AMERICAN UNIVERSITY OF BEIRUT

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FACULTY OF ENGINEERING

FALL TERM 2013-14

Name:....................................
ID: ....

Oct. 9, 2013

TEST ID: 1000

(EECE380) ENGINEERING ELECTROMAGNETICS

CLOSED BOOK (1 ½ HRS)

- Programmable Calculators are not allowed
- Provide your answers on the computer's card only
- Return the computer's card attached to the question sheet
- Mark with a pencil your last name, first name initial (FI) and father's name initial (MI).
- Mark your AUB ID NO. in the box titled "Social Security No."
- The test ID No. is your exam version. Mark it in the box titled ' Test ID".
- Use pencil for marking your answers
- When using eraser, be sure that you have erased well

1. Let $\overrightarrow{F}(r,\varphi,z) = r\cos\varphi\ \overrightarrow{a_r} + z\sin\varphi\ \overrightarrow{a_z}$. Find $\oint \overrightarrow{F} \cdot \overrightarrow{dl}$ along the path shown in Fig.1

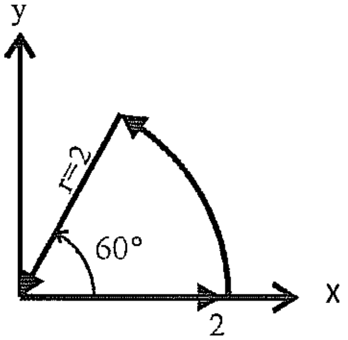


Figure 1

a. 3
b. ½
c. 1
d. √3/2
e. None of the above

2. Given V(R,θ,φ)=R $\sin^2\theta \cos\varphi$, Calculate $|\nabla V(2,\pi/6,\pi/2)|$.
a. 0
b. 1 /2
c. 1
d. √3/2
e. None of the above

3. Three point charges are located on the x-axis. The first charge q_1 = 10 μC is at x = -1m. The second charge q_2 = 20 μC is at the origin. The third charge q_3 = - 30 μC is located at x = 2.0 m. What is the force on q_2?

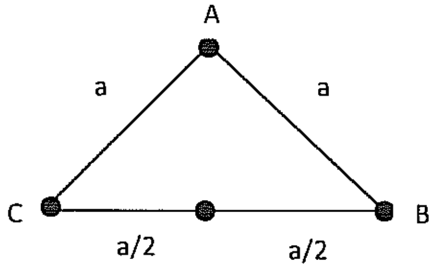
a. 1.65 N in the negative x- direction
b. 3.15 N in the positive x- direction
c. 1.50 N in the negative x- direction
d. 4.80 N in the positive x- direction
e. None of the above

4. A charge Q exerts a 12N force on another charge q. If the distance between the charges is doubled, what is the magnitude of the force on Q by q?
a. 3N
b. 6N
c. 24N
d. 36N
e. None of the above

5. At what separation will two charges, each of magnitude 6 μC, exert a force of 1.4 N ?
   a. 5.1 μm
   b. 0.23m
   c. 0.48m
   d. 2m
   e. None of the above

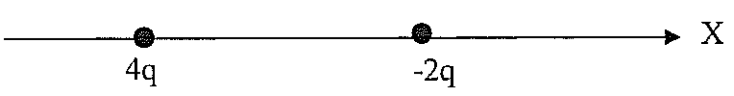
6. The figure below shows an equilateral triangle **ABC**. A positive point charge $+q$ is located at each of the three vertices **A**, **B**, and **C**. Each side of the triangle is of length a. A point charge Q (that may be positive or negative) is placed at the mid-point between **B** and **C**.

   Is it possible to choose the value of Q (*that is non-zero*) such that the force on Q is zero?



