Chapter 2: Transmission Lines

Lesson #4

Chapter — Section: 2-1, 2-2

Topics: Lumped-element model

Highlights:

- TEM lines
- General properties of transmission lines
- *L*, *C*, *R*, *G*

Lesson #5

Chapter — Section: 2-3, 2-4

Topics: Transmission-line equations, wave propagation

Highlights:

- Wave equation
- Characteristic impedance
- General solution

Special Illustrations:

- Example 2-1

Lesson #6

Chapter — Section: 2-5

Topics: Lossless line

Highlights:

- General wave propagation properties
- Reflection coefficient
- Standing waves
- Maxima and minima

Special Illustrations:

- Example 2-2
- Example 2-5

Lesson #7

Chapter — Section: 2-6

Topics: Input impedance

Highlights:

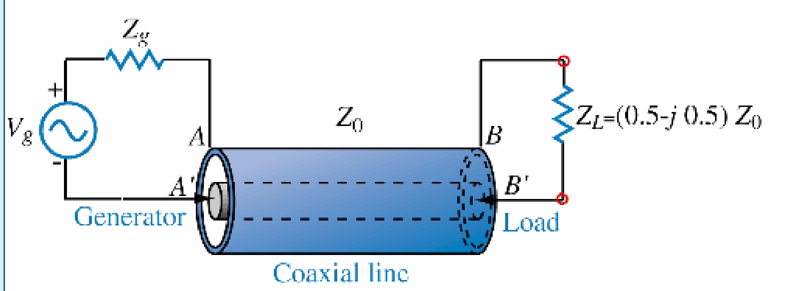
- Thévenin equivalent
- Solution for V and I at any location

Special Illustrations:

- Example 2-6
- CD-ROM Modules 2.1-2.4, Configurations A-C
- CD-ROM Demos 2.1-2.4, Configurations A-C

Module 2.4B: $Z_L=(0.5 - j\,0.5)Z_0$

Given: A coaxial line connected as shown.



$V_g = 10 \cos \omega t$
$Z_g = 50\ \Omega$
$Z_0 = 50\ \Omega$
$\omega = 3\pi \times 10^9$ rad/s
$\varepsilon_r = 4$
Line length $l = 24$ cm
$Z_L = (0.5 - j\,0.5)\,Z_0$
Diameter ratio $b/a = 5$

* From Exercise 2.1B: $\Gamma=0.45\ \angle -116^{\circ}$

* From Exercise 2.2B: $\beta l=4.8\pi$ (rad), and $Z_i = (71 - j56)\ \Omega$

* From Exercise 2.3B: $V_0^+ = 5\text{ V} \angle -144^{\circ}$

Q. Obtain a complete expression for $v(z,t)$. The solution has the general form:

$$v(z,t) = A\cos(3\pi\times 10^9 t - 20\pi z + \phi_1) + B\cos(3\pi\times 10^9 t + 20\pi z + \phi_2) \quad \text{V},$$

with $z = 0$ being located at the load.

A = [ ] check answer | I give up

B = [ ] check answer | I give up

ϕ_1 = [ ] $^{\circ}$ check answer | I give up

ϕ_2 = [ ] $^{\circ}$ check answer | I give up

Lessons #8 and 9

Chapter — Section: 2-7, 2-8

Topics: Special cases, power flow

Highlights:

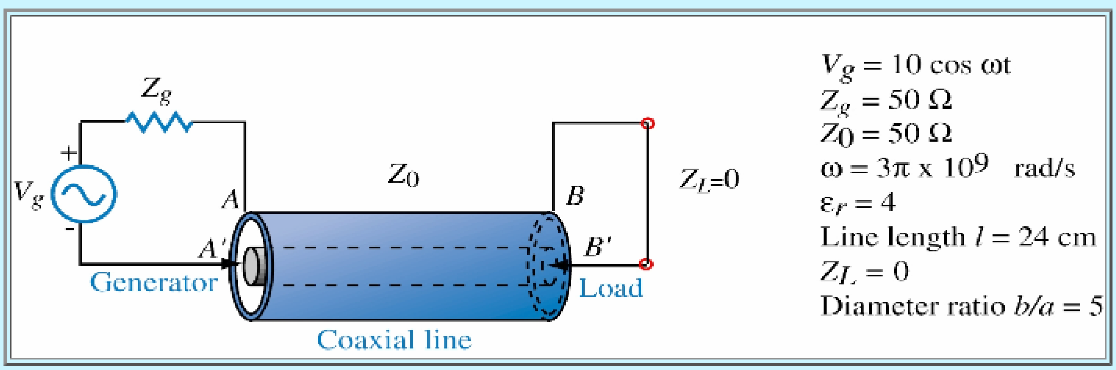
- Sorted line
- Open line
- Matched line
- Quarter-wave transformer
- Power flow

Special Illustrations:

- Example 2-8
- CD-ROM Modules 2.1-2.4, Configurations D and E
- CD-ROM Demos 2.1-2.4, Configurations D and E

Demo 2.2D: $Z_L = 0\ \Omega$

Given: A coaxial line connected as shown.



Display $\rho_l(z, t)$ for $Z_L = 0$

Lessons #10 and 11

Chapter — Section: 2-9

Topics: Smith chart

Highlights:

- Structure of Smith chart
- Calculating impedances, admittances, transformations
- Locations of maxima and minima

Special Illustrations:

- Example 2-10
- Example 2-11

Lesson #12

Chapter — Section: 2-10

Topics: Matching

Highlights:

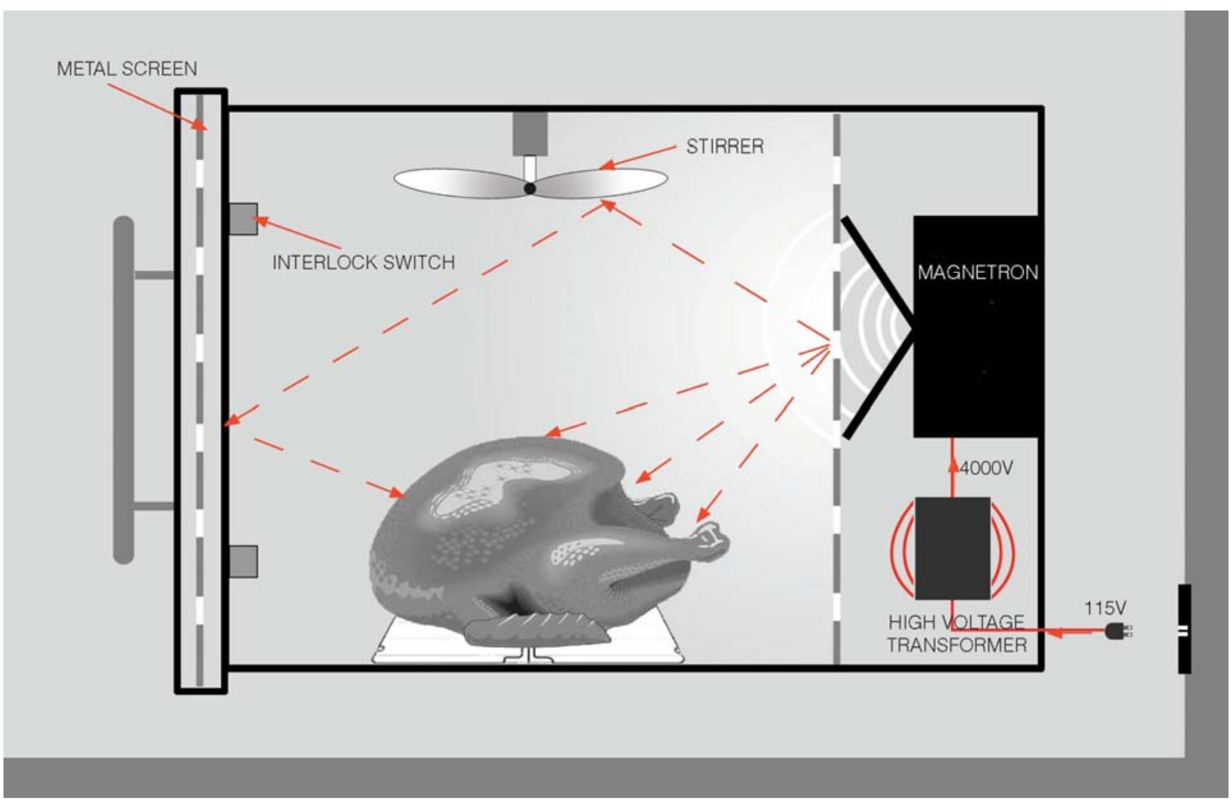
- Matching network
- Double-stub tuning

Special Illustrations:

- Example 2-12
- Technology Brief on "Microwave Oven" (CD-ROM)

Microwave Ovens

Percy Spencer, while working for Raytheon in the 1940s on the design and construction of magnetrons for radar, observed that a chocolate bar that had unintentionally been exposed to microwaves had melted in his pocket. The process of cooking by microwave was patented in 1946, and by the 1970s microwave ovens had become standard household items.



Lesson #13

Chapter — Section: 2-11

Topics: Transients

Highlights:

- Step function
- Bounce diagram

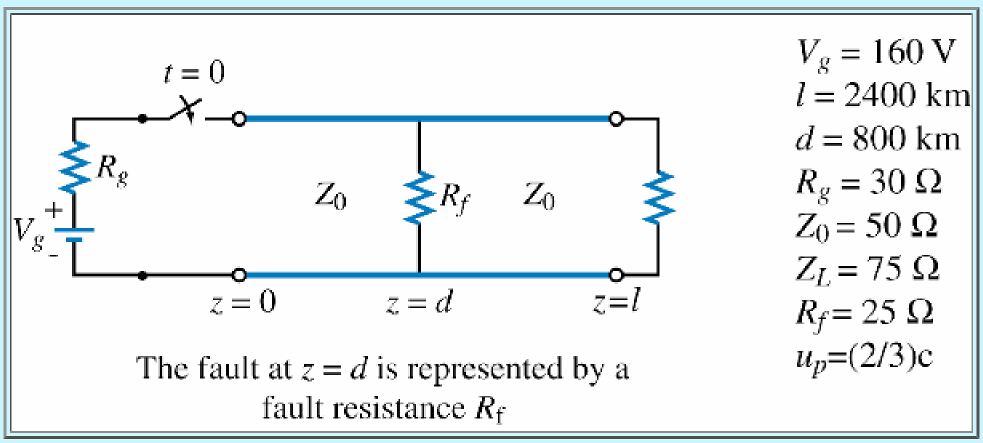
Special Illustrations:

- CD-ROM Modules 2.5-2.9
- CD-ROM Demos 2.5-2.13

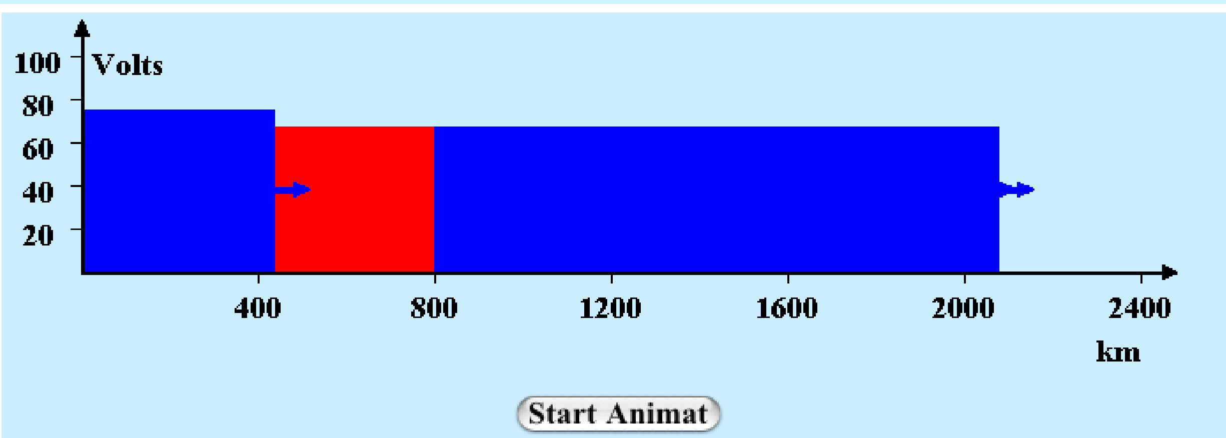
Demo 2.13

Demo 8.13: $R_g = 0.6\, Z_0$, $Z_L = 1.5\, Z_0$

Given: A fault, represented by a 25 Ω shunt resistance, is located at a distance of 800 km from the sending end of a 2400-km long transmission line with $u_p = 2c/3$. The switch is closed at $t = 0$ and the line is not properly matched at either end ($R_g = 0.6Z_0$ and $Z_L = 1.5\, Z_0$).



Display the voltage along the line as a function of time for $t \geq 0$.



Chapter 2

Sections 2-1 to 2-4: Transmission-Line Model

Problem 2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \le 0.01$.

(a) $\dfrac{l}{\lambda} = \dfrac{lf}{u_p} = \dfrac{(20\times10^{-2}\text{ m})\times(20\times10^{3}\text{ Hz})}{3\times10^{8}\text{ m/s}} = 1.33\times10^{-5}$ (negligible).

(b) $\dfrac{l}{\lambda} = \dfrac{lf}{u_p} = \dfrac{(50\times10^{3}\text{ m})\times(60\times10^{0}\text{ Hz})}{3\times10^{8}\text{ m/s}} = 0.01$ (borderline).

(c) $\dfrac{l}{\lambda} = \dfrac{lf}{u_p} = \dfrac{(20\times10^{-2}\text{ m})\times(600\times10^{6}\text{ Hz})}{3\times10^{8}\text{ m/s}} = 0.40$ (nonnegligible).

(d) $\dfrac{l}{\lambda} = \dfrac{lf}{u_p} = \dfrac{(1\times10^{-3}\text{ m})\times(100\times10^{9}\text{ Hz})}{3\times10^{8}\text{ m/s}} = 0.33$ (nonnegligible).

Problem 2.2 Calculate the line parameters R', L', G', and C' for a coaxial line with an inner conductor diameter of 0.5 cm and an outer conductor diameter of 1 cm, filled with an insulating material where $\mu = \mu_0$, $\varepsilon_r = 4.5$, and $\sigma = 10^{-3}$ S/m. The conductors are made of copper with $\mu_c = \mu_0$ and $\sigma_c = 5.8\times10^{7}$ S/m. The operating frequency is 1 GHz.

Solution: Given

$$a = (0.5/2)\text{ cm} = 0.25\times10^{-2}\text{ m},$$
$$b = (1.0/2)\text{ cm} = 0.50\times10^{-2}\text{ m},$$

combining Eqs. (2.5) and (2.6) gives

$$\begin{aligned} R' &= \frac{1}{2\pi}\sqrt{\frac{\pi f\mu_c}{\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right) \\ &= \frac{1}{2\pi}\sqrt{\frac{\pi(10^{9}\text{ Hz})(4\pi\times10^{-7}\text{ H/m})}{5.8\times10^{7}\text{ S/m}}}\left(\frac{1}{0.25\times10^{-2}\text{ m}}+\frac{1}{0.50\times10^{-2}\text{ m}}\right) \\ &= 0.788\ \Omega/\text{m}. \end{aligned}$$

From Eq. (2.7),

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times 10^{-7}\text{ H/m}}{2\pi}\ln 2 = 139\text{ nH/m}.$$

From Eq. (2.8),

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi\times 10^{-3}\text{ S/m}}{\ln 2} = 9.1\text{ mS/m}.$$

From Eq. (2.9),

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\varepsilon_r\varepsilon_0}{\ln(b/a)} = \frac{2\pi\times 4.5\times(8.854\times 10^{-12}\text{ F/m})}{\ln 2} = 362\text{ pF/m}.$$

Problem 2.3 A 1-GHz parallel-plate transmission line consists of 1.2-cm-wide copper strips separated by a 0.15-cm-thick layer of polystyrene. Appendix B gives $\mu_c = \mu_0 = 4\pi\times 10^{-7}$ (H/m) and $\sigma_c = 5.8\times 10^7$ (S/m) for copper, and $\varepsilon_r = 2.6$ for polystyrene. Use Table 2-1 to determine the line parameters of the transmission line. Assume $\mu = \mu_0$ and $\sigma \simeq 0$ for polystyrene.

Solution:

$$R' = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f\mu_c}{\sigma_c}} = \frac{2}{1.2\times 10^{-2}}\left(\frac{\pi\times 10^9\times 4\pi\times 10^{-7}}{5.8\times 10^7}\right)^{1/2} = 1.38 \quad (\Omega/\text{m}),$$

$$L' = \frac{\mu d}{w} = \frac{4\pi\times 10^{-7}\times 1.5\times 10^{-3}}{1.2\times 10^{-2}} = 1.57\times 10^{-7} \quad (\text{H/m}),$$

$$G' = 0 \qquad \text{because } \sigma = 0,$$

$$C' = \frac{\varepsilon w}{d} = \varepsilon_0\varepsilon_r\frac{w}{d} = \frac{10^{-9}}{36\pi}\times 2.6\times\frac{1.2\times 10^{-2}}{1.5\times 10^{-3}} = 1.84\times 10^{-10} \quad (\text{F/m}).$$

Problem 2.4 Show that the transmission line model shown in Fig. 2-37 (P2.4) yields the same telegrapher's equations given by Eqs. (2.14) and (2.16).

Solution: The voltage at the central upper node is the same whether it is calculated from the left port or the right port:

$$\begin{aligned} v(z+\tfrac{1}{2}\Delta z,t) &= v(z,t) - \tfrac{1}{2}R'\Delta z\, i(z,t) - \tfrac{1}{2}L'\Delta z\frac{\partial}{\partial t}i(z,t) \\ &= v(z+\Delta z,t) + \tfrac{1}{2}R'\Delta z\, i(z+\Delta z,t) + \tfrac{1}{2}L'\Delta z\frac{\partial}{\partial t}i(z+\Delta z,t). \end{aligned}$$

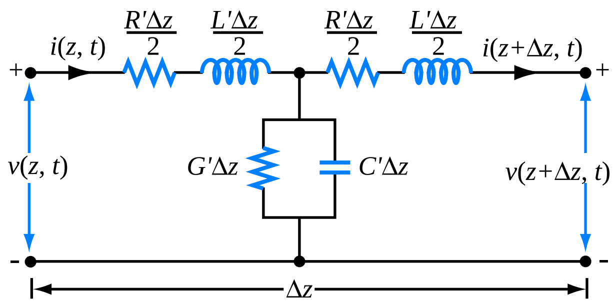


Figure P2.4: Transmission line model.

Recognizing that the current through the $G' \parallel C'$ branch is $i(z,t) - i(z+\Delta z,t)$ (from Kirchhoff's current law), we can conclude that

$$i(z,t) - i(z+\Delta z,t) = G'\Delta z\, v(z+\tfrac{1}{2}\Delta z,t) + C'\Delta z \frac{\partial}{\partial t} v(z+\tfrac{1}{2}\Delta z,t).$$

From both of these equations, the proof is completed by following the steps outlined in the text, ie. rearranging terms, dividing by Δz, and taking the limit as $\Delta z \to 0$.

Problem 2.5 Find $\alpha, \beta, u_{\rm p}$, and Z_0 for the coaxial line of Problem 2.2.

Solution: From Eq. (2.22),

$$\begin{aligned}
\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{(0.788\ \Omega/\text{m}) + j(2\pi \times 10^9\ \text{s}^{-1})(139 \times 10^{-9}\ \text{H/m})} \\
&\quad \times \sqrt{(9.1 \times 10^{-3}\ \text{S/m}) + j(2\pi \times 10^9\ \text{s}^{-1})(362 \times 10^{-12}\ \text{F/m})} \\
&= (109 \times 10^{-3} + j44.5)\ \text{m}^{-1}.
\end{aligned}$$

Thus, from Eqs. (2.25a) and (2.25b), $\alpha = 0.109$ Np/m and $\beta = 44.5$ rad/m.

