Quiz 2 . V. 1000 .

1. Region $y \leq 0$ is a perfect conductor while region $y \geq 0$ is a dielectric medium with $\varepsilon_{1r} = 2.5$. If there is a surface charge of $3nC/m^2$ on the conductor, Determine D at P (-6, 1, 7).

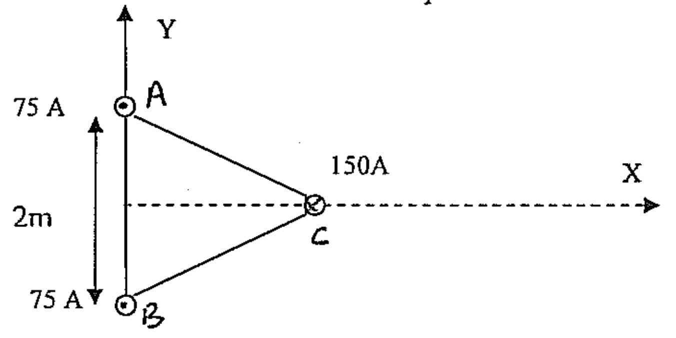
   a. $-1.2\ nC/m^2\ a_y$
   b. $3\ nC/m^2\ a_y$
   c. $3nC/m^2\ a_z$
   d. $-7.5\ nC/m^2\ a_x$
   e. None of the above

2. A straight wire of diameter 0.5 mm carrying a current of 1 A is replaced by another wire of 1 mm diameter carrying same current. The strength of the magnetic field far away is?

   a. twice the earlier value
   b. same as the earlier value
   c. one-half of the earlier value
   d. one-quarter of the earlier value
   e. None of the above

3. A three phase transmission line consists of 3 conductors that are supported at points A, B, C to form an equilateral triangle as shown. At one instant, conductors A and B both carry a current of 75A while conductor C carries a return current of 150A. Find the force per meter on conductor C at that instant.

   a. $1.366\ a_x$ mN/m
   b. $0.974\ -a_x$ mN/m
   c. $1.949\ a_x$ mN/m
   d. $1.672\ -a_x$ mN/m
   e. None of the above



4. A coil of one turn is made of a wire of certain length and then from the same length a coil of two turns is made. If the same current is passed in both the cases, then the ratio of the magnetic field intensity at their centers will be?

   a. 2:1
   b. 1:4
   c. 4:1
   d. 1:2
   e. None of the above

5. The magnetic field density in the centre of a circular coil of 50 turns, radius 0.5 m and carrying a current of 2A is ?

a. 0.5 x 10^{-5} T

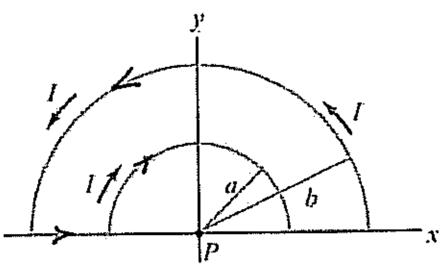
b. 1.25x 10^{-4} T

c. 3x 10^{-5} T

d. 4 x 10^{-5} T

e. None of the above

6. Find the magnetic field density at point P due to the following current distributions



a. $\frac{-\mu_o I}{4}(\frac{1}{b}-\frac{1}{a})a_z$

b. $\frac{-\mu_o I}{4}(\frac{1}{a}+\frac{1}{b})a_z$

c. $\frac{-\mu_o I}{4}(\frac{1}{a}-\frac{1}{b})a_z$

d. $\frac{-\mu_o I}{4}(\frac{ab}{a-b})a_z$

e. None of the above

7. Determine the magnetic moment of an electric circuit formed by the triangular loop of the following figure.

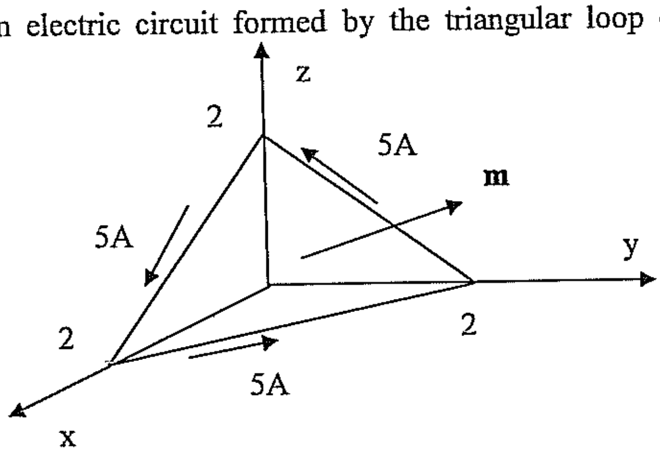
a. $10\left(\vec{a_x}-\vec{a_y}-\vec{a_z}\right)\mathrm{A}\cdot\mathrm{m}^2$