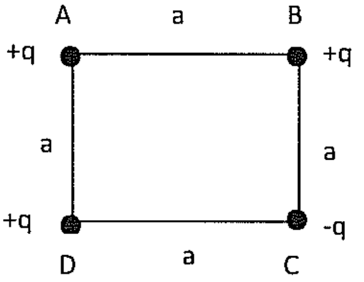
   a. Yes, because the forces on Q are vectors and three vectors can add to zero.
   b. Yes, because the electric force at the mid-point between **B** and **C** is zero whether a charge is placed there or not.
   c. No, because the forces on Q due to the charges at **B** and **C** point in the same direction.
   d. No, because a fourth charge would be needed to cancel the force on Q due to the charge at **A**.
   e. None of the above

7. At which point (or points) is the electric field zero for the two point charges shown on the x axis?



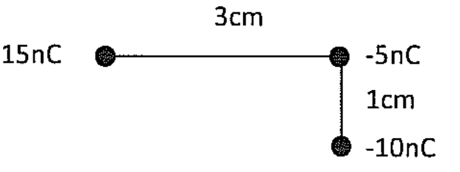
   a. The electric field is never zero in the vicinity of these charges.
   b. The electric field is zero somewhere on the x axis to the left of the $+4q$ charge.
   c. The electric field is zero somewhere on the x axis to the right of the $-2q$ charge.
   d. The electric field is zero somewhere on the x axis between the two charges, but this point is nearer to the $-2q$ charge.
   e. None of the above

8. Consider a square with side a. Four charges +q, +q, −q, and +q are placed at the corners A, B, C, and D, respectively. The **magnitude** of the electric field at D due to the charges at A, B, and C is given by: (consider $1/4\pi\varepsilon_o$ to be K)



a. $|E|=(3/2)\,K.q/a^2$
b. $|E|=(7/2)\,K.q/a^2$
c. $|E|=(5/2)\,K.q/a^2$
d. $|E|=(9/4)\,K.q/a^2$
e. None of the above

9. The charge in the bottom right corner of the figure is Q =- 10 nC. What is the **magnitude** of the force on Q?



a. 3.32×10^{-3} N
b. 5.29×10^{-3} N
c. 4.27×10^{-3} N
d. 7.94×10^{-3} N
e. None of the above

10. Given $E=(3x^2+y)\,a_x + x\,a_y$ (kV/m). Find the work done in moving a -2μC charge from (0, 5,0) to (2, -1, 0) by taking the path y=5-3x.

a. 6mJ
b. 12mJ
c. 18mJ
d. 8mJ
e. None of the above

11. Two dipoles with dipole moments $-5a_z$ nC.m and $9a_z$ nC.m are located at points (0,0,-2) and (0,0,3) respectively. Find the potential at the origin.

a. -10.2 V
b. -15.4 V
c. -20.2 V
d. -26.3V
e. None of the above

12. Consider $D=(2y^2+z)\,a_x + 4xy\,a_y + x\,a_z$ C/m^2. Find the flux $\oint_s \vec{D}\cdot\vec{ds}$ through the cube defined by $0\le x\le 1$, $0\le y\le 1$, $0\le z\le 1$.

a. 1C
b. 4C
c. 3C
d. 2C
e. None of the above

GOOD LUCK !

$$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{2a} \ln \frac{x-a}{x+a} + C, & x^2 > a^2 \\ \frac{1}{2a} \ln \frac{a-x}{a+x} + C, & x^2 < a^2 \end{cases}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right) + C$$

$$\int \frac{x\,dx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2} + C$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x/a^2}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{x^2\,dx}{(x^2 + a^2)^{3/2}} = \ln\left(\frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a}\right) - \frac{x}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{dx}{(x^2 + a^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2 + a^2} + \frac{1}{a} \tan^{-1} \frac{x}{a}\right) + C$$

$$\int \sqrt{1 - x^2}\, dx = \frac{1}{2} x\sqrt{1 - x^2} + \sin^{-1} x$$

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \dfrac{\partial}{\partial R} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