From Eq. (2.29),

$$\begin{aligned}
Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} &= \sqrt{\frac{(0.788\ \Omega/\text{m}) + j(2\pi \times 10^9\ \text{s}^{-1})(139 \times 10^{-9}\ \text{H/m})}{(9.1 \times 10^{-3}\ \text{S/m}) + j(2\pi \times 10^9\ \text{s}^{-1})(362 \times 10^{-12}\ \text{F/m})}} \\
&= (19.6 + j0.030)\ \Omega.
\end{aligned}$$

From Eq. (2.33),

$$u_{\rm p} = \frac{\omega}{\beta} = \frac{2\pi \times 10^9}{44.5} = 1.41 \times 10^8\ \text{m/s}.$$

Section 2-5: The Lossless Line

Problem 2.6 In addition to not dissipating power, a lossless line has two important features: (1) it is dispertionless (μ_{p} is independent of frequency) and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad \text{(distortionless line)}.$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \qquad \beta = \omega\sqrt{L'C'}, \qquad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned}
\gamma = \alpha + j\beta &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{G'}{C'} + j\omega\right)} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{R'}{L'} + j\omega\right)} \\
&= \sqrt{L'C'}\left(\frac{R'}{L'} + j\omega\right) = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}.
\end{aligned}$$

Hence,

$$\alpha = \mathfrak{Re}(\gamma) = R'\sqrt{\frac{C'}{L'}}, \qquad \beta = \mathfrak{Im}(\gamma) = \omega\sqrt{L'C'}, \qquad u_{\text{p}} = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}\sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$

Problem 2.7 For a distortionless line with $Z_0 = 50\ \Omega$, $\alpha = 20$ (mNp/m), $u_{\text{p}} = 2.5 \times 10^8$ (m/s), find the line parameters and λ at 100 MHz.

Solution: The product of the expressions for α and Z_0 given in Problem 2.6 gives

$$R' = \alpha Z_0 = 20 \times 10^{-3} \times 50 = 1 \quad (\Omega/\text{m}),$$

and taking the ratio of the expression for Z_0 to that for $u_{\text{p}} = \omega/\beta = 1/\sqrt{L'C'}$ gives

$$L' = \frac{Z_0}{u_{\text{p}}} = \frac{50}{2.5 \times 10^8} = 2 \times 10^{-7} \text{ (H/m)} = 200 \quad \text{(nH/m)}.$$

With L' known, we use the expression for Z_0 to find C':

$$C' = \frac{L'}{Z_0^2} = \frac{2 \times 10^{-7}}{(50)^2} = 8 \times 10^{-11} \text{ (F/m)} = 80 \quad \text{(pF/m)}.$$

The distortionless condition given in Problem 2.6 is then used to find G'.

$$G' = \frac{R'C'}{L'} = \frac{1 \times 80 \times 10^{-12}}{2 \times 10^{-7}} = 4 \times 10^{-4} \text{ (S/m)} = 400 \quad (\mu\text{S/m}),$$

and the wavelength is obtained by applying the relation

$$\lambda = \frac{\mu_{\text{p}}}{f} = \frac{2.5 \times 10^8}{100 \times 10^6} = 2.5 \text{ m}.$$

Problem 2.8 Find α and Z_0 of a distortionless line whose $R' = 2\ \Omega$/m and $G' = 2 \times 10^{-4}$ S/m.

Solution: From the equations given in Problem 2.6,

$$\alpha = \sqrt{R'G'} = [2 \times 2 \times 10^{-4}]^{1/2} = 2 \times 10^{-2} \quad \text{(Np/m)},$$

$$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{R'}{G'}} = \left(\frac{2}{2 \times 10^{-4}}\right)^{1/2} = 100\ \Omega.$$

Problem 2.9 A transmission line operating at 125 MHz has $Z_0 = 40\ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters R', L', G', and C'.

Solution: Given an arbitrary transmission line, $f = 125$ MHz, $Z_0 = 40\ \Omega$, $\alpha = 0.02$ Np/m, and $\beta = 0.75$ rad/m. Since Z_0 is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.6, $\beta = \omega\sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \text{ nH/m}.$$

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \text{ nH/m}}{40^2} = 23.9 \text{ pF/m}.$$

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'}\sqrt{\frac{R'}{G'}} = \sqrt{R'G'}\sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \text{ Np/m} \times 40\ \Omega = 0.6\ \Omega\text{/m}$$

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \text{ Np/m})^2}{0.8\ \Omega\text{/m}} = 0.5 \text{ mS/m}.$$

Problem 2.10 Using a slotted line, the voltage on a lossless transmission line was found to have a maximum magnitude of 1.5 V and a minimum magnitude of 0.6 V. Find the magnitude of the load's reflection coefficient.

Solution: From the definition of the Standing Wave Ratio given by Eq. (2.59),

$$S = \frac{|\widetilde{V}|_{\text{max}}}{|\widetilde{V}|_{\text{min}}} = \frac{1.5}{0.6} = 2.5.$$

Solving for the magnitude of the reflection coefficient in terms of S, as in Example 2-4,

$$|\Gamma| = \frac{S-1}{S+1} = \frac{2.5-1}{2.5+1} = 0.43.$$

Problem 2.11 Polyethylene with $\varepsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with characteristic impedance of 50 Ω. The radius of the inner conductor is 1.2 mm.

(a) What is the radius of the outer conductor?

(b) What is the phase velocity of the line?

Solution: Given a lossless coaxial line, $Z_0 = 50\ \Omega$, $\varepsilon_r = 2.25$, $a = 1.2$ mm:

(a) From Table 2-2, $Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$ which can be rearranged to give

$$b = ae^{Z_0\sqrt{\varepsilon_r}/60} = (1.2 \text{ mm})e^{50\sqrt{2.25}/60} = 4.2 \text{ mm}.$$

(b) Also from Table 2-2,

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25}} = 2.0 \times 10^8 \text{ m/s}.$$

Problem 2.12 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)$ Ω. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,
(b) the standing-wave ratio on the line,
(c) the position of the voltage maximum nearest the load,
(d) the position of the current maximum nearest the load.

Solution:

(a) From Eq. (2.49a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57e^{-j79.8^\circ}.$$

(b) From Eq. (2.59),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

(c) From Eq. (2.56)

$$l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8 \text{ cm}}{4\pi} \frac{\pi \text{ rad}}{180^\circ} + \frac{n \times 8 \text{ cm}}{2}$$
$$= -0.89 \text{ cm} + 4.0 \text{ cm} = 3.11 \text{ cm}.$$

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.58),

$$l_{min} = l_{max} - \lambda/4 = 3.11 \text{ cm} - 8 \text{ cm}/4 = 1.11 \text{ cm}.$$

Problem 2.13 On a 150-Ω lossless transmission line, the following observations were noted: distance of first voltage minimum from the load = 3 cm; distance of first voltage maximum from the load = 9 cm; $S = 3$. Find Z_L.

Solution: Distance between a minimum and an adjacent maximum = $\lambda/4$. Hence,

$$9 \text{ cm} - 3 \text{ cm} = 6 \text{ cm} = \lambda/4,$$

or $\lambda = 24$ cm. Accordingly, the first voltage minimum is at $\ell_{\text{min}} = 3\text{ cm} = \frac{\lambda}{8}$. Application of Eq. (2.57) with $n = 0$ gives

$$\theta_r - 2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = -\pi,$$

which gives $\theta_r = -\pi/2$.

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5.$$

Hence, $\Gamma = 0.5e^{-j\pi/2} = -j0.5$.

Finally,

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 150 \left[\frac{1-j0.5}{1+j0.5}\right] = (90 - j120)\ \Omega.$$

Problem 2.14 Using a slotted line, the following results were obtained: distance of first minimum from the load = 4 cm; distance of second minimum from the load = 14 cm, voltage standing-wave ratio = 1.5. If the line is lossless and $Z_0 = 50\ \Omega$, find the load impedance.

Solution: Following Example 2.5: Given a lossless line with $Z_0 = 50\ \Omega$, $S = 1.5$, $l_{\text{min}(0)} = 4$ cm, $l_{\text{min}(1)} = 14$ cm. Then

$$l_{\text{min}(1)} - l_{\text{min}(0)} = \frac{\lambda}{2}$$

or

$$\lambda = 2 \times (l_{\text{min}(1)} - l_{\text{min}(0)}) = 20\text{ cm}$$

and

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi\text{ rad/cycle}}{20\text{ cm/cycle}} = 10\pi\text{ rad/m}.$$

From this we obtain

$$\begin{aligned}\theta_r = 2\beta l_{\text{min}(n)} - (2n+1)\pi\text{ rad} &= 2 \times 10\pi\text{ rad/m} \times 0.04\text{ m} - \pi\text{ rad} \\ &= -0.2\pi\text{ rad} = -36.0^\circ.\end{aligned}$$

Also,

$$|\Gamma| = \frac{S-1}{S+1} = \frac{1.5-1}{1.5+1} = 0.2.$$

So

$$Z_L = Z_0 \left(\frac{1+\Gamma}{1-\Gamma} \right) = 50 \left(\frac{1+0.2e^{-j36.0^\circ}}{1-0.2e^{-j36.0^\circ}} \right) = (67.0 - j16.4)\ \Omega.$$

Problem 2.15 A load with impedance $Z_L = (25 - j50)\ \Omega$ is to be connected to a lossless transmission line with characteristic impedance Z_0, with Z_0 chosen such that the standing-wave ratio is the smallest possible. What should Z_0 be?

Solution: Since S is monotonic with $|\Gamma|$ (i.e., a plot of S *vs.* $|\Gamma|$ is always increasing), the value of Z_0 which gives the minimum possible S also gives the minimum possible $|\Gamma|$, and, for that matter, the minimum possible $|\Gamma|^2$. A necessary condition for a minimum is that its derivative be equal to zero:

$$0 = \frac{\partial}{\partial Z_0} |\Gamma|^2 = \frac{\partial}{\partial Z_0} \frac{|R_L + jX_L - Z_0|^2}{|R_L + jX_L + Z_0|^2}$$

$$= \frac{\partial}{\partial Z_0} \frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2} = \frac{4R_L(Z_0^2 - (R_L^2 + X_L^2))}{((R_L + Z_0)^2 + X_L^2)^2} .$$

Therefore, $Z_0^2 = R_L^2 + X_L^2$ or

$$Z_0 = |Z_L| = \sqrt{(25^2 + (-50)^2)} = 55.9\ \Omega.$$

A mathematically precise solution will also demonstrate that this point is a minimum (by calculating the second derivative, for example). Since the endpoints of the range may be local minima or maxima without the derivative being zero there, the endpoints (namely $Z_0 = 0\ \Omega$ and $Z_0 = \infty\ \Omega$) should be checked also.

Problem 2.16 A 50-Ω lossless line terminated in a purely resistive load has a voltage standing wave ratio of 3. Find all possible values of Z_L.

Solution:

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5.$$

For a purely resistive load, $\theta_r = 0$ or π. For $\theta_r = 0$,

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma} \right] = 50 \left[\frac{1+0.5}{1-0.5} \right] = 150\ \Omega.$$

For $\theta_r = \pi$, $\Gamma = -0.5$ and

$$Z_L = 50 \left[\frac{1-0.5}{1+0.5} \right] = 15\ \Omega.$$

Section 2-6: Input Impedance

Problem 2.17 At an operating frequency of 300 MHz, a lossless 50-Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)$ Ω. Find the input impedance.

Solution: Given a lossless transmission line, $Z_0 = 50$ Ω, $f = 300$ MHz, $l = 2.5$ m, and $Z_L = (40 + j20)$ Ω. Since the line is air filled, $u_p = c$ and therefore, from Eq. (2.38),

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \text{ rad/m}.$$

Since the line is lossless, Eq. (2.69) is valid:

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) = 50 \left(\frac{(40 + j20) + j50 \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})}{50 + j(40 + j20) \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})} \right)$$
$$= 50 \left(\frac{(40 + j20) + j50 \times 0}{50 + j(40 + j20) \times 0} \right) = (40 + j20) \ \Omega.$$

Problem 2.18 A lossless transmission line of electrical length $l = 0.35\lambda$ is terminated in a load impedance as shown in Fig. 2-38 (P2.18). Find Γ, S, and Z_{in}.

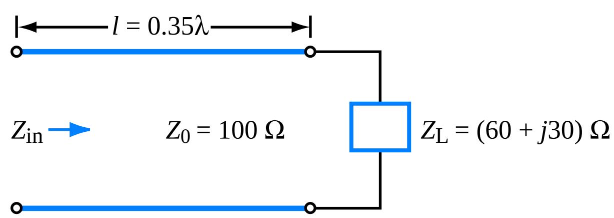


Figure P2.18: Loaded transmission line.

Solution: From Eq. (2.49a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(60 + j30) - 100}{(60 + j30) + 100} = 0.307e^{j132.5^\circ}.$$

From Eq. (2.59),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.307}{1 - 0.307} = 1.89.$$

From Eq. (2.63)

$$Z_{\text{in}} = Z_0 \left(\frac{Z_{\text{L}} + jZ_0 \tan\beta l}{Z_0 + jZ_{\text{L}} \tan\beta l} \right)$$
$$= 100 \left(\frac{(60 + j30) + j100 \tan\left(\frac{2\pi \text{ rad}}{\lambda} 0.35\lambda\right)}{100 + j(60 + j30) \tan\left(\frac{2\pi \text{ rad}}{\lambda} 0.35\lambda\right)} \right) = (64.8 - j38.3)\ \Omega.$$

Problem 2.19 Show that the input impedance of a quarter-wavelength long lossless line terminated in a short circuit appears as an open circuit.

Solution:

$$Z_{\text{in}} = Z_0 \left(\frac{Z_{\text{L}} + jZ_0 \tan\beta l}{Z_0 + jZ_{\text{L}} \tan\beta l} \right).$$

For $l = \frac{\lambda}{4}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$. With $Z_{\text{L}} = 0$, we have

$$Z_{\text{in}} = Z_0 \left(\frac{jZ_0 \tan \pi/2}{Z_0} \right) = j\infty \quad \text{(open circuit)}.$$

Problem 2.20 Show that at the position where the magnitude of the voltage on the line is a maximum the input impedance is purely real.

Solution: From Eq. (2.56), $l_{\max} = (\theta_{\text{r}} + 2n\pi)/2\beta$, so from Eq. (2.61), using polar representation for Γ,

$$Z_{\text{in}}(-l_{\max}) = Z_0 \left(\frac{1 + |\Gamma| e^{j\theta_{\text{r}}} e^{-j2\beta l_{\max}}}{1 - |\Gamma| e^{j\theta_{\text{r}}} e^{-j2\beta l_{\max}}} \right)$$
$$= Z_0 \left(\frac{1 + |\Gamma| e^{j\theta_{\text{r}}} e^{-j(\theta_{\text{r}} + 2n\pi)}}{1 - |\Gamma| e^{j\theta_{\text{r}}} e^{-j(\theta_{\text{r}} + 2n\pi)}} \right) = Z_0 \left(\frac{1 + |\Gamma|}{1 - |\Gamma|} \right),$$

which is real, provided Z_0 is real.

Problem 2.21 A voltage generator with $v_{\text{g}}(t) = 5\cos(2\pi \times 10^9 t)$ V and internal impedance $Z_{\text{g}} = 50\ \Omega$ is connected to a 50-Ω lossless air-spaced transmission line. The line length is 5 cm and it is terminated in a load with impedance $Z_{\text{L}} = (100 - j100)\ \Omega$. Find

(a) Γ at the load.

(b) Z_{in} at the input to the transmission line.

(c) the input voltage $\widetilde{V}_{\text{i}}$ and input current $\tilde{I}_{\text{i}}$.

Solution:

(a) From Eq. (2.49a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62e^{-j29.7^\circ}.$$

(b) All formulae for Z_{in} require knowledge of $\beta = \omega/u_p$. Since the line is an air line, $u_p = c$, and from the expression for $v_g(t)$ we conclude $\omega = 2\pi \times 10^9$ rad/s. Therefore

$$\beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m}.$$

Then, using Eq. (2.63),

$$\begin{aligned} Z_{in} &= Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right) \\ &= 50 \left(\frac{(100 - j100) + j50 \tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)}{50 + j(100 - j100) \tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)} \right) \\ &= 50 \left(\frac{(100 - j100) + j50 \tan\left(\frac{\pi}{3} \text{ rad}\right)}{50 + j(100 - j100) \tan\left(\frac{\pi}{3} \text{ rad}\right)} \right) = (12.5 - j12.7)\ \Omega. \end{aligned}$$

An alternative solution to this part involves the solution to part (a) and Eq. (2.61).

(c) In phasor domain, $\widetilde{V}_g = 5 \text{ V } e^{j0^\circ}$. From Eq. (2.64),

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40e^{-j34.0^\circ} \quad \text{(V)},$$

and also from Eq. (2.64),

$$\widetilde{I}_i = \frac{\widetilde{V}_i}{Z_{in}} = \frac{1.4e^{-j34.0^\circ}}{(12.5 - j12.7)} = 78.4e^{j11.5^\circ} \quad \text{(mA)}.$$

Problem 2.22 A 6-m section of 150-Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30^\circ) \quad \text{(V)}$$

and $Z_g = 150\ \Omega$. If the line, which has a relative permittivity $\varepsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)\ \Omega$, find

(a) λ on the line,

(b) the reflection coefficient at the load,

(c) the input impedance,

(d) the input voltage $\widetilde{V}_i$,
(e) the time-domain input voltage $v_i(t)$.

Solution:

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \text{ V},$$
$$\widetilde{V}_g = 5e^{-j30°} \text{ V}.$$

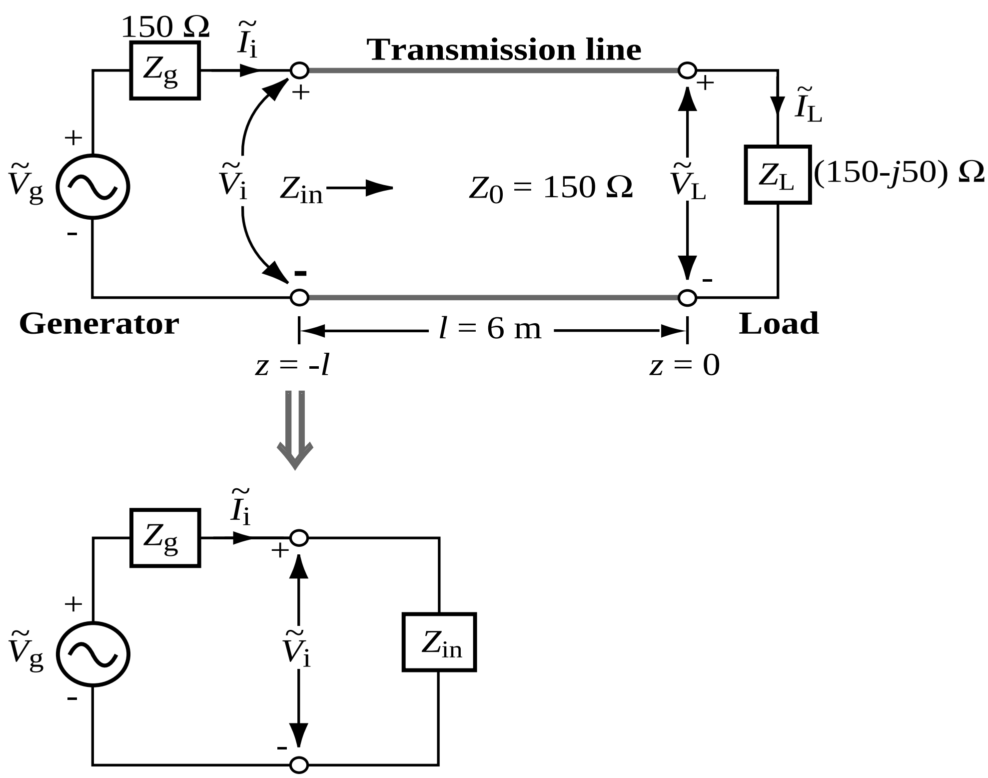


Figure P2.22: Circuit for Problem 2.22.

(a)

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \quad \text{(m/s)},$$
$$\lambda = \frac{u_p}{f} = \frac{2\pi u_p}{\omega} = \frac{2\pi \times 2 \times 10^8}{8\pi \times 10^7} = 5 \text{ m},$$
$$\beta = \frac{\omega}{u_p} = \frac{8\pi \times 10^7}{2 \times 10^8} = 0.4\pi \quad \text{(rad/m)},$$
$$\beta l = 0.4\pi \times 6 = 2.4\pi \quad \text{(rad)}.$$

Since this exceeds 2π (rad), we can subtract 2π, which leaves a remainder $\beta l = 0.4\pi$ (rad).

(b) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{150 - j50 - 150}{150 - j50 + 150} = \dfrac{-j50}{300 - j50} = 0.16 e^{-j80.54^\circ}$.

