=25 Ω; V_L=4 cos(wt+180º)

Section II: Comprehensive Problems

Problem 1 (20 pts):

An electric field propagating in the +Z direction is given by:

$$E(z,t) = 100 e^{-0.010z} \cos(\pi \times 10^7 t - \pi z - \frac{\pi}{4}) \frac{V}{m}$$

a) Determine the attenuation constant, the wave frequency, the wavelength, the propagation velocity (phase velocity) and the phase shift (reference phase) (10 pts):

(a) From inspection of the given equation, we see
Attenuation constant: α = 0.010 Np/m
Wave frequency: since $\omega = 2\pi f$, we have $f = \omega/2\pi = 5\times10^6$ 1/s = 5 MHz
Wavelength: since $\beta = 2\pi/\lambda$, we have $\lambda = 2\pi/\beta = 2\pi/\pi$ =2 m
Propagation velocity: $u_p = \lambda f$ = (2m)(5MHz) = 1×10^7 m/s
Phase shift: $\phi = -\pi/4$ radians

b) How far must the wave travel before its amplitude is reduced to 1 V/m (5pts)

(b)

$$1\frac{V}{m} = 100.e^{-.010z}\frac{V}{m}, \text{ so } e^{-.010z} = \frac{1}{100} = 0.01,$$

$$-0.010z = \ln(0.01), \quad z = \frac{\ln(0.01)}{-0.01\,rad/m} = 460m.$$

c) Express the propagating electric field in phasor form (5 pts)

$$E(z,t) = 100.e^{-.010z}\cos\left(\pi\times10^{7}t - \pi z - \frac{\pi}{4}\right)\frac{V}{m}.$$

This is equivalent to:

$$E(z,t) = \mathrm{Re}\left[100.e^{-.010z}e^{-j\pi z}e^{-j\pi/4}e^{j\pi\times10^{7}t}\right]\frac{V}{m}.$$

Suppressing the "Re" and the "$e^{j\omega t}$", the phasor is

$$E_s = 100.e^{-.010z}e^{-j\pi z}e^{-j\pi/4}\frac{V}{m}$$

Problem 2 (20 pts):

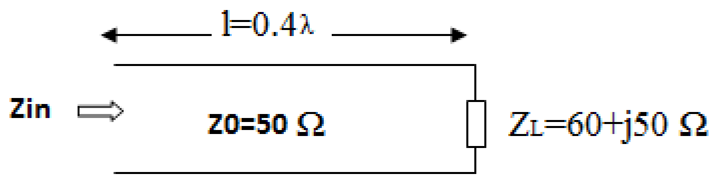
Use the **Smith Chart** to answer the following questions:

a) Find the shortest lengths of a short-circuited 75 Ω line to give the following impedance (2 Pts):
   1- $Z_1=0$ (1/2 pt)
   2- $Z_2=\infty$ (1/2 pt)
   3- $Z_3=j15\Omega$ (1 pt)

   1- L=0 or 0.5λ
   2- L=0.25λ
   3- L=0.032λ

b) Find the following parameters for the transmission line circuit below (18 pts):

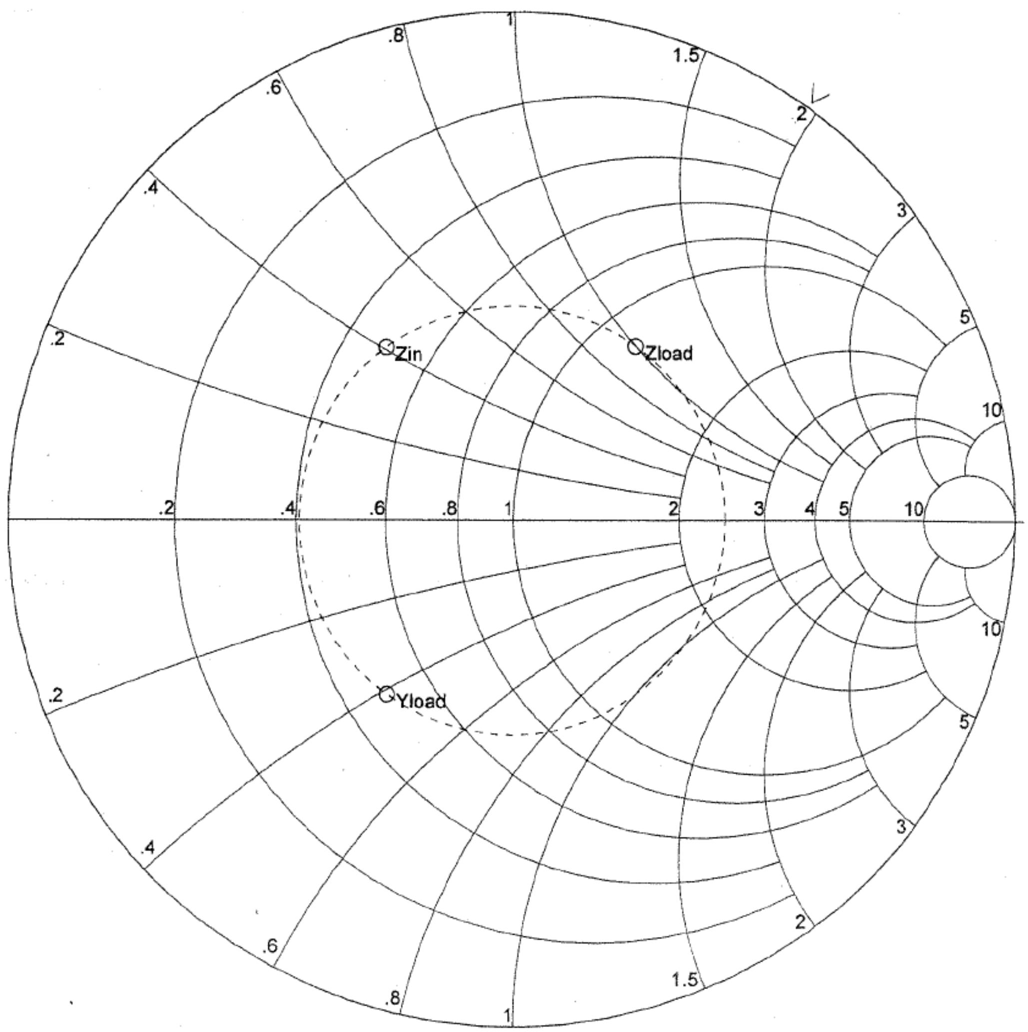
1- The SWR on the line
2- The reflection coefficient at the load
3- The load admittance
4- The input impedance of the line
5- The distance from the load to the first voltage minimum
6- The distance from the load to the first voltage maximum