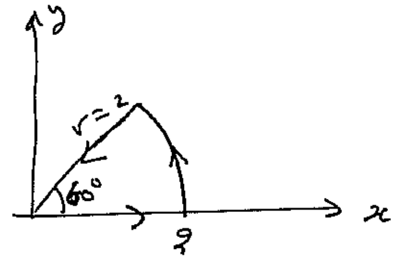
$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, A =	$\hat{x}A_x + \hat{y}A_y + \hat{z}A_z$	$\hat{r}A_r + \hat{\phi}A_\phi + \hat{z}A_z$	$\hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$
Magnitude of A, $\|A\|$ =	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1}$ =	$\hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1$, for $P(x_1, y_1, z_1)$	$\hat{r}r_1 + \hat{z}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{R}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$ $\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{z} = \hat{z}\cdot\hat{x} = 0$ $\hat{x}\times\hat{y} = \hat{z}$ $\hat{y}\times\hat{z} = \hat{x}$ $\hat{z}\times\hat{x} = \hat{y}$	$\hat{r}\cdot\hat{r} = \hat{\phi}\cdot\hat{\phi} = \hat{z}\cdot\hat{z} = 1$ $\hat{r}\cdot\hat{\phi} = \hat{\phi}\cdot\hat{z} = \hat{z}\cdot\hat{r} = 0$ $\hat{r}\times\hat{\phi} = \hat{z}$ $\hat{\phi}\times\hat{z} = \hat{r}$ $\hat{z}\times\hat{r} = \hat{\phi}$	$\hat{R}\cdot\hat{R} = \hat{\theta}\cdot\hat{\theta} = \hat{\phi}\cdot\hat{\phi} = 1$ $\hat{R}\cdot\hat{\theta} = \hat{\theta}\cdot\hat{\phi} = \hat{\phi}\cdot\hat{R} = 0$ $\hat{R}\times\hat{\theta} = \hat{\phi}$ $\hat{\theta}\times\hat{\phi} = \hat{R}$ $\hat{\phi}\times\hat{R} = \hat{\theta}$
Dot product, A · B =	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, A × B =	$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l}$ =	$\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$	$\hat{r}\,dr + \hat{\phi}r\,d\phi + \hat{z}\,dz$	$\hat{R}\,dR + \hat{\theta}R\,d\theta + \hat{\phi}R\sin\theta\,d\phi$
Differential surface areas	$ds_x = \hat{x}\,dy\,dz$ $ds_y = \hat{y}\,dx\,dz$ $ds_z = \hat{z}\,dx\,dy$	$ds_r = \hat{r}r\,d\phi\,dz$ $ds_\phi = \hat{\phi}\,dr\,dz$ $ds_z = \hat{z}r\,dr\,d\phi$	$ds_R = \hat{R}R^2\sin\theta\,d\theta\,d\phi$ $ds_\theta = \hat{\theta}R\sin\theta\,dR\,d\phi$ $ds_\phi = \hat{\phi}R\,dR\,d\theta$
Differential volume, $d\mathcal{V}$ =	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

EECE 380 – Quiz #1 – solution
OCT. 9, 2013

1. Find $\oint \vec{F} \cdot d\vec{\ell}$ if $\vec{F}(r,\varphi,z) = r\cos\varphi\,\vec{a_r} + z\sin\phi\,\vec{a_z}$



$$\int_0^2 r\cos\varphi\,dr + 0 + \int_2^0 r\cos\varphi\,dr$$

$$= \frac{r^2}{2}\cos 0 \Big/_0^2 + \frac{r^2}{2}\Big/_2^0 \cos 60$$

$$= \frac{4}{2} - \frac{4}{2}\cdot\frac{1}{2} = 1$$

2- $V(R,\theta,\varphi) = R\sin^2\theta\cos\varphi$ find $|\nabla V(2, \frac{\pi}{6}, \frac{\pi}{2})|$.

$$\nabla V = \vec{a_R}\,(\sin^2\theta\cos\varphi) + \vec{a_\theta}\,\frac{1}{R}(2R\sin\theta\cos\theta\cos\varphi) +$$

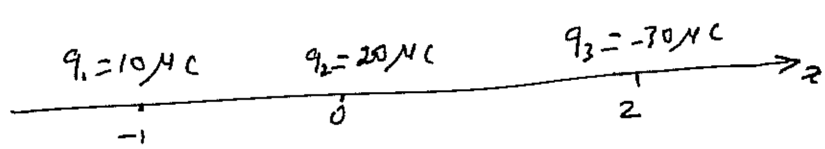
$$\vec{a_\varphi}\,(-R\sin^2\theta\sin\varphi)$$

$$|\nabla V| = \sqrt{(\sin^2\theta\cos\varphi)^2 + \left(\frac{1}{R}(R\sin 2\theta\cos\varphi)\right)^2 + (-R\sin^2\theta\sin\varphi)^2}$$

$$= \sqrt{0 + 0 + \left(2\cdot\left(\frac{1}{4}\right)\cdot 1\right)^2}$$

$$= \frac{1}{2}$$

3- Find F_{q_2}?