(c)

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$
$$= 150 \left[\frac{(150 - j50) + j150 \tan(0.4\pi)}{150 + j(150 - j50)\tan(0.4\pi)}\right] = (115.70 + j27.42)\ \Omega.$$

(d)

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5e^{-j30^\circ}(115.7 + j27.42)}{150 + 115.7 + j27.42}$$
$$= 5e^{-j30^\circ}\left(\frac{115.7 + j27.42}{265.7 + j27.42}\right)$$
$$= 5e^{-j30^\circ} \times 0.44 e^{j7.44^\circ} = 2.2 e^{-j22.56^\circ} \quad \text{(V)}.$$

(e)

$$v_i(t) = \mathfrak{Re}[\widetilde{V}_i e^{j\omega t}] = \mathfrak{Re}[2.2 e^{-j22.56^\circ} e^{j\omega t}] = 2.2\cos(8\pi \times 10^7 t - 22.56^\circ)\ \text{V}.$$

Problem 2.23 Two half-wave dipole antennas, each with impedance of 75 Ω, are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. 2.39 (P2.23(a)). All lines are 50 Ω and lossless.

(a) Calculate Z_{in_1}, the input impedance of the antenna-terminated line, at the parallel juncture.

(b) Combine Z_{in_1} and Z_{in_2} in parallel to obtain Z'_L, the effective load impedance of the feedline.

(c) Calculate Z_{in} of the feedline.

Solution:

(a)

$$Z_{in_1} = Z_0 \left[\frac{Z_{L_1} + jZ_0 \tan\beta l_1}{Z_0 + jZ_{L_1} \tan\beta l_1}\right]$$
$$= 50 \left\{\frac{75 + j50 \tan[(2\pi/\lambda)(0.2\lambda)]}{50 + j75 \tan[(2\pi/\lambda)(0.2\lambda)]}\right\} = (35.20 - j8.62)\ \Omega.$$

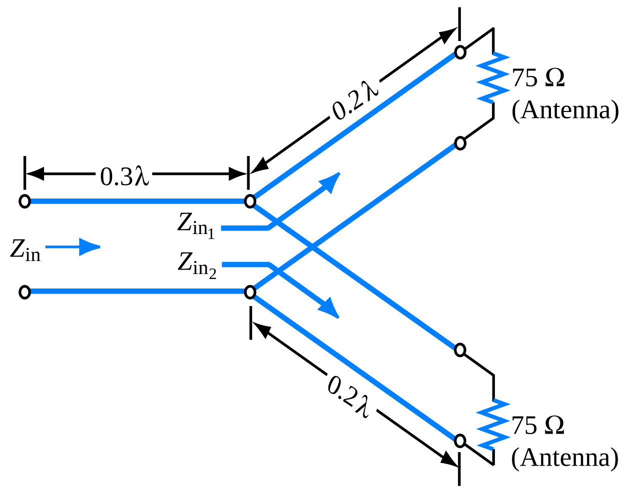


Figure P2.23: (a) Circuit for Problem 2.23.

(b)

$$Z'_L = \frac{Z_{in_1} Z_{in_2}}{Z_{in_1} + Z_{in_2}} = \frac{(35.20 - j8.62)^2}{2(35.20 - j8.62)} = (17.60 - j4.31)\ \Omega.$$

(c)

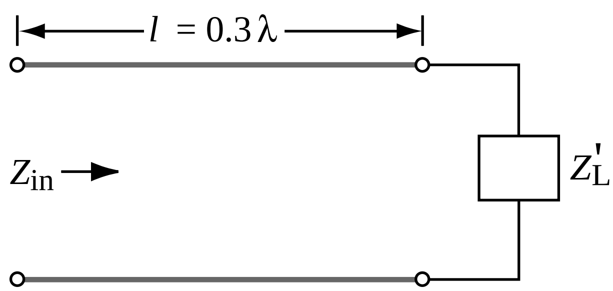


Figure P2.23: (b) Equivalent circuit.

$$Z_{in} = 50 \left\{ \frac{(17.60 - j4.31) + j50 \tan[(2\pi/\lambda)(0.3\lambda)]}{50 + j(17.60 - j4.31) \tan[(2\pi/\lambda)(0.3\lambda)]} \right\} = (107.57 - j56.7)\ \Omega.$$

Section 2-7: Special Cases

Problem 2.24 At an operating frequency of 300 MHz, it is desired to use a section of a lossless 50-Ω transmission line terminated in a short circuit to construct an equivalent load with reactance $X = 40\ \Omega$. If the phase velocity of the line is $0.75c$, what is the shortest possible line length that would exhibit the desired reactance at its input?

Solution:

$$\beta = \omega/u_{\rm p} = \frac{(2\pi \text{ rad/cycle}) \times (300 \times 10^6 \text{ cycle/s})}{0.75 \times (3 \times 10^8 \text{ m/s})} = 8.38 \text{ rad/m}.$$

On a lossless short-circuited transmission line, the input impedance is always purely imaginary; i.e., $Z_{\rm in}^{\rm sc} = jX_{\rm in}^{\rm sc}$. Solving Eq. (2.68) for the line length,

$$l = \frac{1}{\beta} \tan^{-1}\left(\frac{X_{\rm in}^{\rm sc}}{Z_0}\right) = \frac{1}{8.38 \text{ rad/m}} \tan^{-1}\left(\frac{40\ \Omega}{50\ \Omega}\right) = \frac{(0.675 + n\pi) \text{ rad}}{8.38 \text{ rad/m}},$$

for which the smallest positive solution is 8.05 cm (with $n = 0$).

Problem 2.25 A lossless transmission line is terminated in a short circuit. How long (in wavelengths) should the line be in order for it to appear as an open circuit at its input terminals?

Solution: From Eq. (2.68), $Z_{\rm in}^{\rm sc} = jZ_0 \tan\beta l$. If $\beta l = (\pi/2 + n\pi)$, then $Z_{\rm in}^{\rm sc} = j\infty\ (\Omega)$. Hence,

$$l = \frac{\lambda}{2\pi}\left(\frac{\pi}{2} + n\pi\right) = \frac{\lambda}{4} + \frac{n\lambda}{2}.$$

This is evident from Figure 2.15(d).

Problem 2.26 The input impedance of a 31-cm-long lossless transmission line of unknown characteristic impedance was measured at 1 MHz. With the line terminated in a short circuit, the measurement yielded an input impedance equivalent to an inductor with inductance of 0.064 μH, and when the line was open circuited, the measurement yielded an input impedance equivalent to a capacitor with capacitance of 40 pF. Find Z_0 of the line, the phase velocity, and the relative permittivity of the insulating material.

Solution: Now $\omega = 2\pi f = 6.28 \times 10^6$ rad/s, so

$$Z_{\rm in}^{\rm sc} = j\omega L = j2\pi \times 10^6 \times 0.064 \times 10^{-6} = j0.4\ \Omega$$

and $Z_{in}^{oc} = 1/j\omega C = 1/(j2\pi \times 10^6 \times 40 \times 10^{-12}) = -j4000\ \Omega$.

From Eq. (2.74), $Z_0 = \sqrt{Z_{in}^{sc} Z_{in}^{oc}} = \sqrt{(j0.4\ \Omega)(-j4000\ \Omega)} = 40\ \ \Omega$. Using Eq. (2.75),

$$u_p = \frac{\omega}{\beta} = \frac{\omega l}{\tan^{-1}\sqrt{-Z_{in}^{sc}/Z_{in}^{oc}}}$$
$$= \frac{6.28 \times 10^6 \times 0.31}{\tan^{-1}\left(\pm\sqrt{-j0.4/(-j4000)}\right)} = \frac{1.95 \times 10^6}{(\pm 0.01 + n\pi)} \text{ m/s},$$

where $n \geq 0$ for the plus sign and $n \geq 1$ for the minus sign. For $n = 0$, $u_p = 1.94 \times 10^8$ m/s $= 0.65c$ and $\varepsilon_r = (c/u_p)^2 = 1/0.65^2 = 2.4$. For other values of n, u_p is very slow and ε_r is unreasonably high.

Problem 2.27 A 75-Ω resistive load is preceded by a $\lambda/4$ section of a 50-Ω lossless line, which itself is preceded by another $\lambda/4$ section of a 100-Ω line. What is the input impedance?

Solution: The input impedance of the $\lambda/4$ section of line closest to the load is found from Eq. (2.77):

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{50^2}{75} = 33.33\ \Omega.$$

The input impedance of the line section closest to the load can be considered as the load impedance of the next section of the line. By reapplying Eq. (2.77), the next section of $\lambda/4$ line is taken into account:

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{100^2}{33.33} = 300\ \Omega.$$

Problem 2.28 A 100-MHz FM broadcast station uses a 300-Ω transmission line between the transmitter and a tower-mounted half-wave dipole antenna. The antenna impedance is 73 Ω. You are asked to design a quarter-wave transformer to match the antenna to the line.

(a) Determine the electrical length and characteristic impedance of the quarter-wave section.

(b) If the quarter-wave section is a two-wire line with $d = 2.5$ cm, and the spacing between the wires is made of polystyrene with $\varepsilon_r = 2.6$, determine the physical length of the quarter-wave section and the radius of the two wire conductors.

Solution:

(a) For a match condition, the input impedance of a load must match that of the transmission line attached to the generator. A line of electrical length $\lambda/4$ can be used. From Eq. (2.77), the impedance of such a line should be

$$Z_0 = \sqrt{Z_{\text{in}} Z_{\text{L}}} = \sqrt{300 \times 73} = 148\ \Omega.$$

(b)

$$\frac{\lambda}{4} = \frac{u_{\text{p}}}{4f} = \frac{c}{4\sqrt{\varepsilon_{\text{r}}}\,f} = \frac{3 \times 10^8}{4\sqrt{2.6} \times 100 \times 10^6} = 0.465 \text{ m},$$

and, from Table 2-2,

$$Z_0 = \frac{120}{\sqrt{\varepsilon}} \ln\left[\left(\frac{d}{2a}\right) + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right] \ \Omega.$$

Hence,

$$\ln\left[\left(\frac{d}{2a}\right) + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right] = \frac{148\sqrt{2.6}}{120} = 1.99,$$

which leads to

$$\left(\frac{d}{2a}\right) + \sqrt{\left(\frac{d}{2a}\right)^2 - 1} = 7.31,$$

and whose solution is $a = d/7.44 = 25 \text{ cm}/7.44 = 3.36$ mm.

Problem 2.29 A 50-MHz generator with $Z_{\text{g}} = 50\ \Omega$ is connected to a load $Z_{\text{L}} = (50 - j25)\ \Omega$. The time-average power transferred from the generator into the load is maximum when $Z_{\text{g}} = Z_{\text{L}}^*$, where Z_{L}^* is the complex conjugate of Z_{L}. To achieve this condition without changing Z_{g}, the effective load impedance can be modified by adding an open-circuited line in series with Z_{L}, as shown in Fig. 2-40 (P2.29). If the line's $Z_0 = 100\ \Omega$, determine the shortest length of line (in wavelengths) necessary for satisfying the maximum-power-transfer condition.

Solution: Since the real part of Z_{L} is equal to Z_{g}, our task is to find l such that the input impedance of the line is $Z_{\text{in}} = +j25\ \Omega$, thereby cancelling the imaginary part of Z_{L} (once Z_{L} and the input impedance the line are added in series). Hence, using Eq. (2.73),

$$-j100 \cot \beta l = j25,$$

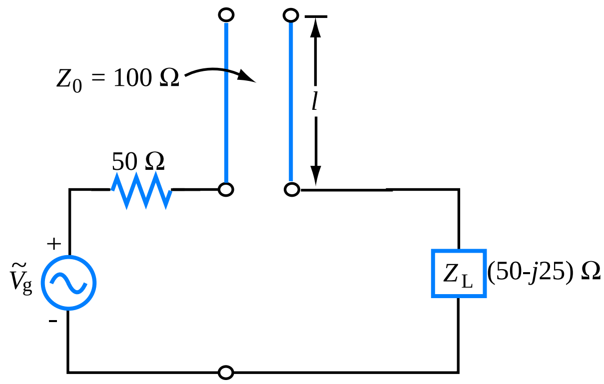


Figure P2.29: Transmission-line arrangement for Problem 2.29.

or

$$\cot\beta l = -\frac{25}{100} = -0.25,$$

which leads to

$$\beta l = -1.326 \text{ or } 1.816.$$

Since l cannot be negative, the first solution is discarded. The second solution leads to

$$l = \frac{1.816}{\beta} = \frac{1.816}{(2\pi/\lambda)} = 0.29\lambda.$$

Problem 2.30 A 50-Ω lossless line of length $l = 0.375\lambda$ connects a 300-MHz generator with $\widetilde{V}_g = 300$ V and $Z_g = 50$ Ω to a load Z_L. Determine the time-domain current through the load for:

(a) $Z_L = (50 - j50)$ Ω,
(b) $Z_L = 50$ Ω,
(c) $Z_L = 0$ (short circuit).

Solution:

(a) $Z_L = (50 - j50)$ Ω, $\beta l = \frac{2\pi}{\lambda} \times 0.375\lambda = 2.36$ (rad) $= 135°$.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j50 - 50}{50 - j50 + 50} = \frac{-j50}{100 - j50} = 0.45e^{-j63.43°}.$$

Application of Eq. (2.63) gives:

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = 50\left[\frac{(50 - j50) + j50\tan 135°}{50 + j(50 - j50)\tan 135°}\right] = (100 + j50) \text{ Ω}.$$

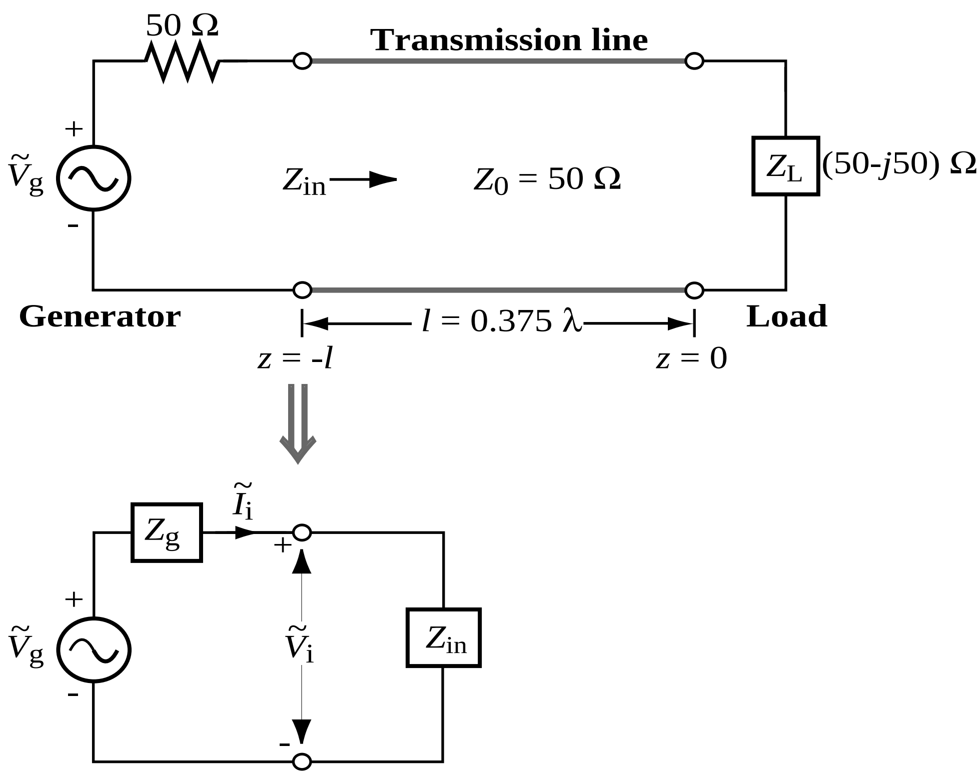


Figure P2.30: Circuit for Problem 2.30(a).

Using Eq. (2.66) gives

$$\begin{aligned}
V_0^+ &= \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) \\
&= \frac{300(100 + j50)}{50 + (100 + j50)}\left(\frac{1}{e^{j135^\circ} + 0.45\, e^{-j63.43^\circ}\, e^{-j135^\circ}}\right) \\
&= 150\, e^{-j135^\circ} \quad \text{(V)}, \\
\widetilde{I}_L &= \frac{V_0^+}{Z_0}(1 - \Gamma) = \frac{150\, e^{-j135^\circ}}{50}(1 - 0.45\, e^{-j63.43^\circ}) = 2.68\, e^{-j108.44^\circ} \quad \text{(A)}, \\
i_L(t) &= \mathfrak{Re}[\widetilde{I}_L e^{j\omega t}] \\
&= \mathfrak{Re}[2.68\, e^{-j108.44^\circ}\, e^{j6\pi\times 10^8 t}] \\
&= 2.68 \cos(6\pi \times 10^8 t - 108.44^\circ) \quad \text{(A)}.
\end{aligned}$$

(b)

$$
\begin{aligned}
Z_{\rm L} &= 50\ \Omega, \\
\Gamma &= 0, \\
Z_{\rm in} &= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad Z_0 = 50\ \Omega, \\
V_0^+ &= \frac{300\times 50}{50+50}\left(\frac{1}{e^{j135^\circ}+0}\right) = 150\,e^{-j135^\circ} \quad \text{(V)}, \\
\widetilde{I}_{\rm L} &= \frac{V_0^+}{Z_0} = \frac{150}{50}\,e^{-j135^\circ} = 3\,e^{-j135^\circ} \quad \text{(A)}, \\
i_{\rm L}(t) &= \mathfrak{Re}[3\,e^{-j135^\circ}e^{j6\pi\times 10^8 t}] = 3\cos(6\pi\times 10^8 t - 135^\circ) \quad \text{(A)}.
\end{aligned}
$$

(c)

$$
\begin{aligned}
Z_{\rm L} &= 0, \\
\Gamma &= -1, \\
Z_{\rm in} &= Z_0\left(\frac{0+jZ_0\tan 135^\circ}{Z_0+0}\right) = jZ_0\tan 135^\circ = -j50 \quad (\Omega), \\
V_0^+ &= \frac{300(-j50)}{50-j50}\left(\frac{1}{e^{j135^\circ}-e^{-j135^\circ}}\right) = 150\,e^{-j135^\circ} \quad \text{(V)}, \\
\widetilde{I}_{\rm L} &= \frac{V_0^+}{Z_0}[1-\Gamma] = \frac{150\,e^{-j135^\circ}}{50}[1+1] = 6e^{-j135^\circ} \quad \text{(A)}, \\
i_{\rm L}(t) &= 6\cos(6\pi\times 10^8 t - 135^\circ) \quad \text{(A)}.
\end{aligned}
$$

Section 2-8: Power Flow on Lossless Line

Problem 2.31 A generator with $\widetilde{V}_{\rm g} = 300$ V and $Z_{\rm g} = 50\ \Omega$ is connected to a load $Z_{\rm L} = 75\ \Omega$ through a 50-Ω lossless line of length $l = 0.15\lambda$.

(a) Compute $Z_{\rm in}$, the input impedance of the line at the generator end.

(b) Compute $\widetilde{I}_{\rm i}$ and $\widetilde{V}_{\rm i}$.

(c) Compute the time-average power delivered to the line, $P_{\rm in} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_{\rm i}\widetilde{I}_{\rm i}^*]$.

(d) Compute $\widetilde{V}_{\rm L}$, $\widetilde{I}_{\rm L}$, and the time-average power delivered to the load, $P_{\rm L} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_{\rm L}\widetilde{I}_{\rm L}^*]$. How does $P_{\rm in}$ compare to $P_{\rm L}$? Explain.

(e) Compute the time average power delivered by the generator, $P_{\rm g}$, and the time average power dissipated in $Z_{\rm g}$. Is conservation of power satisfied?