b. $5\left(2\vec{a_x}+1.3\vec{a_y}+\vec{a_z}\right)\mathrm{A}\cdot\mathrm{m}^2$

c. $10\left(\vec{a_x}+\vec{a_y}+\vec{a_z}\right)\mathrm{A}\cdot\mathrm{m}^2$

d. $5\left(\vec{a_x}-1.3\vec{a_y}-\vec{a_z}\right)\mathrm{A}\cdot\mathrm{m}^2$

e. None of the above

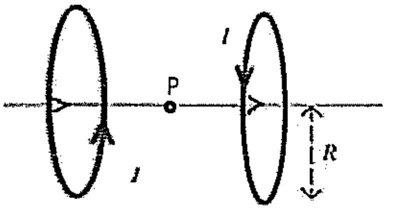


8. Given the magnetic vector potential $\vec{\mathbf{A}} = -r^2/4\ \overline{a_z}$ Wb/m, calculate the total magnetic flux crossing the surface $\phi = \pi/2,\ 1 \le r \le 2\text{m},\ \ 0 \le z \le 5\text{m}$

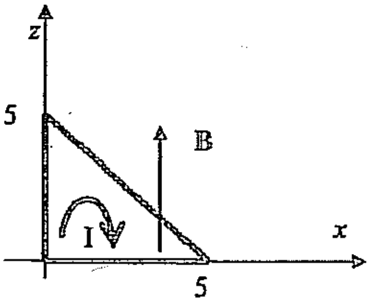
a. 2.25 Wb
b. 5.73 Wb
c. 7.21 Wb
d. 3.75 Wb.
e. None of the above

9. Two identical coaxial circular coils carry the same current I but in opposite directions. The magnitude of the magnetic field density B at a point on the axis midway between the coils is:

a. Zero
b. The same as that produced by one coil
c. Twice that produced by one coil
d. Half that produced by one coil
e. None of the above

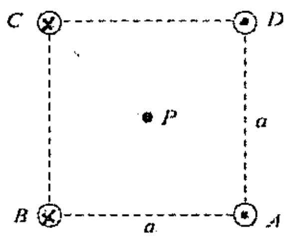


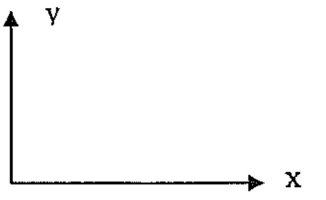
10. A triangular loop is placed in the x-z plane as shown in the figure below. Assume that a dc current I=2 A flows in the loop and that B=30 a_z mWb/m exists in the region. Find the total force and torque on the loop.



a. 0 N & 0.9 a_x N.m
b. 200 N a_x & 0.2 a_x N.m
c. 0 N and 0.75 a_x N.m
d. 150 a_x mN and 0 N.m
e. None of the above

11. Four infinitely long parallel wires carrying equal current I are arranged in such a way that when looking at the cross section, they are at the corners of a square, as shown in the figure below. Currents in A and D point out of the page, and into the page at B and C. What is the magnetic field density at the center P of the square?





a. $\dfrac{-\mu_o I}{4\pi a} a_y$

b. $\dfrac{-2\mu_o I}{\pi a} a_y$

c. $\dfrac{-\mu_o I}{\pi a^2} a_y$

d. $\dfrac{-4\mu_o I}{\sqrt{2}\pi a} a_y$

e. None of the above

12. A unit normal vector from region 2 (μ=$2\mu_o$) to region 1 (μ=μ_o) is $a_{n21} = (6\overline{a_x} + 2\overline{a_y} - 3\overline{a_z})/7$. If H_1=10 a_x +a_y +12 a_z A/m and H_2= H_{2x} – 5a_y +4 a_z A/m. Find H_{2x}.

a. 3.44
b. 5.83
c. 7.92
d. 8.82
e. None of the above

13. In the above problem, find the surface current density J on the interface

a. $4.86\overline{a_x} - 8.64\overline{a_y} + 3.95\overline{a_z}$ A/m
b. $3.82\overline{a_x} - 3.24\overline{a_y} + 8.67\overline{a_z}$ A/m
c. $7.67\overline{a_x} + 2.64\overline{a_y} + 6.23\overline{a_z}$ A/m
d. $5.36\overline{a_x} - 9.21\overline{a_y} + 4.88\overline{a_z}$ A/m
e. None of the above

14. A cylindrical capacitor has radii a=1cm and b=2.5cm. If the space between the plates is filled with an inhomogeneous dielectric with $\varepsilon_r = (10+r)/r$, where r is in centimeters. Determine the capacitance per meter of the capacitor.

a. 377.8 pF/m
b. 256.8 pF/m
c. 673.2 pF/m
d. 434.6 pF/m
e. None of the above.