$$F_{q_2} = \frac{20\times10^{-6}}{4\pi\varepsilon_0}\left[\frac{10\times10^{-6}(0-(-1))}{|0-(-1)|^3} - \frac{30\times10^{-6}(0-2)}{|0-(2)|^3}\right]\vec{a_x}$$

$$= \frac{20\times10^{-6}\times10^{-6}}{4\pi\varepsilon_0}\left[\frac{10}{1} + \frac{60}{8}\right] = \frac{20\times10^{-12}}{\cancel{4\pi}\cdot\frac{1}{\cancel{36\pi}_{9}}\times10^{-9}}\left[\frac{80+60}{8}\right]\vec{a_x}$$

$$= 20\times9\times10^{-3}\left[\frac{140}{8}\right]\vec{a_x}$$

$$= 3.15\,\vec{a_x}$$

4. $F = \frac{qQ}{4\pi\varepsilon_0 R^2} = 12\,N$

if $R = 2R \rightarrow F_{New} = \frac{qQ}{4\pi\varepsilon_0 (2R)^2} = \frac{qQ}{4\pi\varepsilon_0 R^2 \cdot 4} = \frac{12}{4} = 3N$

12N

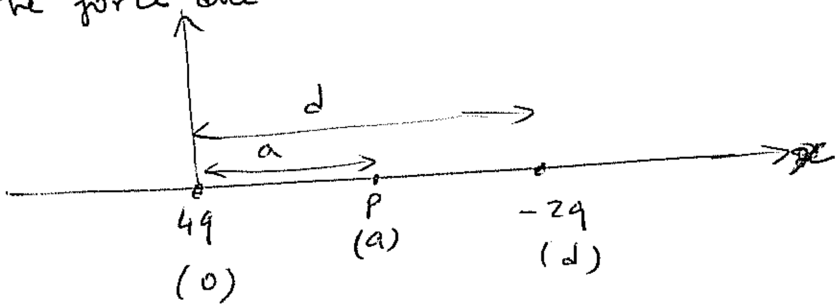
5- $F = 1.4\,N = \frac{6\times10^{-6}\times6\times10^{-6}}{4\pi\varepsilon_0 d^2} \Rightarrow d^2 = \frac{10^{-12}\times\cancel{36}}{1.4\times\cancel{4}\times\cancel{\pi}\cdot\frac{1}{\cancel{36\pi}}\times10^{-9}}$ 9

$= \frac{10^{-12}\times\cancel{36}\times9\times10^{9}}{1.4} = \frac{10^{-3}\times36\times9}{1.4}$

$\Rightarrow d = 0.48\,m$

6- the force on Q from B & C will cancel each other. So the answer is no, because a fourth charge will be needed to cancel the force due to charge on A.

7-



$E = \frac{4q}{4\pi\varepsilon_0 a^2}\vec{a}_{R_1} + \frac{-2q}{4\pi\varepsilon_0 (d-a)^2}\vec{a}_{R_2}$

$\vec{a}_R = \frac{\vec{R}}{|R|} = \frac{a\,\vec{a}_x}{|a|}$ & $\vec{a}_{R_2} = \frac{\vec{R}}{|R|} = \frac{(a-d)\,\vec{a}_x}{|a-d|} = \frac{d-a}{|a-d|}\vec{a}$

$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{4a}{a^3}\vec{a}_x - \frac{2(d-a)\,\vec{a}_x}{|a-d|^3}\right]$

$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4}{a^2} - \frac{2}{(d-a)^2}\right]\vec{a}_x = 0$

$\Rightarrow \frac{4}{a^2} - \frac{2}{(d-a)^2} = 0 \Rightarrow 4d^2 + 4a^2 - 8da = 2a^2$