Solution:

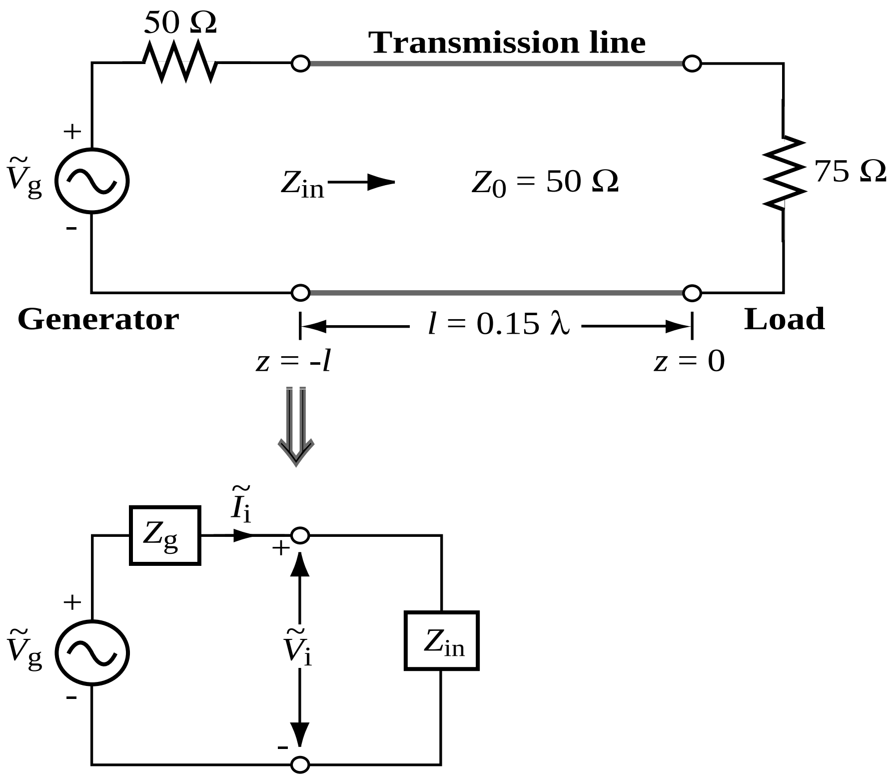


Figure P2.31: Circuit for Problem 2.31.

(a)

$$\beta l = \frac{2\pi}{\lambda} \times 0.15\lambda = 54°,$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = 50 \left[\frac{75 + j50 \tan 54°}{50 + j75 \tan 54°}\right] = (41.25 - j16.35)\ \Omega.$$

(b)

$$\tilde{I}_i = \frac{\tilde{V}_g}{Z_g + Z_{in}} = \frac{300}{50 + (41.25 - j16.35)} = 3.24 e^{j10.16°} \quad \text{(A)},$$

$$\tilde{V}_i = \tilde{I}_i Z_{in} = 3.24 e^{j10.16°} (41.25 - j16.35) = 143.6 e^{-j11.46°} \quad \text{(V)}.$$

(c)

$$P_{\text{in}} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_{\text{i}}\widetilde{I}_{\text{i}}^*] = \frac{1}{2}\mathfrak{Re}[143.6\,e^{-j11.46^\circ} \times 3.24\,e^{-j10.16^\circ}]$$
$$= \frac{143.6 \times 3.24}{2}\cos(21.62^\circ) = 216 \quad \text{(W)}.$$

(d)

$$\Gamma = \frac{Z_{\text{L}} - Z_0}{Z_{\text{L}} + Z_0} = \frac{75 - 50}{75 + 50} = 0.2,$$
$$V_0^+ = \widetilde{V}_{\text{i}}\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) = \frac{143.6\,e^{-j11.46^\circ}}{e^{j54^\circ} + 0.2\,e^{-j54^\circ}} = 150e^{-j54^\circ} \quad \text{(V)},$$
$$\widetilde{V}_{\text{L}} = V_0^+(1+\Gamma) = 150e^{-j54^\circ}(1+0.2) = 180e^{-j54^\circ} \quad \text{(V)},$$
$$\widetilde{I}_{\text{L}} = \frac{V_0^+}{Z_0}(1-\Gamma) = \frac{150e^{-j54^\circ}}{50}(1-0.2) = 2.4\,e^{-j54^\circ} \quad \text{(A)},$$
$$P_{\text{L}} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_{\text{L}}\widetilde{I}_{\text{L}}^*] = \frac{1}{2}\mathfrak{Re}[180e^{-j54^\circ} \times 2.4\,e^{j54^\circ}] = 216 \quad \text{(W)}.$$

$P_{\text{L}} = P_{\text{in}}$, which is as expected because the line is lossless; power input to the line ends up in the load.

(e)

Power delivered by generator:

$$P_{\text{g}} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_{\text{g}}\widetilde{I}_{\text{i}}] = \frac{1}{2}\mathfrak{Re}[300 \times 3.24\,e^{j10.16^\circ}] = 486\cos(10.16^\circ) = 478.4 \quad \text{(W)}.$$

Power dissipated in Z_{g}*:*

$$P_{Z_{\text{g}}} = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_{\text{i}}\widetilde{V}_{Z_{\text{g}}}] = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_{\text{i}}\widetilde{I}_{\text{i}}^* Z_{\text{g}}] = \frac{1}{2}|\widetilde{I}_{\text{i}}|^2 Z_{\text{g}} = \frac{1}{2}(3.24)^2 \times 50 = 262.4 \quad \text{(W)}.$$

Note 1: $P_{\text{g}} = P_{Z_{\text{g}}} + P_{\text{in}} = 478.4$ W.

Problem 2.32 If the two-antenna configuration shown in Fig. 2-41 (P2.32) is connected to a generator with $\widetilde{V}_{\text{g}} = 250$ V and $Z_{\text{g}} = 50\ \Omega$, how much average power is delivered to each antenna?

Solution: Since line 2 is $\lambda/2$ in length, the input impedance is the same as $Z_{\text{L}_1} = 75\ \Omega$. The same is true for line 3. At junction C–D, we now have two 75-Ω impedances in parallel, whose combination is $75/2 = 37.5\ \Omega$. Line 1 is $\lambda/2$ long. Hence at A–C, input impedance of line 1 is 37.5 Ω, and

$$\widetilde{I}_{\text{i}} = \frac{\widetilde{V}_{\text{g}}}{Z_{\text{g}} + Z_{\text{in}}} = \frac{250}{50 + 37.5} = 2.86 \quad \text{(A)},$$

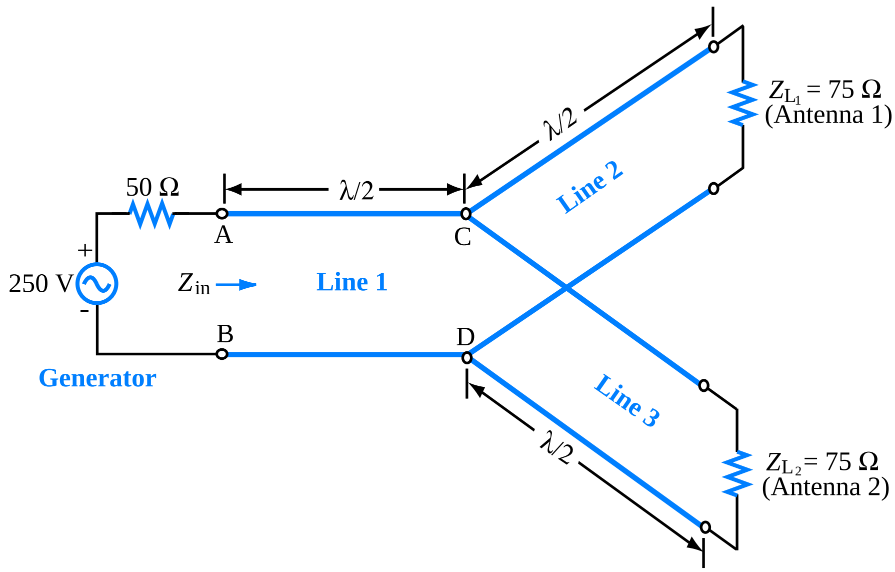


Figure P2.32: Antenna configuration for Problem 2.32.

$$P_{in} = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{V}_i^*] = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{I}_i^* \widetilde{Z}_{in}^*] = \frac{(2.86)^2 \times 37.5}{2} = 153.37 \quad \text{(W)}.$$

This is divided equally between the two antennas. Hence, each antenna receives $\frac{153.37}{2} = 76.68$ (W).

Problem 2.33 For the circuit shown in Fig. 2-42 (P2.33), calculate the average incident power, the average reflected power, and the average power transmitted into the infinite 100-Ω line. The $\lambda/2$ line is lossless and the infinitely long line is slightly lossy. (Hint: The input impedance of an infinitely long line is equal to its characteristic impedance so long as $\alpha \neq 0$.)

Solution: Considering the semi-infinite transmission line as equivalent to a load (since all power sent down the line is lost to the rest of the circuit), $Z_L = Z_1 = 100$ Ω. Since the feed line is $\lambda/2$ in length, Eq. (2.76) gives $Z_{in} = Z_L = 100$ Ω and $\beta l = (2\pi/\lambda)(\lambda/2) = \pi$, so $e^{\pm j\beta l} = -1$. From Eq. (2.49a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3}.$$

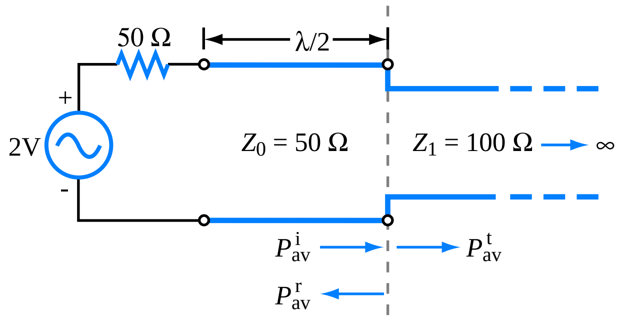


Figure P2.33: Line terminated in an infinite line.

Also, converting the generator to a phasor gives $\widetilde{V}_{\rm g} = 2e^{j0^\circ}$ (V). Plugging all these results into Eq. (2.66),

$$V_0^+ = \left(\frac{\widetilde{V}_{\rm g} Z_{\rm in}}{Z_{\rm g} + Z_{\rm in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) = \left(\frac{2\times 100}{50+100}\right)\left(\frac{1}{(-1)+\frac{1}{3}(-1)}\right)$$
$$= 1e^{j180^\circ} = -1 \quad \text{(V)}.$$

From Eqs. (2.84), (2.85), and (2.86),

$$P_{\rm av}^{\rm i} = \frac{|V_0^+|^2}{2Z_0} = \frac{|1e^{j180^\circ}|^2}{2\times 50} = 10.0 \text{ mW},$$
$$P_{\rm av}^{\rm r} = -|\Gamma|^2 P_{\rm av}^{\rm i} = -\left|\frac{1}{3}\right|^2 \times 10 \text{ mW} = -1.1 \text{ mW},$$
$$P_{\rm av}^{\rm t} = P_{\rm av} = P_{\rm av}^{\rm i} + P_{\rm av}^{\rm r} = 10.0 \text{ mW} - 1.1 \text{ mW} = 8.9 \text{ mW}.$$

Problem 2.34 An antenna with a load impedance $Z_{\rm L} = (75 + j25)$ Ω is connected to a transmitter through a 50-Ω lossless transmission line. If under matched conditions (50-Ω load), the transmitter can deliver 20 W to the load, how much power does it deliver to the antenna? Assume $Z_{\rm g} = Z_0$.

Solution: From Eqs. (2.66) and (2.61),

$$\begin{aligned}
V_0^+ &= \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) \\
&= \left(\frac{\widetilde{V}_g Z_0\left[(1+\Gamma e^{-j2\beta l})/(1-\Gamma e^{-j2\beta l})\right]}{Z_0 + Z_0\left[(1+\Gamma e^{-j2\beta l})/(1-\Gamma e^{-j2\beta l})\right]}\right)\left(\frac{e^{-j\beta l}}{1+\Gamma e^{-j2\beta l}}\right) \\
&= \frac{\widetilde{V}_g e^{-j\beta l}}{(1-\Gamma e^{-j2\beta l}) + (1+\Gamma e^{-j2\beta l})} \\
&= \frac{\widetilde{V}_g e^{-j\beta l}}{(1-\Gamma e^{-j2\beta l}) + (1+\Gamma e^{-j2\beta l})} = \tfrac{1}{2}\widetilde{V}_g e^{-j\beta l}.
\end{aligned}$$

Thus, in Eq. (2.86),

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma|^2) = \frac{|\frac{1}{2}\widetilde{V}_g e^{-j\beta l}|^2}{2Z_0}(1-|\Gamma|^2) = \frac{|\widetilde{V}_g|^2}{8Z_0}(1-|\Gamma|^2).$$

Under the matched condition, $|\Gamma| = 0$ and $P_L = 20$ W, so $|\widetilde{V}_g|^2/8Z_0 = 20$ W.

When $Z_L = (75 + j25)\ \Omega$, from Eq. (2.49a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(75 + j25)\ \Omega - 50\ \Omega}{(75 + j25)\ \Omega + 50\ \Omega} = 0.277e^{j33.6^\circ},$$

so $P_{av} = 20\text{ W}\,(1-|\Gamma|^2) = 20\text{ W}\,(1-0.277^2) = 18.46$ W.

Section 2-9: Smith Chart

Problem 2.35 Use the Smith chart to find the reflection coefficient corresponding to a load impedance:

(a) $Z_L = 3Z_0$,
(b) $Z_L = (2-2j)Z_0$,
(c) $Z_L = -2jZ_0$,
(d) $Z_L = 0$ (short circuit).

Solution: Refer to Fig. P2.35.

(a) Point A is $z_L = 3 + j0$. $\Gamma = 0.5e^{0^\circ}$
(b) Point B is $z_L = 2 - j2$. $\Gamma = 0.62e^{-29.7^\circ}$
(c) Point C is $z_L = 0 - j2$. $\Gamma = 1.0e^{-53.1^\circ}$
(d) Point D is $z_L = 0 + j0$. $\Gamma = 1.0e^{180.0^\circ}$

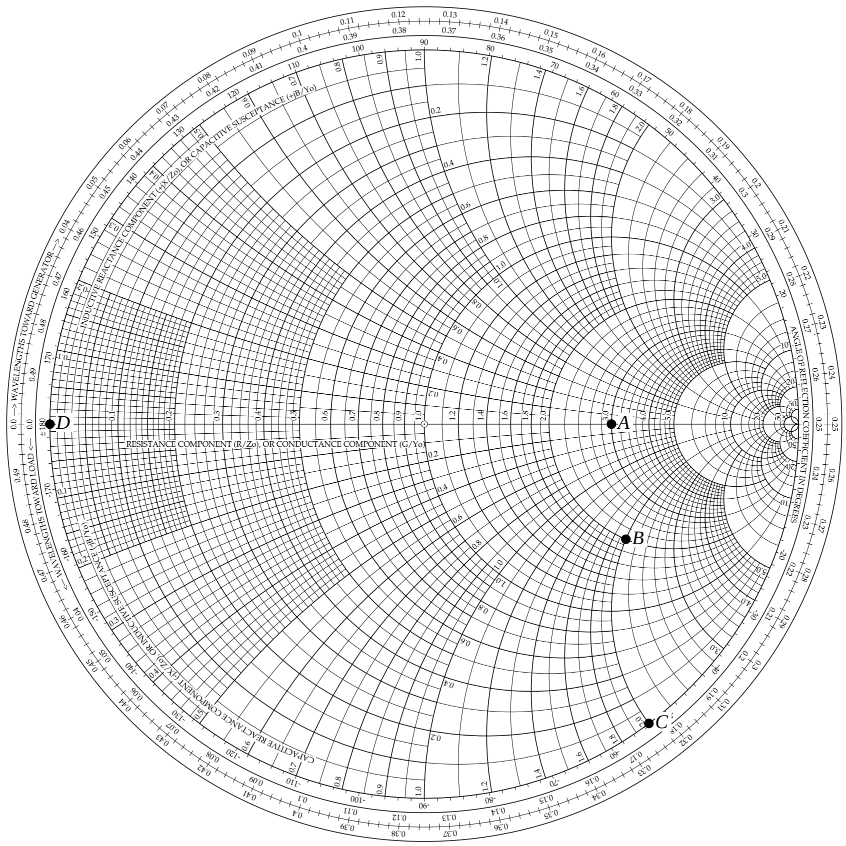


Figure P2.35: Solution of Problem 2.35.

Problem 2.36 Use the Smith chart to find the normalized load impedance corresponding to a reflection coefficient:

(a) $\Gamma = 0.5$,
(b) $\Gamma = 0.5\angle 60^\circ$,
(c) $\Gamma = -1$,
(d) $\Gamma = 0.3\angle -30^\circ$,
(e) $\Gamma = 0$,
(f) $\Gamma = j$.

Solution: Refer to Fig. P2.36.

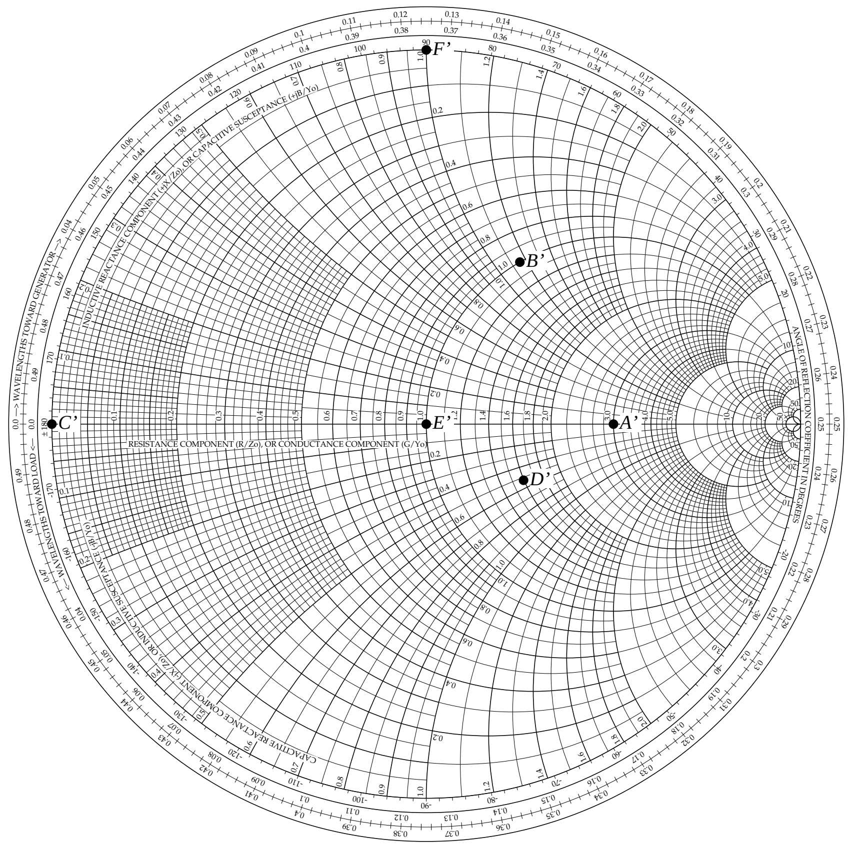


Figure P2.36: Solution of Problem 2.36.

(a) Point A' is $\Gamma = 0.5$ at $z_{\rm L} = 3 + j0$.
(b) Point B' is $\Gamma = 0.5e^{j60^\circ}$ at $z_{\rm L} = 1 + j1.15$.
(c) Point C' is $\Gamma = -1$ at $z_{\rm L} = 0 + j0$.
(d) Point D' is $\Gamma = 0.3e^{-j30^\circ}$ at $z_{\rm L} = 1.60 - j0.53$.
(e) Point E' is $\Gamma = 0$ at $z_{\rm L} = 1 + j0$.
(f) Point F' is $\Gamma = j$ at $z_{\rm L} = 0 + j1$.

Problem 2.37 On a lossless transmission line terminated in a load $Z_{\rm L} = 100\ \Omega$, the standing-wave ratio was measured to be 2.5. Use the Smith chart to find the two possible values of Z_0.

Solution: Refer to Fig. P2.37. $S = 2.5$ is at point $L1$ and the constant SWR circle is shown. z_L is real at only two places on the SWR circle, at $L1$, where $z_L = S = 2.5$, and $L2$, where $z_L = 1/S = 0.4$. so $Z_{01} = Z_L/z_{L1} = 100\ \Omega/2.5 = 40\ \Omega$ and $Z_{02} = Z_L/z_{L2} = 100\ \Omega/0.4 = 250\ \Omega$.