EECE 380
Quiz #2

1- the point $P(-6,1,7)$ is in the dielectric medium since $y = 1 > 0$ at P. So

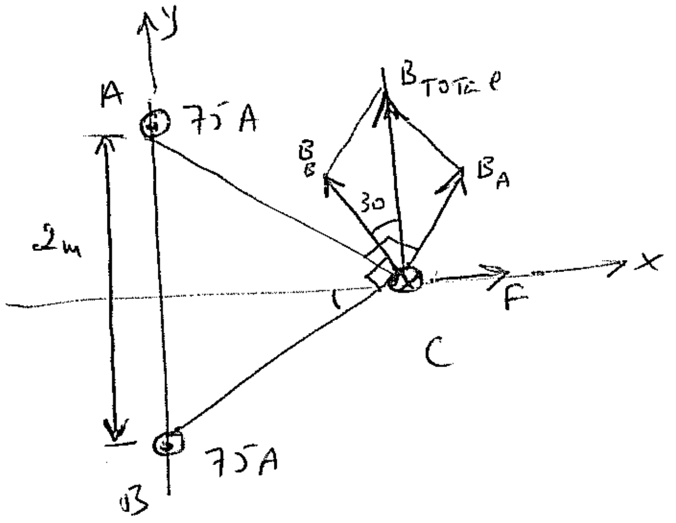
$$D_{yn} = \rho_s = 3\,nC/m^2$$

so $\vec{D} = 3\vec{a}_y\ nC/m^2$

2- Apply $\oint \vec{H}\cdot d\vec{l} = \vec{I}$ for a contour outside the conductor to find out that the diameter size is of no importance

answer: same as earlier value

3-



$$\vec{B}_A = \frac{\mu_0 I}{2\pi r}\vec{a}_\phi$$

$$= \frac{\mu_0 . 75}{2\pi . 2}\cos 30\ \vec{a}_y$$

$$= \frac{\mu_0 . 75 . \sqrt{3}}{8\pi}\vec{a}_y$$

$$\vec{B}_B = \frac{\mu_0 . 75 . \sqrt{3}}{8\pi}\vec{a}_y$$

$$B_R = \frac{\mu_0 . 75 . \sqrt{3}}{4\pi} = \frac{\cancel{4\pi} \times 10^{-7} \times 75 \times \sqrt{3}}{\cancel{4\pi}}\vec{a}_y$$

$$F/m = I\oint d\vec{l} \times \vec{B} = 150\left(-\vec{a}_z \times 10^{-7} \times 75 . \sqrt{3}\,\vec{a}_y\right)$$

$$= \vec{a}_x\left(150 \times 75 \times \sqrt{3} \times 10^{-7}\right)$$

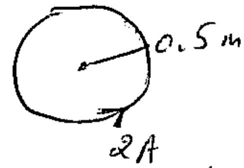
$$\simeq 1.949\ mN/m.$$

4- $\oint \vec{H}\cdot d\vec{l} = NI$

case 1: $H = \frac{I}{2\pi a}$ & case 2: $I = 2I$ & $a = \frac{a}{2}$ so $H = \frac{4I}{2\pi a}$

ratio: 4:1

5 -



$N = 50$ turns

$$\vec{H} = \frac{N I r^2}{2(r^2+h^2)^{\frac{3}{2}}} \vec{a}_z$$

$$h = 0 \Rightarrow |H| = \frac{N I \cancel{r^2}}{2 \cancel{r^3}} = \frac{NI}{2r}$$

$$B = \mu H = \frac{4\pi \times 10^{-7} \times 50 \times 2}{2 \times 0.5} = \frac{12.56 \times 10^{-5}}{1}$$

$$= 1.25 \times 10^{-4} \text{ T}.$$

6 - there is no magnetic field due to straight segments because point P is along the lines.

By Biot-Savart - law : $B = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \vec{\ }}{R^2}$

$d\vec{\ell} \times \vec{a}_R = a\, d\phi\ \vec{a}_z$ & $|R| = a$

so $B_{outer} = \frac{\mu_0 . I}{4\pi} \int_0^{\pi} \frac{b\, d\phi}{b^2} \vec{a}_z = \frac{\mu_0 I}{4 b}$ (out of the page $(\vec{a}_z)$)

$$B_{inner} = \frac{\mu_0 I}{4 a} \quad \text{(into the page)} \ (-a_z)$$

so $B = \frac{\mu_0 I}{4}\left(\frac{1}{b} - \frac{1}{a}\right) \vec{a}_z$

7 - $\vec{m} = I A \vec{a}_n$

$A = \frac{1}{2}$ base $\times$ height $= \frac{1}{2}(2\sqrt{2})(2\sqrt{2}) \sin 60 = 4 . \frac{\sqrt{3}}{2} = 2\sqrt{3}$

eq. of plane ; $f = x + y + z - 2 = 0$

$$\vec{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\vec{a}_x + \vec{a}_y + \vec{a}_z}{\sqrt{3}}$$

or $m = 5 \times 2\sqrt{3} \cdot \dfrac{\vec{a}_x + \vec{a}_y + \vec{a}_z}{\sqrt{3}}$