$\Rightarrow 2a^2 - 8d\,a + 4d^2 = 0$

$\Delta = (8d)^2 - 4.2.\,4d^2 = 64d^2 - 32d^2 = 32d^2$

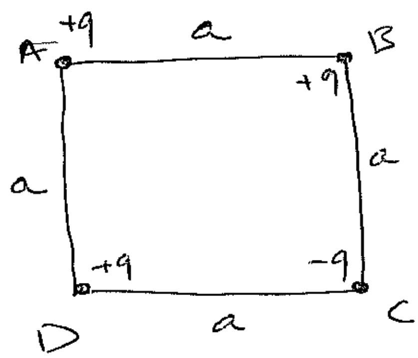
$$\Rightarrow \quad a_1 = \frac{+8d + d\sqrt{32}}{4} = 3.14\,d$$

$$a_2 = \frac{8d - d\sqrt{32}}{4} = 0.585\,d$$

So the electric field is zero on the axis to the righ

8 -

find E at D due to charges at A, B & C.



the magnitude of E at D due to A & C are

$$E_A = E_C = \frac{kq}{a^2} \qquad \text{where } k = \frac{1}{4\pi\varepsilon_0}$$

whereas $$E_B = \frac{kq}{(a\sqrt{2})^2} = \frac{kq}{2a^2}$$

As for directions, $\vec{E}_A$ (downwards) ~~since~~ and $\vec{E}_C$ (right) are at right angles & can be added:

$$|\vec{E}_A + \vec{E}_C| = \frac{kq}{a^2}\sqrt{1+1} = \sqrt{2}\,\frac{kq}{a^2}$$

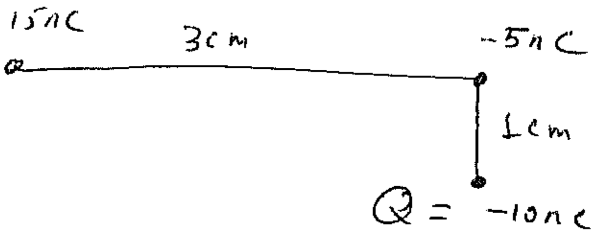
this resultant vector points 45° down & the vector $\vec{E}_B$ points at 135°. the angle between these is 90° so

$$|(\vec{E}_A + \vec{E}_C) + \vec{E}_B| = \sqrt{(\sqrt{2})^2 + (\tfrac{1}{2})^2}\;\frac{kq}{a^2}$$

or $$|\vec{E}| = \sqrt{\frac{9}{4}}\;\frac{kq}{a^2} = \frac{3}{2}\,\frac{kq}{a^2}.$$

9 -

find F on Q.



$$E_{15} = \frac{15\times10^{-9}}{4\pi\varepsilon_0 R^2}\,\vec{a}_R$$

$$\vec{R} = 0.03\,\vec{a}_x - 0.01\,\vec{a}_y \quad \& \ |R| = \sqrt{(0.03)^2+(0.01)^2}$$

$$\text{so } E_{15} = \frac{15\times9}{\left[(0.03)^2+(0.01)^2\right]^{3/2}}\left(0.03\,\vec{a}_x - 0.01\,\vec{a}_y\right)$$

$$\& \ E_{-5} = \frac{-5\times10^{-9}}{4\pi\varepsilon_0 R^2}\,\vec{a}_R \ ; \ \vec{R} = -0.01\,\vec{a}_y$$

$$|R| = 0.01$$

$$= \frac{45}{(0.01)^2}\,\vec{a}_y$$

$$\text{or } E_T = E_{15} + E_{-5} = \left(1.28\times10^5\,\vec{a}_x + 4.08\,\vec{a}_y\times10^5\right)$$

$$\Rightarrow |E_T| = 4.27\times10^5 \ \& \ F = -10\times10^{-9}\times4.27\times10^5$$

$$= -4.27\times10^{-3}\,N.$$

10. $d\vec{l} = \vec{a}_x\,dx + \vec{a}_y\,dy \quad ; \quad y = 5-3x \Rightarrow dy = -3\,dx$