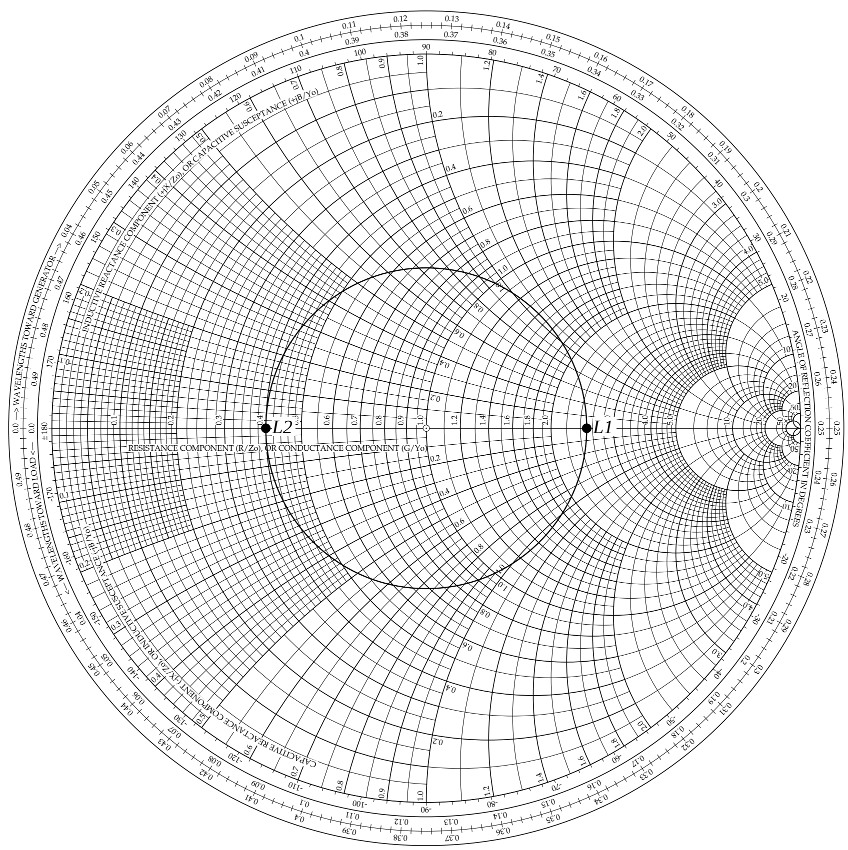


Figure P2.37: Solution of Problem 2.37.

Problem 2.38 A lossless 50-Ω transmission line is terminated in a load with $Z_L = (50 + j25)\ \Omega$. Use the Smith chart to find the following:

(a) the reflection coefficient Γ,
(b) the standing-wave ratio,
(c) the input impedance at 0.35λ from the load,

(d) the input admittance at 0.35λ from the load,

(e) the shortest line length for which the input impedance is purely resistive,

(f) the position of the first voltage maximum from the load.

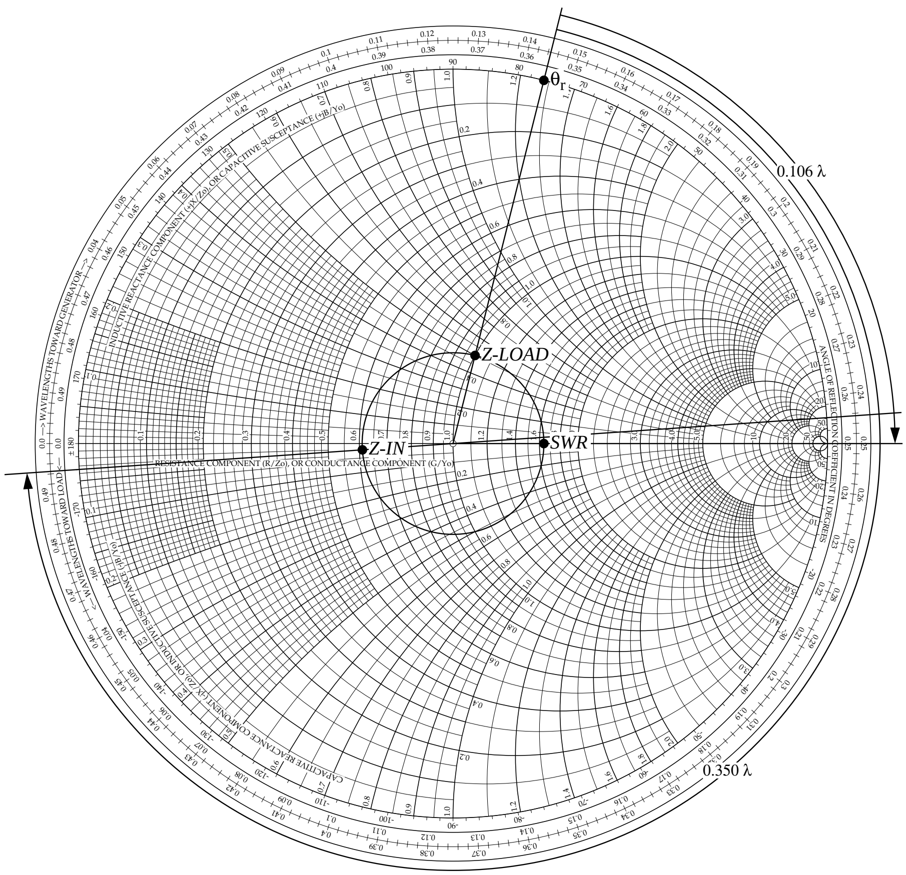


Figure P2.38: Solution of Problem 2.38.

Solution: Refer to Fig. P2.38. The normalized impedance

$$z_L = \frac{(50 + j25)\ \Omega}{50\ \Omega} = 1 + j0.5$$

is at point *Z-LOAD*.

(a) $\Gamma = 0.24e^{j76.0^\circ}$ The angle of the reflection coefficient is read of that scale at the point θ_r.

(b) At the point *SWR*: $S = 1.64$.

(c) Z_{in} is 0.350λ from the load, which is at 0.144λ on the wavelengths to generator scale. So point *Z-IN* is at $0.144\lambda + 0.350\lambda = 0.494\lambda$ on the WTG scale. At point *Z-IN*:

$$Z_{\text{in}} = z_{\text{in}} Z_0 = (0.61 - j0.022) \times 50\ \Omega = (30.5 - j1.09)\ \Omega.$$

(d) At the point on the SWR circle opposite *Z-IN*,

$$Y_{\text{in}} = \frac{y_{\text{in}}}{Z_0} = \frac{(1.64 + j0.06)}{50\ \Omega} = (32.7 + j1.17)\ \text{mS}.$$

(e) Traveling from the point *Z-LOAD* in the direction of the generator (clockwise), the SWR circle crosses the $x_{\text{L}} = 0$ line first at the point *SWR*. To travel from *Z-LOAD* to *SWR* one must travel $0.250\lambda - 0.144\lambda = 0.106\lambda$. (Readings are on the wavelengths to generator scale.) So the shortest line length would be 0.106λ.

(f) The voltage max occurs at point *SWR*. From the previous part, this occurs at $z = -0.106\lambda$.

Problem 2.39 A lossless 50-Ω transmission line is terminated in a short circuit. Use the Smith chart to find

(a) the input impedance at a distance 2.3λ from the load,

(b) the distance from the load at which the input admittance is $Y_{\text{in}} = -j0.04$ S.

Solution: Refer to Fig. P2.39.

(a) For a short, $z_{\text{in}} = 0 + j0$. This is point *Z-SHORT* and is at 0.000λ on the WTG scale. Since a lossless line repeats every $\lambda/2$, traveling 2.3λ toward the generator is equivalent to traveling 0.3λ toward the generator. This point is at A : *Z-IN*, and

$$Z_{\text{in}} = z_{\text{in}} Z_0 = (0 - j3.08) \times 50\ \Omega = -j154\ \Omega.$$

(b) The admittance of a short is at point *Y-SHORT* and is at 0.250λ on the WTG scale:

$$y_{\text{in}} = Y_{\text{in}} Z_0 = -j0.04\ \text{S} \times 50\ \Omega = -j2,$$

which is point B : *Y-IN* and is at 0.324λ on the WTG scale. Therefore, the line length is $0.324\lambda - 0.250\lambda = 0.074\lambda$. Any integer half wavelengths farther is also valid.

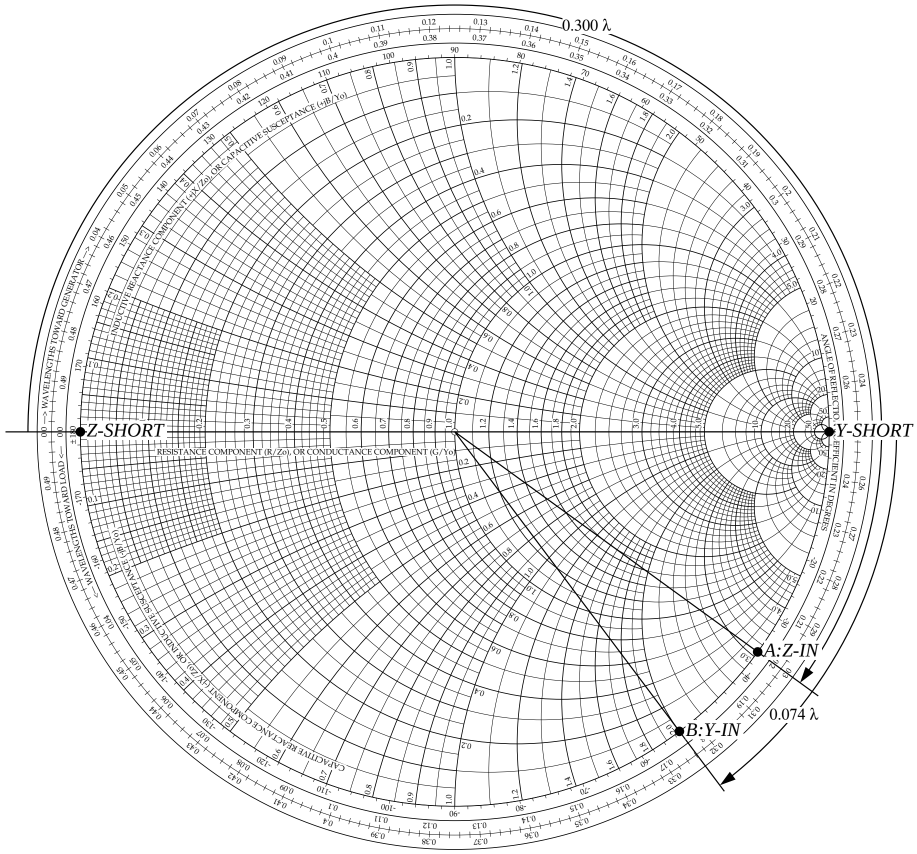


Figure P2.39: Solution of Problem 2.39.

Problem 2.40 Use the Smith chart to find y_{L} if $z_{\mathrm{L}} = 1.5 - j0.7$.

Solution: Refer to Fig. P2.40. The point Z represents $1.5 - j0.7$. The reciprocal of point Z is at point Y, which is at $0.55 + j0.26$.

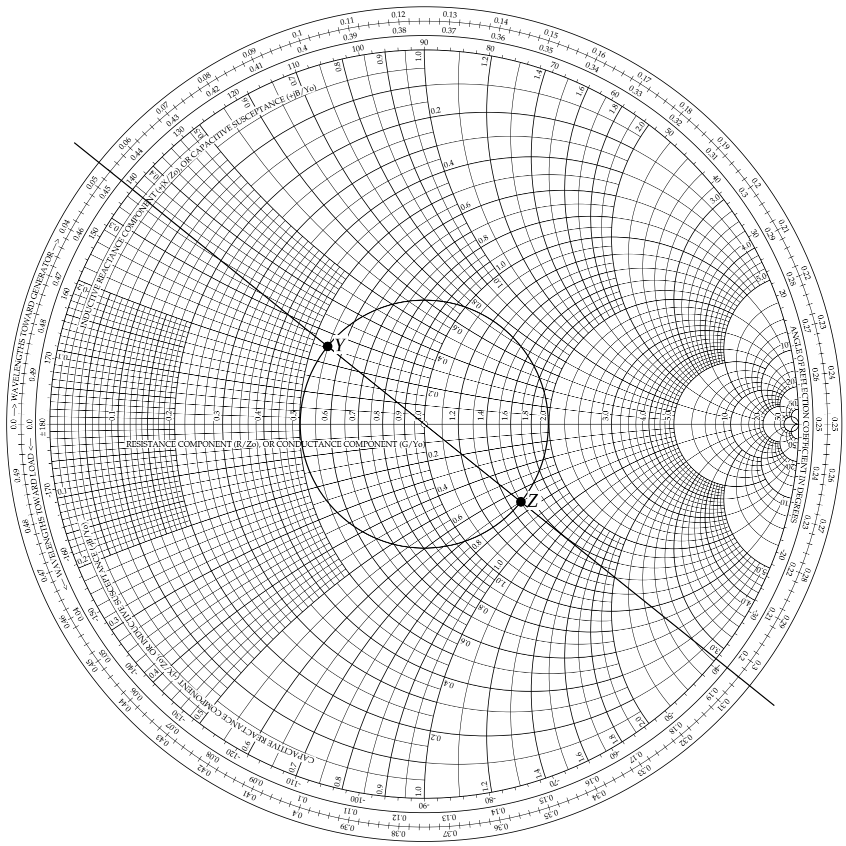


Figure P2.40: Solution of Problem 2.40.

Problem 2.41 A lossless 100-Ω transmission line $3\lambda/8$ in length is terminated in an unknown impedance. If the input impedance is $Z_{\text{in}} = -j2.5\ \Omega$,

(a) use the Smith chart to find Z_{L}.

(b) What length of open-circuit line could be used to replace Z_{L}?

Solution: Refer to Fig. P2.41. $z_{\text{in}} = Z_{\text{in}}/Z_0 = -j2.5\ \Omega/100\ \Omega = 0.0 - j0.025$ which is at point *Z-IN* and is at 0.004λ on the wavelengths to load scale.

(a) Point *Z-LOAD* is 0.375λ toward the load from the end of the line. Thus, on the wavelength to load scale, it is at $0.004\lambda + 0.375\lambda = 0.379\lambda$.

$$Z_{\text{L}} = z_{\text{L}} Z_0 = (0 + j0.95) \times 100\ \Omega = j95\ \Omega.$$

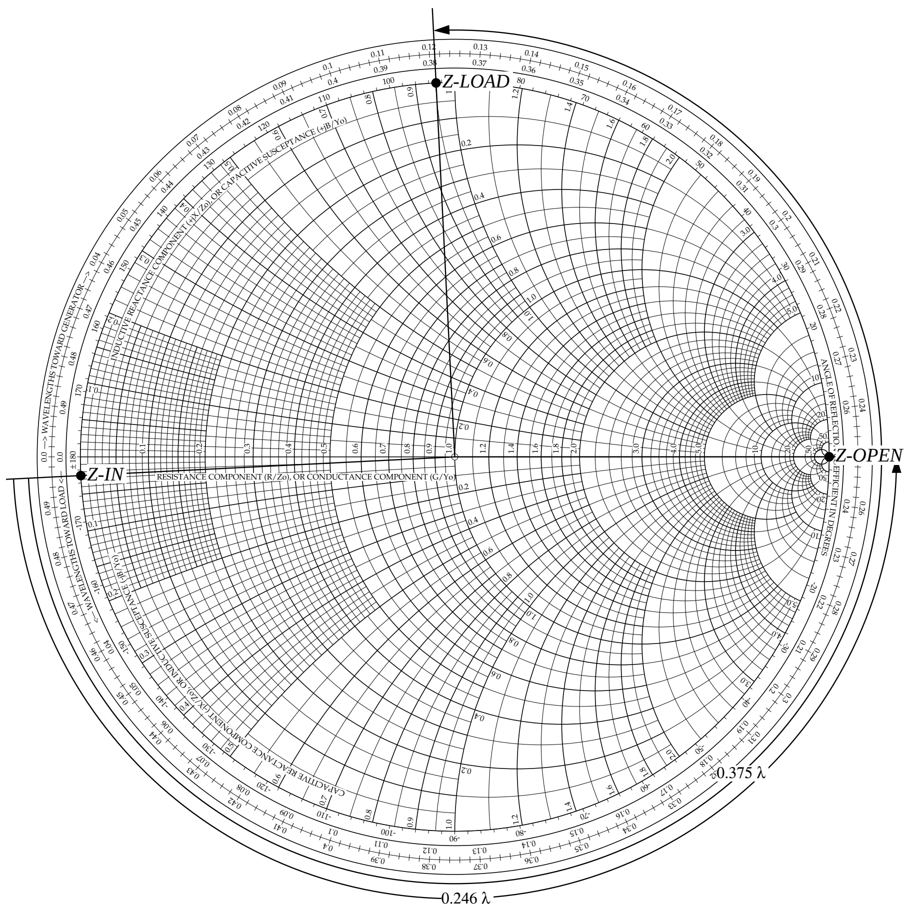


Figure P2.41: Solution of Problem 2.41.

(b) An open circuit is located at point *Z-OPEN*, which is at 0.250λ on the wavelength to load scale. Therefore, an open circuited line with $Z_{\text{in}} = -j0.025$ must have a length of $0.250\lambda - 0.004\lambda = 0.246\lambda$.

Problem 2.42 A 75-Ω lossless line is 0.6λ long. If $S = 1.8$ and $\theta_r = -60°$, use the Smith chart to find $|\Gamma|$, Z_L, and Z_{in}.

Solution: Refer to Fig. P2.42. The SWR circle must pass through $S = 1.8$ at point *SWR*. A circle of this radius has

$$|\Gamma| = \frac{S-1}{S+1} = 0.29.$$

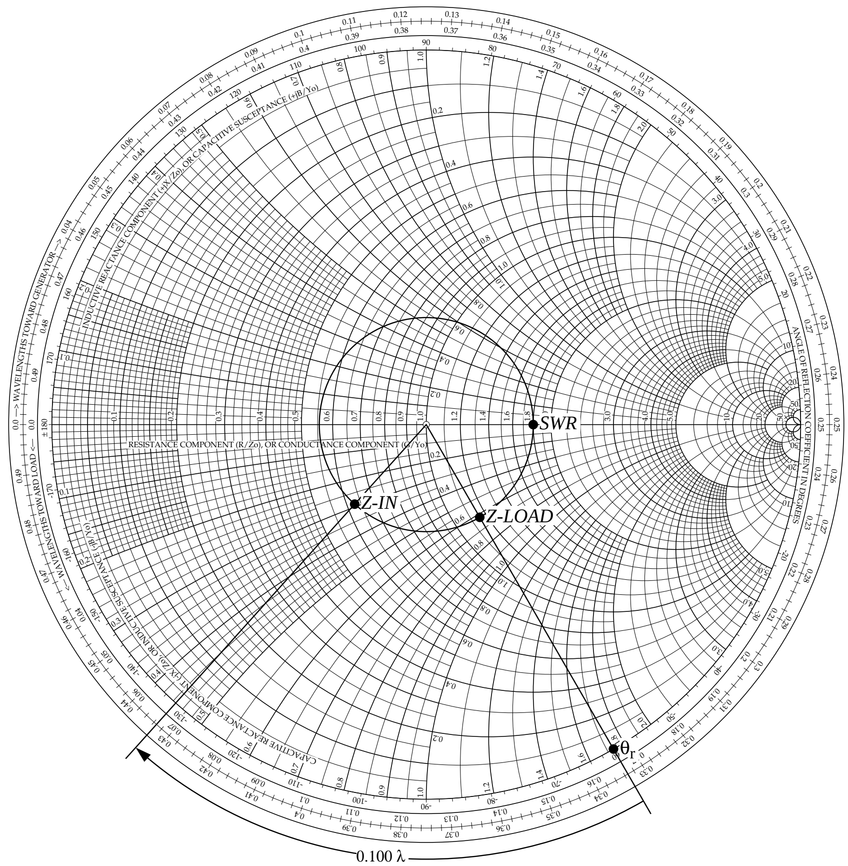


Figure P2.42: Solution of Problem 2.42.

The load must have a reflection coefficient with $\theta_r = -60^\circ$. The angle of the reflection coefficient is read off that scale at the point θ_r. The intersection of the circle of constant $|\Gamma|$ and the line of constant θ_r is at the load, point *Z-LOAD*, which has a value $z_L = 1.15 - j0.62$. Thus,

$$Z_L = z_L Z_0 = (1.15 - j0.62) \times 75\ \Omega = (86.5 - j46.6)\ \Omega.$$

A 0.6λ line is equivalent to a 0.1λ line. On the WTG scale, *Z-LOAD* is at 0.333λ, so *Z-IN* is at $0.333\lambda + 0.100\lambda = 0.433\lambda$ and has a value

$$z_{in} = 0.63 - j0.29.$$

Therefore $Z_{\text{in}} = z_{\text{in}} Z_0 = (0.63 - j0.29) \times 75\ \Omega = (47.0 - j21.8)\ \Omega$.