$= 10\,(\vec{a}_x + \vec{a}_y + \vec{a}_z)$

8 - $A = -\dfrac{r^2}{4}\vec{a}_z$

$B = \nabla \times A = -\dfrac{\partial A_z}{\partial r}\vec{a}_\phi = \dfrac{r}{2}\vec{a}_\phi$

& $\varphi = \int \vec{B} \cdot d\vec{s} = \int_{z=0}^{5}\int_{r=1}^{2} \dfrac{r}{2}\vec{a}_\phi \cdot (dr\,dz)\vec{a}_\phi$

$= \dfrac{r^2}{4}\Big|_1^2 \cdot (5) = \dfrac{15}{4} = 3.75$ Wb.

9 - Zero

10 - $F = 0$ (closed loop)

$T = \vec{m} \times \vec{B}$

$m = I A \vec{a}_n = 2 \times \dfrac{5 \times 5}{2} = 25\,(+\vec{a}_y)$

so $T = +25\vec{a}_y \times \underset{\substack{(mWb/m) \\ (T)}}{30\,\vec{a}_z} = 0{,}750\,\vec{a}_x$ (N.m)

11.

$$B = \frac{\mu_0 I}{2\pi r} \vec{a}_\phi$$

$$B_A = \frac{\mu_0 I}{2\pi \left(\frac{a}{\sqrt{2}}\right)} \left(-\frac{1}{\sqrt{2}} \vec{a}_x - \frac{1}{\sqrt{2}} \vec{a}_y \right)$$

$$B_B = \frac{\mu_0 I}{2\pi \left(\frac{a}{\sqrt{2}}\right)} \left(\frac{1}{\sqrt{2}} \vec{a}_x - \frac{1}{\sqrt{2}} \vec{a}_y \right)$$

$$B_C = \frac{\mu_0 I}{2\pi \left(\frac{a}{\sqrt{2}}\right)} \left(-\frac{1}{\sqrt{2}} \vec{a}_x - \frac{1}{\sqrt{2}} \vec{a}_y \right)$$

$$B_D = \frac{\mu_0 I}{2\pi \left(\frac{a}{\sqrt{2}}\right)} \left(\frac{1}{\sqrt{2}} \vec{a}_x - \frac{1}{\sqrt{2}} \vec{a}_y \right)$$

$$B_T = B_A + B_B + B_C + B_D = \frac{-2\mu_0 I}{\pi a} \vec{a}_y$$

12. $H_{1n} = (\vec{H_1} \cdot \vec{a}_n)\vec{a}_n = \frac{1}{49}(156\vec{a}_x + 52\vec{a}_y - 78\vec{a}_z)$

$H_{2n} = (\vec{H_2} \cdot \vec{a}_n)\vec{a}_n = \frac{1}{49}\left[(6H_{2x} - 22)(6\vec{a}_x + 2\vec{a}_y - 3\vec{a}_z)\right]$

So $B_{1n} = B_{2n}$

or $\mu_1 H_1 = \mu_2 H_2$

or $\cancel{\mu_0}\left(\underline{156\vec{a}_x}\right) = \left(\underline{36 H_{2x} - 132}\right) \cdot 2\cancel{\mu_0}$

or

$156 + 264 = 72 H_{2x}$

$420 = 72 H_{2x}$ or $H_{2x} = 5.83$

13. $\vec{H}_1 = 10\vec{a}_x + \vec{a}_y + 12\vec{a}_z$

$\vec{H}_2 = 5.83\vec{a}_x - 5\vec{a}_y + 4\vec{a}_z$

$\vec{a}_{n_{21}} = \left(6\vec{a}_x + 2\vec{a}_y - 3\vec{a}_z\right)/7.$

$$\vec{a}_{n_{21}} \times (\vec{H}_1 - \vec{H}_2) = J_s$$

$$J_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \\ 4.17 & 6 & 8 \end{vmatrix}$$

$$J_s = 4.86\vec{a}_x - 8.64\vec{a}_y + 3.95\vec{a}_z$$

14. $$V = -\int_b^a \frac{Q}{2\pi\varepsilon_0\varepsilon_r rL}\,dr = \frac{Q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r\left(\frac{10+r}{r}\right)}$$

$$= -\frac{Q}{2\pi\varepsilon_0 L}\ln(10+r)\Big|_b^a = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{10+b}{10+a}$$

$$C = \frac{Q}{V} = 434.6\ \mathrm{pF/m}.$$