$Z_0=50\ \Omega$, $Z_L=60+j50\Omega$, $l=0.4\lambda$

From Smith chart, $z_l=1.2+j1.0$

1- SWR=2.46
2- $\Gamma=0.422\angle 54^\circ$
3- $Y_L=(0.492-j0.410)/50=9.84-j8.2$ mS
4- $Z_{in}=24.5+j20.3\ \Omega$
5- Lmin=0.325λ
6- Lmax=0.075λ

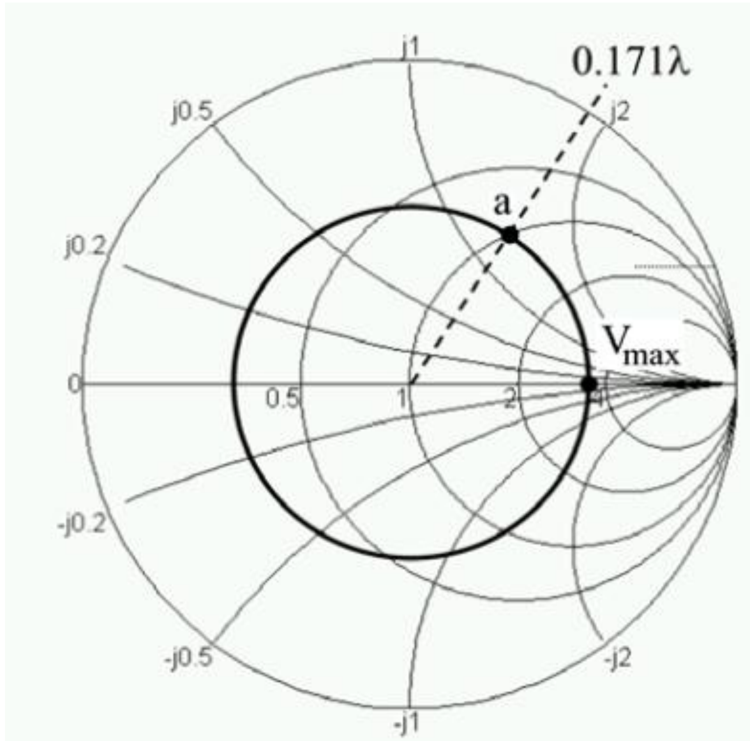


Problem 3 (20 pts):

As an RF engineer working for a very famous RF design corporation, you are being asked to determine an unknown load connected to a 50Ω lossless transmission line. Being an experimentalist you have decided to execute some measurements and come up with your conclusions. Your first observations indicated that the measured VSWR is 3.4 and that a voltage maximum is located at 0.079λ away from the load.

a) Determine the load (10 pts):

We can use the given VSWR to draw a constant Γ circle as shown in the figure. Then we move from V_{max} at WTG = 0.250λ to point a at WTG = 0.250 λ - 0.079 λ = 0.171 λ. At this point we have $z_L = 1 + j1.3$, or $Z_L = 50 + j65\ \Omega$.



b) Your boss was impressed with your results so he gives you another task. He asks you to use another 50Ω transmission line that is terminated by 100+j150 Ω load and determine where will the line impedance be 50+j110Ω (10 pts)?

$$z_L = \frac{Z_L}{Z_o} = \frac{100 + j150}{50} = 2 + j3$$

$$z_{in} = \frac{Z_{in}}{Z_o} = \frac{50 + j110}{50} = 1 + j2.2$$

$$d = 0.2133\lambda - 0.1913\lambda = 0.022\lambda \text{ toward the generator}$$

$$= 0.478\lambda \text{ toward the load}$$