Problem 2.43 Using a slotted line on a 50-Ω air-spaced lossless line, the following measurements were obtained: $S = 1.6$, $|\widetilde{V}|_{\max}$ occurred only at 10 cm and 24 cm from the load. Use the Smith chart to find Z_{L}.

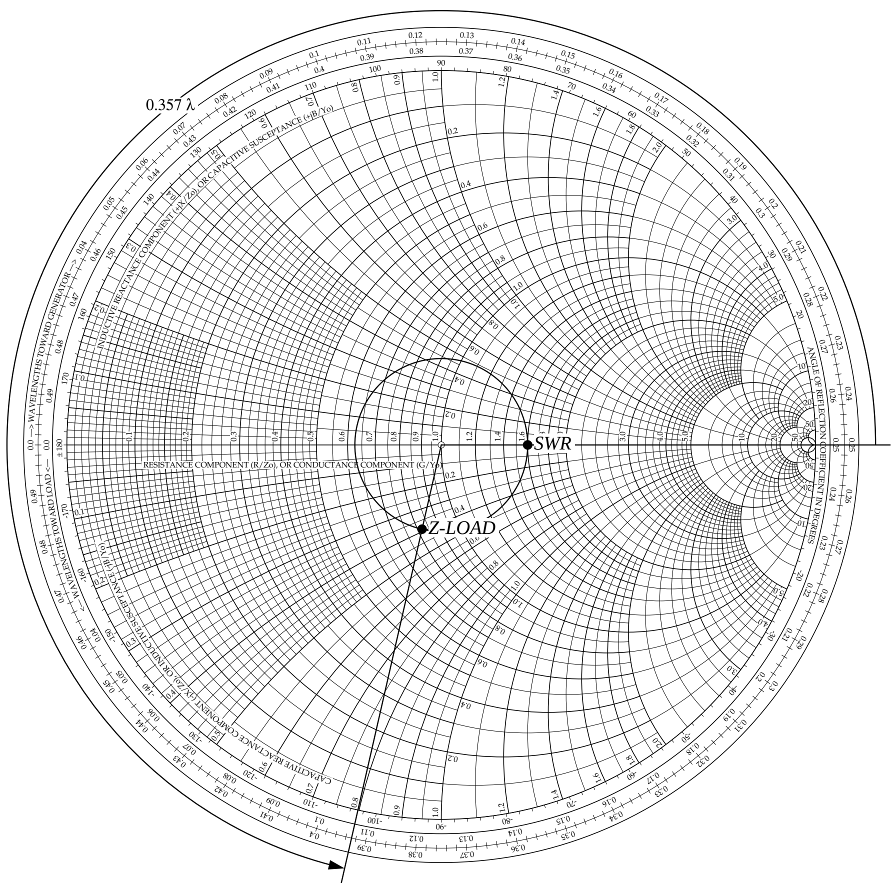


Figure P2.43: Solution of Problem 2.43.

Solution: Refer to Fig. P2.43. The point *SWR* denotes the fact that $S = 1.6$. This point is also the location of a voltage maximum. From the knowledge of the locations of adjacent maxima we can determine that $\lambda = 2(24\text{ cm} - 10\text{ cm}) = 28$ cm. Therefore, the load is $\frac{10\text{ cm}}{28\text{ cm}}\lambda = 0.357\lambda$ from the first voltage maximum, which is at 0.250λ on the WTL scale. Traveling this far on the SWR circle we find point *Z-LOAD* at $0.250\lambda + 0.357\lambda - 0.500\lambda = 0.107\lambda$ on the WTL scale, and here

$$z_L = 0.82 - j0.39.$$

Therefore $Z_L = z_L Z_0 = (0.82 - j0.39) \times 50\ \Omega = (41.0 - j19.5)\ \Omega$.

Problem 2.44 At an operating frequency of 5 GHz, a 50-Ω lossless coaxial line with insulating material having a relative permittivity $\varepsilon_r = 2.25$ is terminated in an antenna with an impedance $Z_L = 150\ \Omega$. Use the Smith chart to find Z_{in}. The line length is 30 cm.

Solution: To use the Smith chart the line length must be converted into wavelengths. Since $\beta = 2\pi/\lambda$ and $u_p = \omega/\beta$,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi u_p}{\omega} = \frac{c}{\sqrt{\varepsilon_r} f} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25} \times (5 \times 10^9 \text{ Hz})} = 0.04 \text{ m}.$$

Hence, $l = \frac{0.30 \text{ m}}{0.04 \text{ m}} \lambda = 7.5\lambda$. Since this is an integral number of half wavelengths,

$$Z_{in} = Z_L = 150\ \Omega.$$

Section 2-10: Impedance Matching

Problem 2.45 A 50-Ω lossless line 0.6λ long is terminated in a load with $Z_L = (50 + j25)\ \Omega$. At 0.3λ from the load, a resistor with resistance $R = 30\ \Omega$ is connected as shown in Fig. 2-43 (P2.45(a)). Use the Smith chart to find Z_{in}.

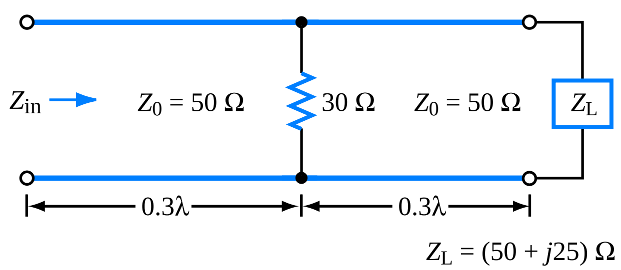


Figure P2.45: (a) Circuit for Problem 2.45.

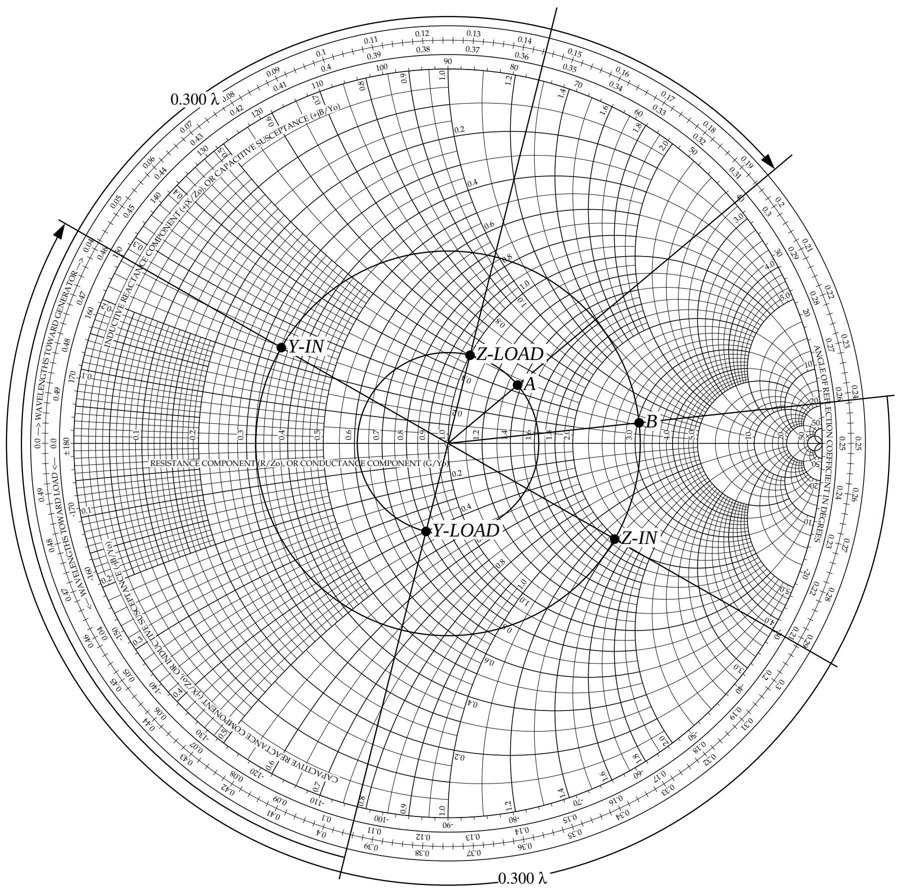


Figure P2.45: (b) Solution of Problem 2.45.

Solution: Refer to Fig. P2.45(b). Since the 30-Ω resistor is in parallel with the input impedance at that point, it is advantageous to convert all quantities to admittances.

$$z_L = \frac{Z_L}{Z_0} = \frac{(50 + j25)\ \Omega}{50\ \Omega} = 1 + j0.5$$

and is located at point *Z-LOAD*. The corresponding normalized load admittance is at point *Y-LOAD*, which is at 0.394λ on the WTG scale. The input admittance of the load only at the shunt conductor is at $0.394\lambda + 0.300\lambda - 0.500\lambda = 0.194\lambda$ and is denoted by point *A*. It has a value of

$$y_{\text{in}A} = 1.37 + j0.45.$$

The shunt conductance has a normalized conductance

$$g = \frac{50\ \Omega}{30\ \Omega} = 1.67.$$

The normalized admittance of the shunt conductance in parallel with the input admittance of the load is the sum of their admittances:

$$y_{\text{in}B} = g + y_{\text{in}A} = 1.67 + 1.37 + j0.45 = 3.04 + j0.45$$

and is located at point B. On the WTG scale, point B is at 0.242λ. The input admittance of the entire circuit is at $0.242\lambda + 0.300\lambda - 0.500\lambda = 0.042\lambda$ and is denoted by point *Y-IN*. The corresponding normalized input impedance is at *Z-IN* and has a value of

$$z_{\text{in}} = 1.9 - j1.4.$$

Thus,

$$Z_{\text{in}} = z_{\text{in}} Z_0 = (1.9 - j1.4) \times 50\ \Omega = (95 - j70)\ \Omega.$$

Problem 2.46 A 50-Ω lossless line is to be matched to an antenna with

$$Z_{\text{L}} = (75 - j20)\ \Omega$$

using a shorted stub. Use the Smith chart to determine the stub length and the distance between the antenna and the stub.

Solution: Refer to Fig. P2.46(a) and Fig. P2.46(b), which represent two different solutions.

$$z_{\text{L}} = \frac{Z_{\text{L}}}{Z_0} = \frac{(75 - j20)\ \Omega}{50\ \Omega} = 1.5 - j0.4$$

and is located at point *Z-LOAD* in both figures. Since it is advantageous to work in admittance coordinates, y_{L} is plotted as point *Y-LOAD* in both figures. *Y-LOAD* is at 0.041λ on the WTG scale.

For the first solution in Fig. P2.46(a), point *Y-LOAD-IN*-1 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-1 is at 0.145λ on the WTG scale, so the stub should be located at $0.145\lambda - 0.041\lambda = 0.104\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-1, $b = 0.52$, so a stub with an input admittance of $y_{\text{stub}} = 0 - j0.52$ is required. This point is *Y-STUB-IN*-1 and is at 0.423λ on the WTG scale. The short circuit admittance

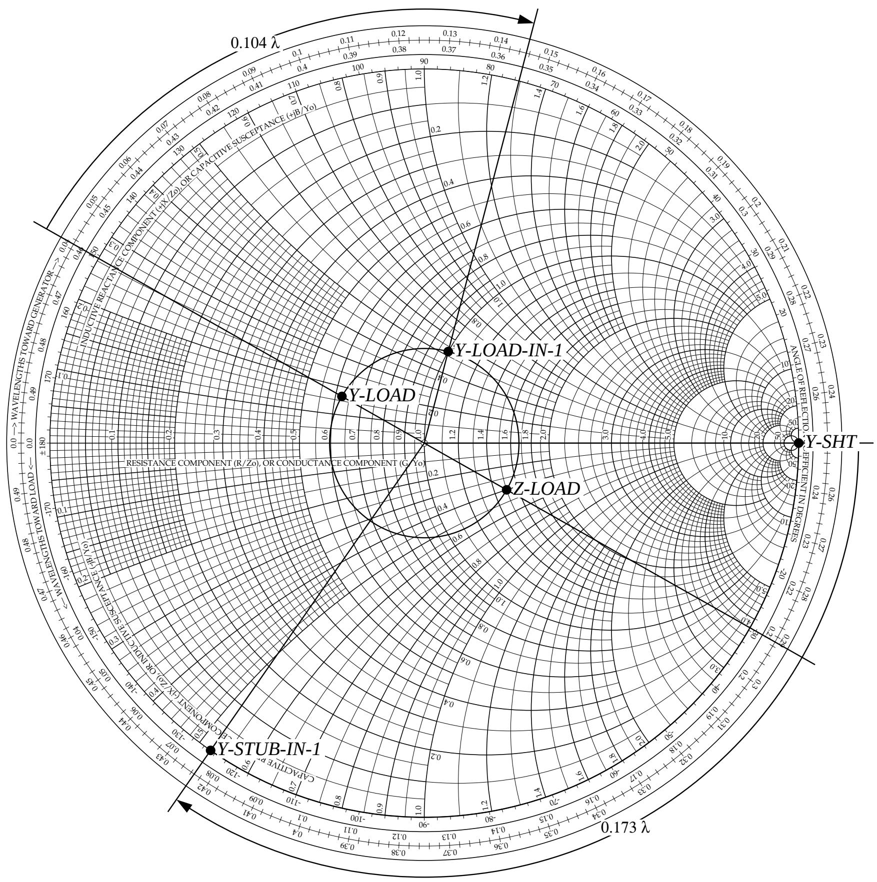


Figure P2.46: (a) First solution to Problem 2.46.

is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.423\lambda - 0.250\lambda = 0.173\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.46(b), point *Y-LOAD-IN-*2 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN-*2 is at 0.355λ on the WTG scale, so the stub should be located at $0.355\lambda - 0.041\lambda = 0.314\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN-*2, $b = -0.52$, so a stub with an input admittance of $y_{\text{stub}} = 0 + j0.52$ is required. This point is *Y-STUB-IN-*2 and is at 0.077λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.077\lambda - 0.250\lambda + 0.500\lambda = 0.327\lambda$ long (or some multiple of a half wavelength

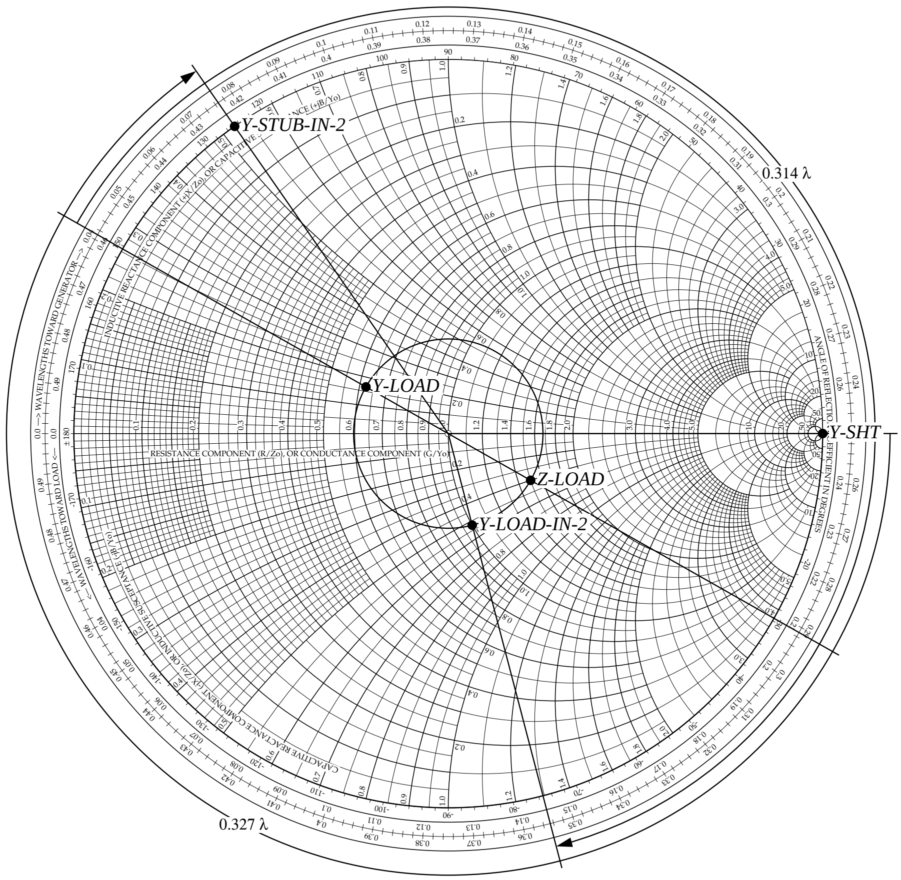


Figure P2.46: (b) Second solution to Problem 2.46.

longer).

Problem 2.47 Repeat Problem 2.46 for a load with $Z_L = (100 + j50)\ \Omega$.

Solution: Refer to Fig. P2.47(a) and Fig. P2.47(b), which represent two different solutions.

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j50\ \Omega}{50\ \Omega} = 2 + j1$$

and is located at point *Z-LOAD* in both figures. Since it is advantageous to work in admittance coordinates, y_L is plotted as point *Y-LOAD* in both figures. *Y-LOAD* is at 0.463λ on the WTG scale.

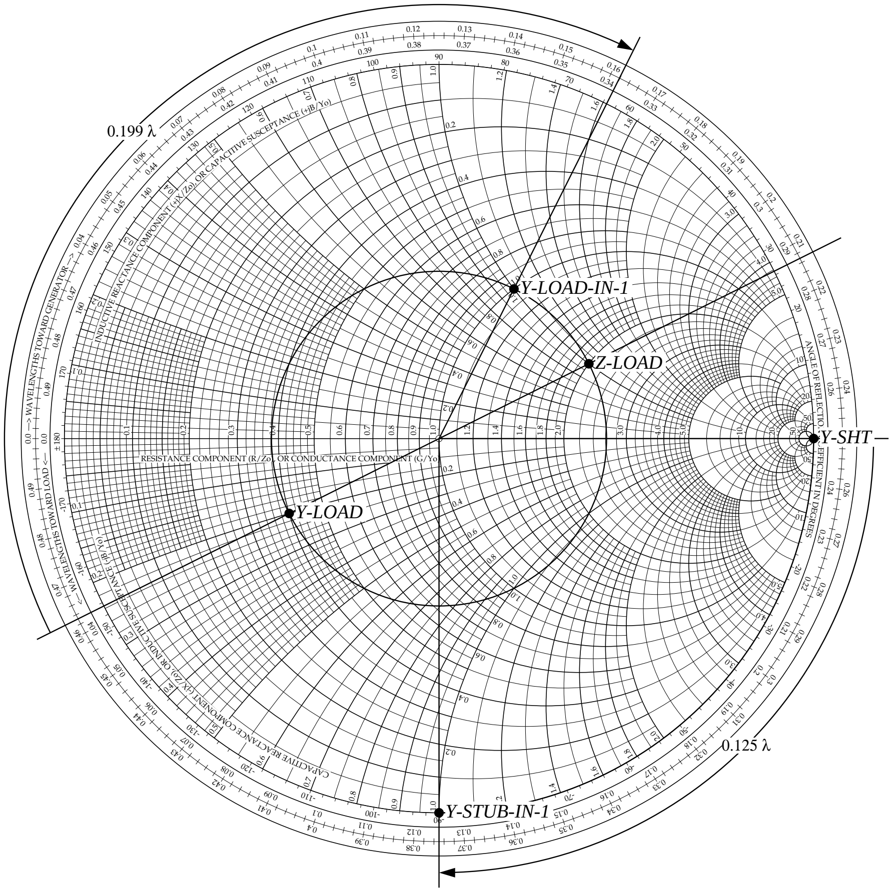


Figure P2.47: (a) First solution to Problem 2.47.