$$W = \left(-2\times10^{-6}\int(3x^2+y)\,dx + \int x\,dy\right)kV$$

$$= \left(-2\times10^{-6}\int_0^2(3x^2+5-3x)\,dx + \int_0^2 x(-3\,dx)\right)kw$$

$$= \left(-2\times10^{-6}\left[\left(\frac{\cancel{3}x^3}{\cancel{3}}+5x-\frac{3x^2}{2}\right)\Big|_0^2 + \frac{-3x^2}{2}\Big|_0^2\right]\right)kV$$

$$= -2\times10^{-6}\left[8+10-6-6\right] = -12\times10^{-6}\times\underset{(V)}{10^3} = -12\,mJ.$$

11 -

$$V = \sum_{k=1}^{2} \frac{\vec{p}_k \cdot \vec{r}_k}{4\pi\varepsilon_0 r_k^3} = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}_1 \cdot \vec{r}_1}{r_1^3} + \frac{\vec{p}_2 \cdot \vec{r}_2}{r_2^3}\right]$$

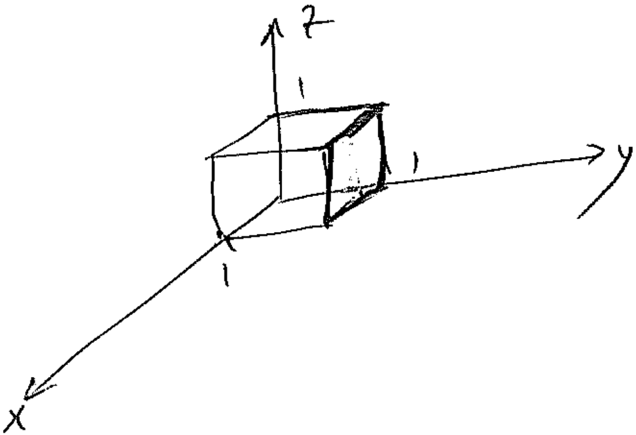
$$\vec{p}_1 = -5\,\vec{a}_z \qquad \vec{r}_1 = (0,0,0) - (0,0,-2) = 2\vec{a}_z \;\; \& \; |r_1| = 2$$

$$p_2 = 9\,\vec{a}_z \qquad \vec{r}_2 = (0,0,0) - (0,0,3) = -3\vec{a}_z \;\; \& \; |r_2| = 3$$

$$\text{so} \quad V = \frac{1}{4\pi \times \frac{1}{36\pi} \times 10^{-9}} \left[\frac{-10}{2^3} - \frac{27}{3^3}\right] \times 10^{-9}$$

$$= -20.25 \text{ V}$$

12.



TOP: $z = 1$.

$\vec{ds} = dx\,dy\,\vec{a}_z$

$\int_{TOP} D \cdot ds = \int x\,dx\,dy$

BOTTOM: $\vec{ds} = -dx\,dy\,\vec{a}_z$

$\int D \cdot ds = -\int x\,dx\,dy$ $\quad$ so $\int_{TOP} + \int_{BOT} = 0$.

Back: $ds = dy\,dz\,(-\vec{a}_x)$ & Front: $ds = dy\,dz\,(\vec{a}_x)$

$\int_{Back} D \cdot ds = \int_B -(2\underbrace{y^2}_{0} + z)\,dx\,dy = \int -z\,dx\,dy$ $\quad$ so $\int_{Back} + \int_{Front} = 0$

$\int_{Front} = \int_F (2\underbrace{y^2}_{0} + z)\,dx\,dy = \int z\,dx\,dy$

Left: $ds = -dx\,dz\,\vec{a}_y$ & $\iint_{Left} \vec{D} \cdot \vec{ds} = \iint 4x\underbrace{y}_{0}\,dx\,dz = 0$

Right: $ds = dx\,dz\,\vec{a}_y$ & $\int_{Right} \vec{D} \cdot \vec{ds} = \iint 4x\underbrace{y}_{=1}\,dx\,dz$

$= 4\frac{x^2}{2}\Big|_0^1 \cdot z\Big|_0^1 = 4\frac{1^2}{2} \cdot 1 = 2$