For the first solution in Fig. P2.47(a), point *Y-LOAD-IN*-1 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-1 is at 0.162λ on the WTG scale, so the stub should be located at $0.162\lambda - 0.463\lambda + 0.500\lambda = 0.199\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-1, $b = 1$, so a stub with an input admittance of $y_{\text{stub}} = 0 - j1$ is required. This point is *Y-STUB-IN*-1 and is at 0.375λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.375\lambda - 0.250\lambda = 0.125\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.47(b), point *Y-LOAD-IN*-2 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-2 is at 0.338λ on the

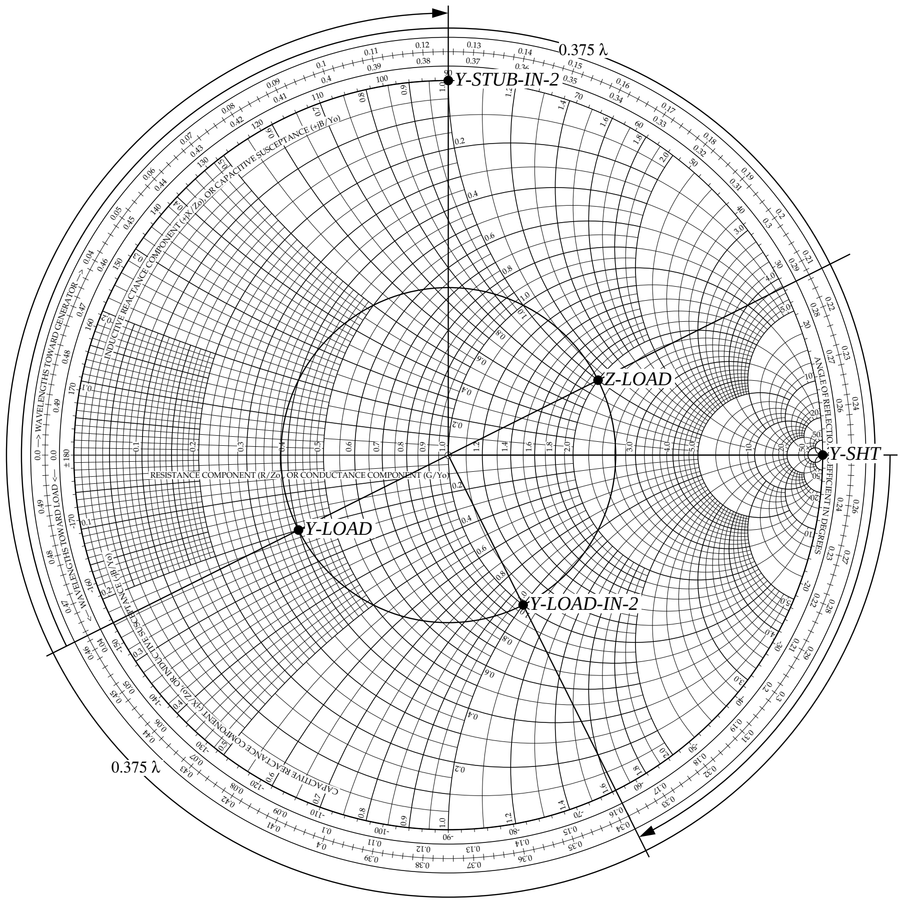


Figure P2.47: (b) Second solution to Problem 2.47.

WTG scale, so the stub should be located at $0.338\lambda - 0.463\lambda + 0.500\lambda = 0.375\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-2, $b = -1$, so a stub with an input admittance of $y_{\text{stub}} = 0 + j1$ is required. This point is *Y-STUB-IN*-2 and is at 0.125λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.125\lambda - 0.250\lambda + 0.500\lambda = 0.375\lambda$ long (or some multiple of a half wavelength longer).

Problem 2.48 Use the Smith chart to find Z_{in} of the feed line shown in Fig. 2-44 (P2.48(a)). All lines are lossless with $Z_0 = 50\ \Omega$.

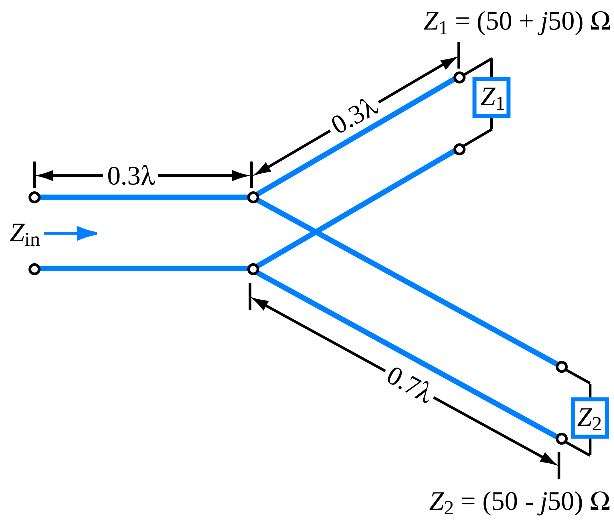


Figure P2.48: (a) Circuit of Problem 2.48.

Solution: Refer to Fig. P2.48(b).

$$z_1 = \frac{Z_1}{Z_0} = \frac{50 + j50\ \Omega}{50\ \Omega} = 1 + j1$$

and is at point *Z-LOAD*-1.

$$z_2 = \frac{Z_2}{Z_0} = \frac{50 - j50\ \Omega}{50\ \Omega} = 1 - j1$$

and is at point *Z-LOAD*-2. Since at the junction the lines are in parallel, it is advantageous to solve the problem using admittances. y_1 is point *Y-LOAD*-1, which is at 0.412λ on the WTG scale. y_2 is point *Y-LOAD*-2, which is at 0.088λ on the WTG scale. Traveling 0.300λ from *Y-LOAD*-1 toward the generator one obtains the input admittance for the upper feed line, point *Y-IN*-1, with a value of $1.97 + j1.02$. Since traveling 0.700λ is equivalent to traveling 0.200λ on any transmission line, the input admittance for the lower line feed is found at point *Y-IN*-2, which has a value of $1.97 - j1.02$. The admittance of the two lines together is the sum of their admittances: $1.97 + j1.02 + 1.97 - j1.02 = 3.94 + j0$ and is denoted *Y-JUNCT*. 0.300λ from *Y-JUNCT* toward the generator is the input admittance of the entire feed line, point *Y-IN*, from which *Z-IN* is found.

$$Z_{\text{in}} = z_{\text{in}} Z_0 = (1.65 - j1.79) \times 50\ \Omega = (82.5 - j89.5)\ \Omega.$$

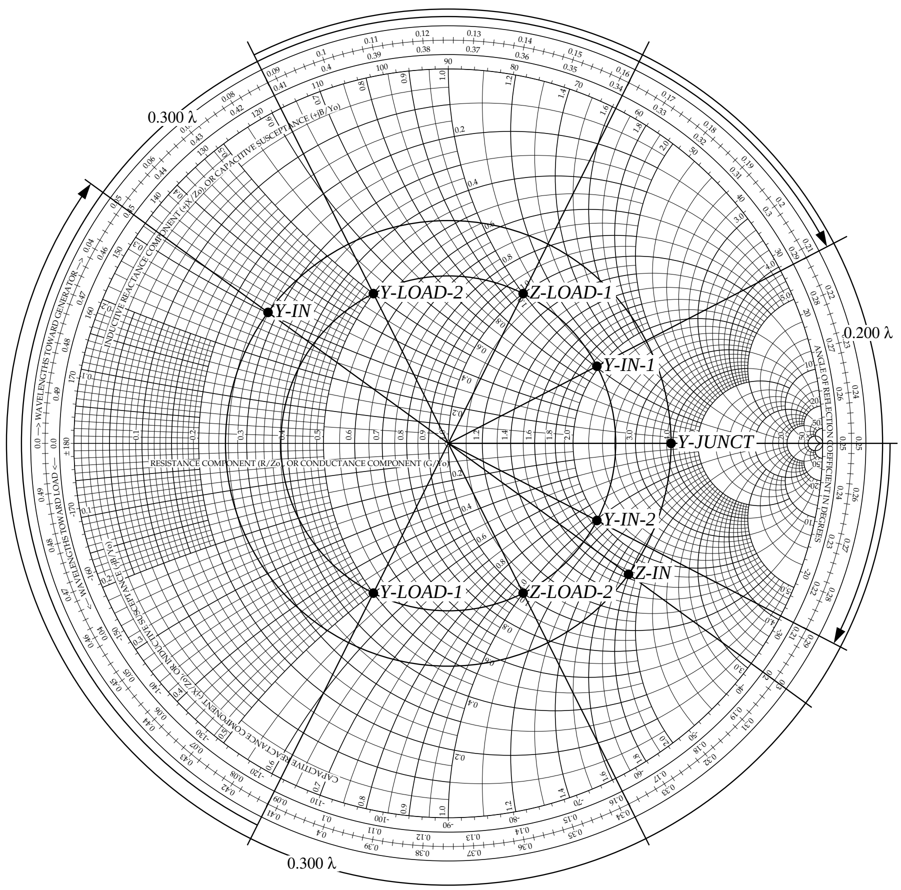


Figure P2.48: (b) Solution of Problem 2.48.

Problem 2.49 Repeat Problem 2.48 for the case where all three transmission lines are $\lambda/4$ in length.

Solution: Since the transmission lines are in parallel, it is advantageous to express loads in terms of admittances. In the upper branch, which is a quarter wave line,

$$Y_{1\text{ in}} = \frac{Y_0^2}{Y_1} = \frac{Z_1}{Z_0^2},$$

and similarly for the lower branch,

$$Y_{2\text{ in}} = \frac{Y_0^2}{Y_2} = \frac{Z_2}{Z_0^2}.$$

Thus, the total load at the junction is

$$Y_{\text{JCT}} = Y_{1\text{ in}} + Y_{2\text{ in}} = \frac{Z_1 + Z_2}{Z_0^2}.$$

Therefore, since the common transmission line is also quarter-wave,

$$Z_{\text{in}} = Z_0^2/Z_{\text{JCT}} = Z_0^2 Y_{\text{JCT}} = Z_1 + Z_2 = (50 + j50)\ \Omega + (50 - j50)\ \Omega = 100\ \Omega.$$

Section 2-11: Transients on Transmission Lines

Problem 2.50 Generate a bounce diagram for the voltage $V(z,t)$ for a 1-m long lossless line characterized by $Z_0 = 50\ \Omega$ and $u_p = 2c/3$ (where c is the velocity of light) if the line is fed by a step voltage applied at $t = 0$ by a generator circuit with $V_g = 60$ V and $R_g = 100\ \Omega$. The line is terminated in a load $Z_L = 25\ \Omega$. Use the bounce diagram to plot $V(t)$ at a point midway along the length of the line from $t = 0$ to $t = 25$ ns.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3},$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = \frac{-1}{3}.$$

From Eq. (2.124b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20\text{ V}.$$

Also,

$$T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5\text{ ns}.$$

The bounce diagram is shown in Fig. P2.50(a) and the plot of $V(t)$ in Fig. P2.50(b).

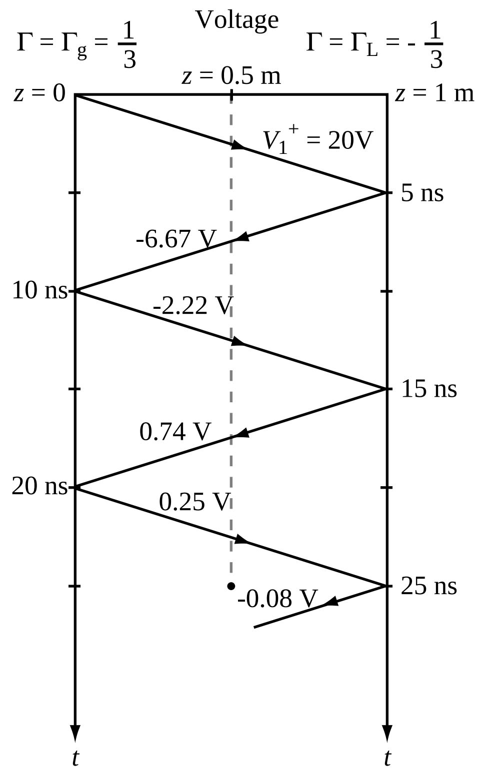


Figure P2.50: (a) Bounce diagram for Problem 2.50.

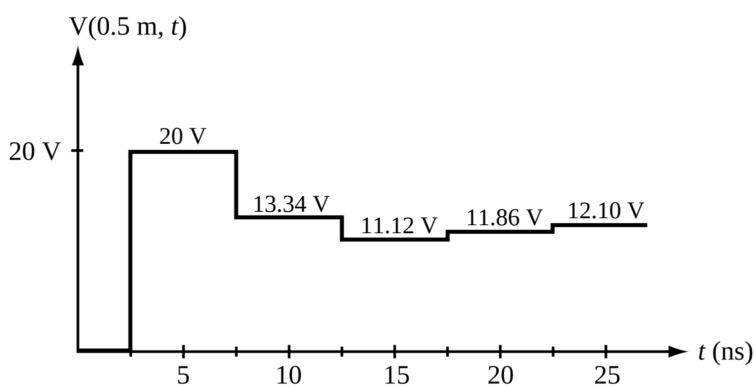


Figure P2.50: (b) Time response of voltage.

Problem 2.51 Repeat Problem 2.50 for the current I on the line.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100-50}{100+50} = \frac{1}{3},$$
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25-50}{25+50} = \frac{-1}{3}.$$

From Eq. (2.124a),

$$I_1^+ = \frac{V_g}{R_g + Z_0} = \frac{60}{100+50} = 0.4 \text{ A}.$$

The bounce diagram is shown in Fig. P2.51(a) and $I(t)$ in Fig. P2.51(b).

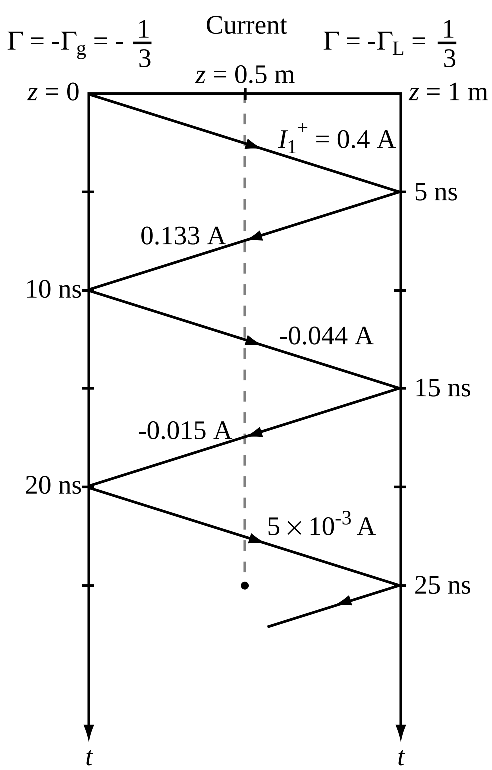


Figure P2.51: (a) Bounce diagram for Problem 2.51.

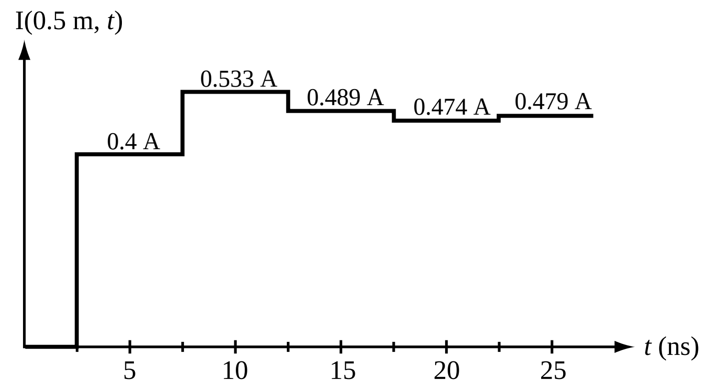


Figure P2.51: (b) Time response of current.

Problem 2.52 In response to a step voltage, the voltage waveform shown in Fig. 2-45 (P2.52) was observed at the sending end of a lossless transmission line with $R_g = 50\ \Omega$, $Z_0 = 50\ \Omega$, and $\varepsilon_r = 2.25$. Determine (a) the generator voltage, (b) the length of the line, and (c) the load impedance.

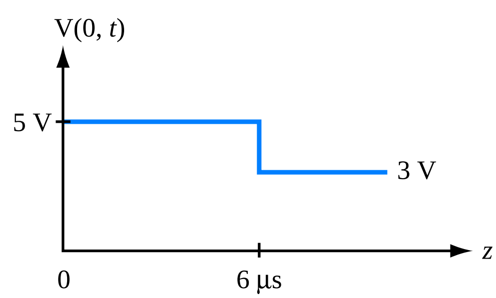


Figure P2.52: Observed voltage at sending end.

Solution:

(a) From the figure, $V_1^+ = 5$ V. Applying Eq. (2.124b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{V_g Z_0}{Z_0 + Z_0} = \frac{V_g}{2},$$

which gives $V_g = 2V_1^+ = 10$ V.

(b) $u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8$ m/s. The first change in the waveform occurs at $\Delta t = 6\ \mu$s. But $\Delta t = 2l/u_p$. Hence,

$$l = \frac{\Delta t \mu_p}{2} = \frac{6 \times 10^{-6}}{2} \times 2 \times 10^8 = 600 \text{ m}.$$

(c) Since $R_g = Z_0$, $\Gamma_g = 0$. Hence $V_2^+ = 0$ and the change in level from 5 V down to 3 V is due to $V_1^- = -2$ V. But

$$V_1^- = \Gamma_L V_1^+, \qquad \text{or} \quad \Gamma_L = \frac{V_1^-}{V_1^+} = \frac{-2}{5} = -0.4.$$

From

$$Z_L = Z_0 \left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) = 50 \left(\frac{1-0.4}{1+0.4}\right) = 21.43\ \Omega.$$

Problem 2.53 In response to a step voltage, the voltage waveform shown in Fig. 2.46 (P2.53) was observed at the sending end of a shorted line with $Z_0 = 50\ \Omega$ and $\varepsilon_r = 4$. Determine V_g, R_g, and the line length.

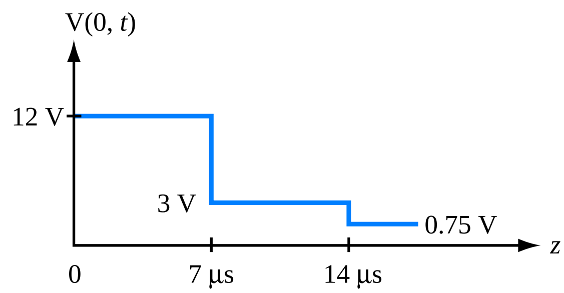


Figure P2.53: Observed voltage at sending end.

Solution:

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{4}} = 1.5 \times 10^8 \text{ m/s},$$

$$7\ \mu\text{s} = 7 \times 10^{-6} \text{ s} = \frac{2l}{u_p} = \frac{2l}{1.5 \times 10^8}.$$

Hence, $l = 525$ m.

From the voltage waveform, $V_1^+ = 12$ V. At $t = 7\mu$s, the voltage at the sending end is

$$V(z=0, t=7\mu\text{s}) = V_1^+ + \Gamma_L V_1^+ + \Gamma_g \Gamma_L V_1^+ = -\Gamma_g V_1^+ \qquad (\text{because } \Gamma_L = -1).$$

Hence, 3 V$= -\Gamma_g \times 12$ V, or $\Gamma_g = -0.25$. From Eq. (2.128),

$$R_g = Z_0 \left(\frac{1+\Gamma_g}{1-\Gamma_g} \right) = 50 \left(\frac{1-0.25}{1+0.25} \right) = 30\ \Omega.$$

Also,

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}, \qquad \text{or} \quad 12 = \frac{V_g \times 50}{30+50},$$

which gives $V_g = 19.2$ V.

Problem 2.54 Suppose the voltage waveform shown in Fig. 2-45 was observed at the sending end of a 50-Ω transmission line in response to a step voltage introduced by a generator with $V_g = 15$ V and an unknown series resistance R_g. The line is 1 km in length, its velocity of propagation is 1×10^8 m/s, and it is terminated in a load $Z_L = 100\ \Omega$.

(a) Determine R_g.

(b) Explain why the drop in level of $V(0,t)$ at $t = 6\ \mu$s cannot be due to reflection from the load.

(c) Determine the shunt resistance R_f and the location of the fault responsible for the observed waveform.

Solution:

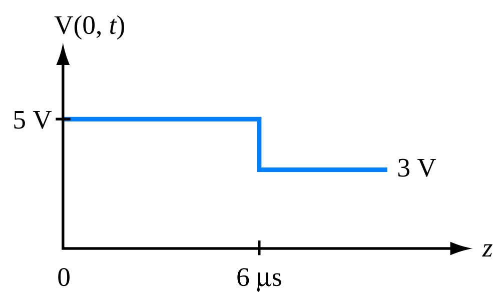


Figure P2.54: Observed voltage at sending end.

(a)

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}.$$

From Fig. 2-45, $V_1^+ = 5$ V. Hence,

$$5 = \frac{15 \times 50}{R_g + 50},$$

which gives $R_g = 100\ \Omega$ and $\Gamma_g = 1/3$.

(b) Roundtrip time delay of pulse return from the load is

$$2T = \frac{2l}{u_p} = \frac{2 \times 10^3}{1 \times 10^8} = 20\ \mu\text{s},$$

which is much longer than 6 μs, the instance at which $V(0,t)$ drops in level.

(c) The new level of 3 V is equal to V_1^+ plus V_1^- plus V_2^+,

$$V_1^+ + V_1^- + V_2^+ = 5 + 5\Gamma_f + 5\Gamma_f\Gamma_g = 3 \quad \text{(V)},$$

which yields $\Gamma_f = -0.3$. But

$$\Gamma_f = \frac{Z_{Lf} - Z_0}{Z_{Lf} + Z_0} = -0.3,$$

which gives $Z_{Lf} = 26.92\ \Omega$. Since Z_{Lf} is equal to R_f and Z_0 in parallel, $R_f = 58.33\ \Omega$.

Problem 2.55 A generator circuit with $V_g = 200$ V and $R_g = 25\ \Omega$ was used to excite a 75-Ω lossless line with a rectangular pulse of duration $\tau = 0.4\ \mu$s. The line is 200 m long, its $u_p = 2 \times 10^8$ m/s, and it is terminated in a load $Z_L = 125\ \Omega$.

(a) Synthesize the voltage pulse exciting the line as the sum of two step functions, $V_{g_1}(t)$ and $V_{g_2}(t)$.

(b) For each voltage step function, generate a bounce diagram for the voltage on the line.

(c) Use the bounce diagrams to plot the total voltage at the sending end of the line.

Solution:

(a) pulse length $= 0.4\ \mu$s.

$$V_g(t) = V_{g_1}(t) + V_{g_2}(t),$$

with

$$V_{g_1}(t) = 200U(t) \quad \text{(V)},$$
$$V_{g_2}(t) = -200U(t - 0.4\ \mu\text{s}) \quad \text{(V)}.$$

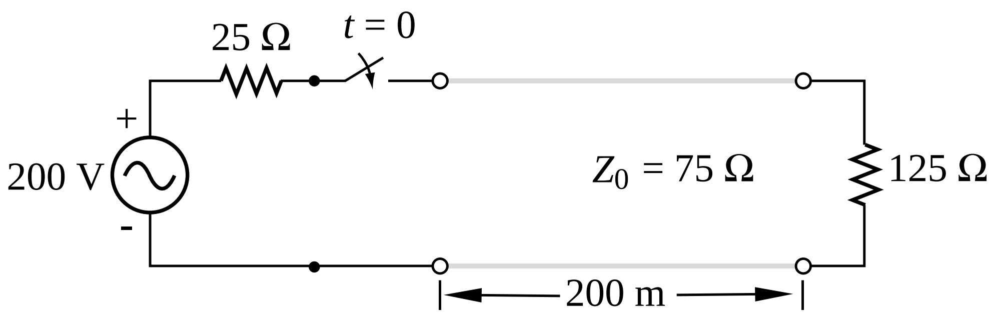


Figure P2.55: (a) Circuit for Problem 2.55.

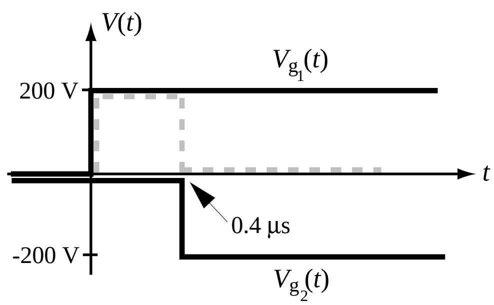


Figure P2.55: (b) Solution of part (a).

(b)

$$T = \frac{l}{u_p} = \frac{200}{2 \times 10^8} = 1\ \mu\text{s}.$$

We will divide the problem into two parts, one for $V_{g_1}(t)$ and another for $V_{g_2}(t)$ and then we will use superposition to determine the solution for the sum. The solution for $V_{g_2}(t)$ will mimic the solution for $V_{g_1}(t)$, except for a reversal in sign and a delay by 0.4 μs.

For $V_{g_1}(t) = 200U(t)$:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{25 - 75}{25 + 75} = -0.5,$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{125 - 75}{125 + 75} = 0.25,$$

$$V_1^+ = \frac{V_1 Z_0}{R_g + Z_0} = \frac{200 \times 75}{25 + 75} = 150 \text{ V},$$
$$V_\infty = \frac{V_g Z_L}{R_g + Z_L} = \frac{200 \times 125}{25 + 125} = 166.67 \text{ V}.$$

(i) $V_1(0,t)$ at sending end due to $V_{g_1}(t)$:

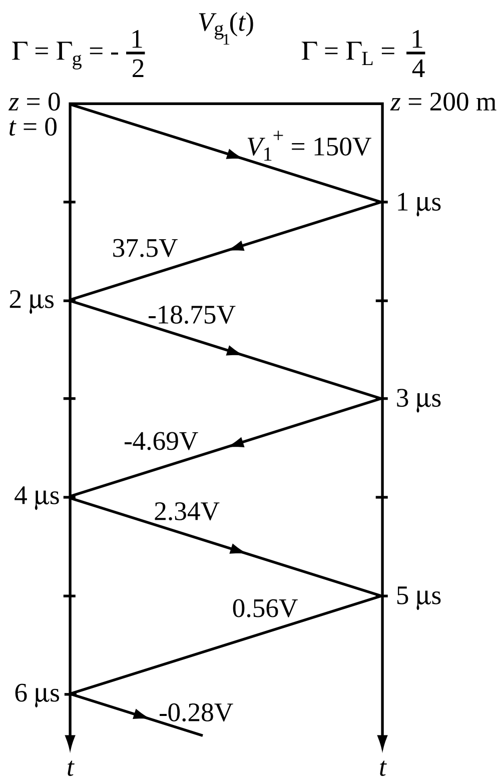


Figure P2.55: (c) Bounce diagram for voltage in reaction to $V_{g_1}(t)$.

(ii) $V_2(0,t)$ at sending end due to $V_{g_2}(t)$:

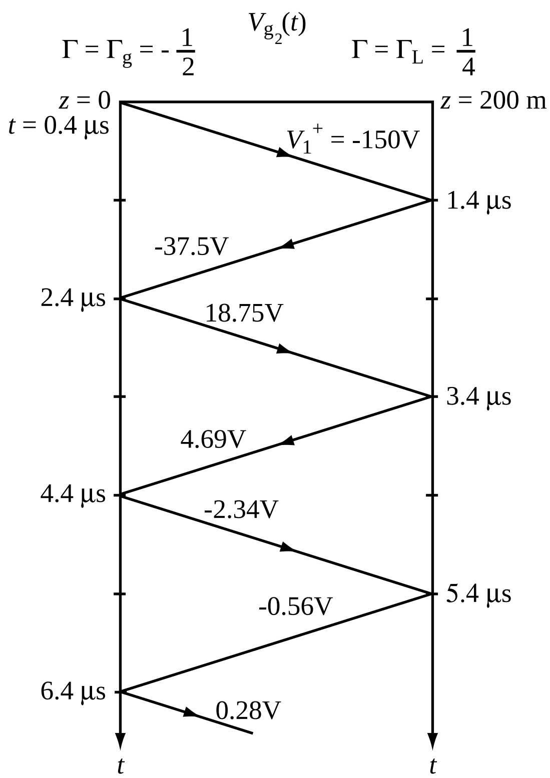


Figure P2.55: (d) Bounce diagram for voltage in reaction to $V_{g_2}(t)$.

(b)

(i) $V_1(0,t)$ at sending end due to $V_{g_1}(t)$:

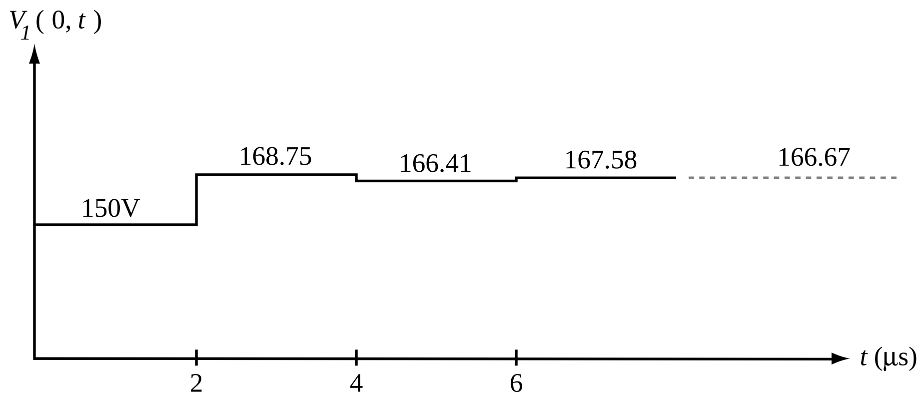


Figure P2.55: (e) $V_1(0,t)$.

(ii) $V_2(0,t)$ at sending end:

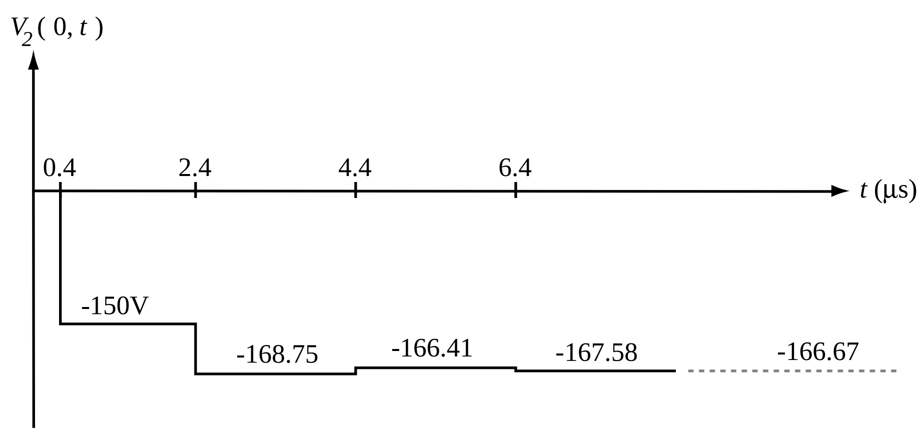


Figure P2.55: (f) $V_2(0,t)$.

(iii) Net voltage $V(0,t) = V_1(0,t) + V_2(0,t)$:

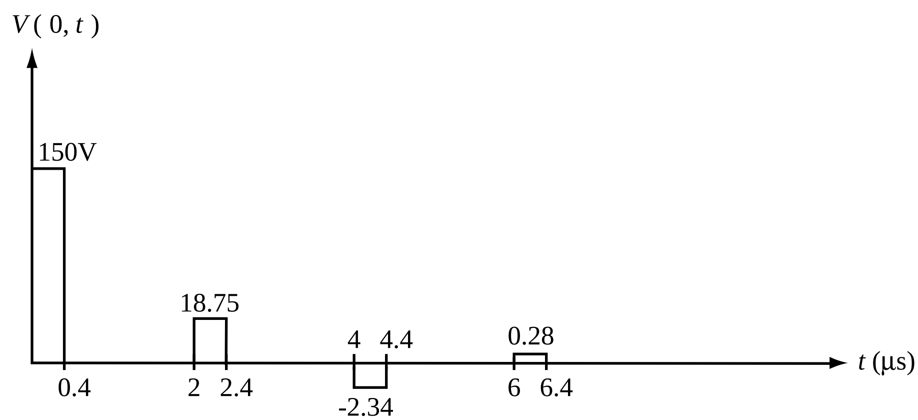


Figure P2.55: (g) Net voltage $V(0,t)$.

Problem 2.56 For the circuit of Problem 2.55, generate a bounce diagram for the current and plot its time history at the middle of the line.

Solution: Using the values for Γ_g and Γ_L calculated in Problem 2.55, we reverse their signs when using them to construct a bounce diagram for the current.

$$I_1^+ = \frac{V_1^+}{Z_0} = \frac{150}{75} = 2 \text{ A},$$
$$I_2^+ = \frac{V_2^+}{Z_0} = \frac{-150}{75} = -2 \text{ A},$$
$$I_\infty^+ = \frac{V_\infty}{Z_L} = 1.33 \text{ A}.$$

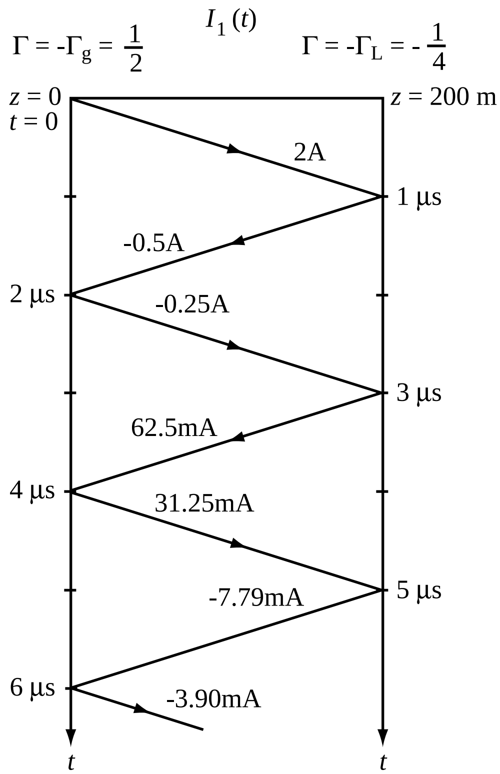


Figure P2.56: (a) Bounce diagram for $I_1(t)$ in reaction to $V_{g_1}(t)$.

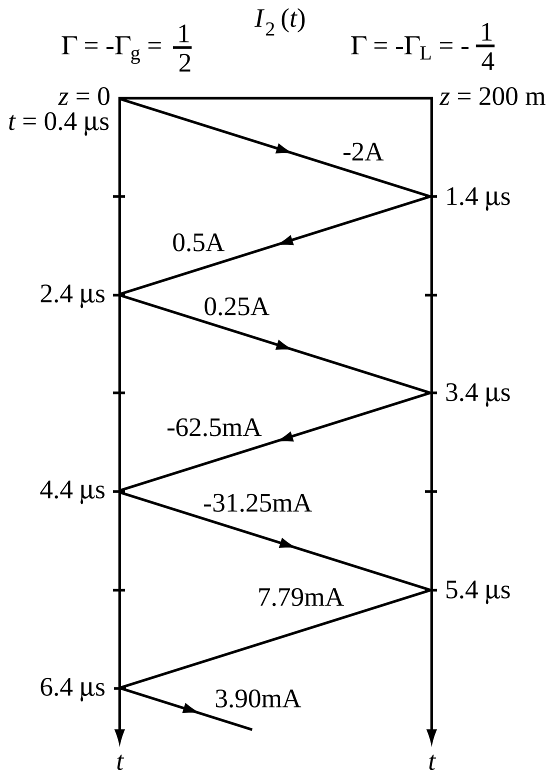


Figure P2.56: (b) Bounce diagram for current $I_2(t)$ in reaction to $V_{g_2}(t)$.

(i) $I_1(l/2,t)$ due to $V_{g_1}(t)$:

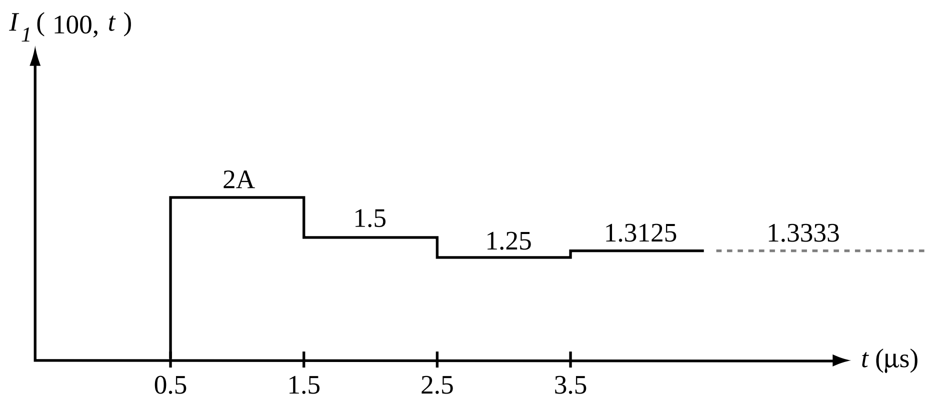


Figure P2.56: (c) $I_1(l/2,t)$.

(ii) $I_2(l/2,t)$ due to $V_{g_2}(t)$:

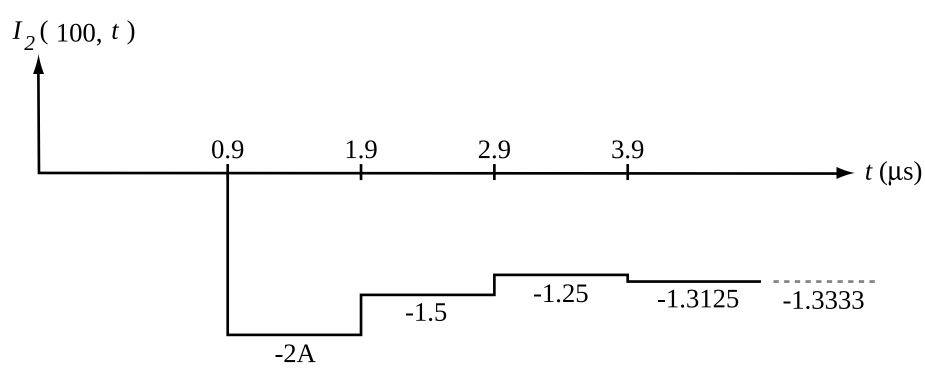


Figure P2.56: (d) $I_2(l/2,t)$.

(iii) Net current $I(l/2,t) = I_1(l/2,t) + I_2(l/2,t)$:

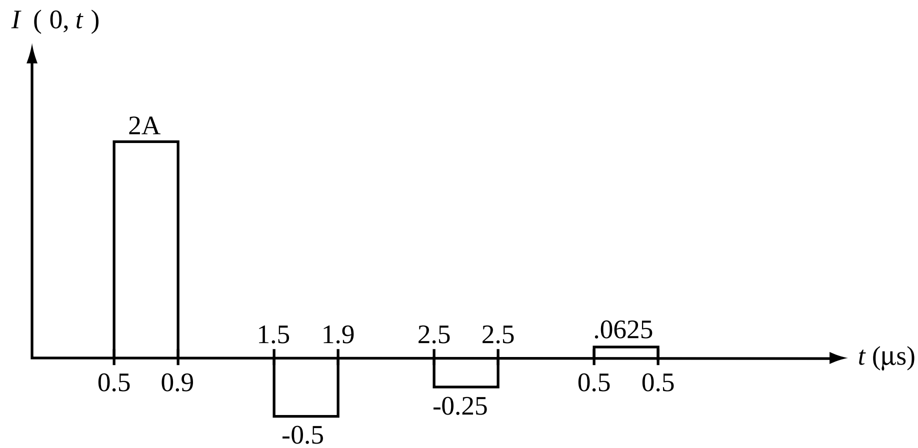


Figure P2.56: (e) Total $I(l/2,t)$.

Problem 2.57 For the parallel-plate transmission line of Problem 2.3, the line parameters are given by:

$$
\begin{aligned}
R' &= 1 \quad (\Omega/\text{m}),\\
L' &= 167 \quad (\text{nH/m}),\\
G' &= 0,\\
C' &= 172 \quad (\text{pF/m}).
\end{aligned}
$$

Find α, β, u_p, and Z_0 at 1 GHz.

Solution: At 1 GHz, $\omega = 2\pi f = 2\pi \times 10^9$ rad/s. Application of (2.22) gives:

$$\begin{aligned}
\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= [(1 + j2\pi \times 10^9 \times 167 \times 10^{-9})(0 + j2\pi \times 10^9 \times 172 \times 10^{-12})]^{1/2} \\
&= [(1 + j1049)(j1.1)]^{1/2} \\
&= \left[\sqrt{1 + (1049)^2}\, e^{j\tan^{-1} 1049} \times 1.1e^{j90^\circ}\right]^{1/2}, \qquad (j = e^{j90^\circ}) \\
&= \left[1049e^{j89.95^\circ} \times 1.1e^{j90^\circ}\right]^{1/2} \\
&= \left[1154e^{j179.95^\circ}\right]^{1/2} \\
&= 34e^{j89.97^\circ} = 34\cos 89.97^\circ + j34 \sin 89.97^\circ = 0.016 + j34.
\end{aligned}$$

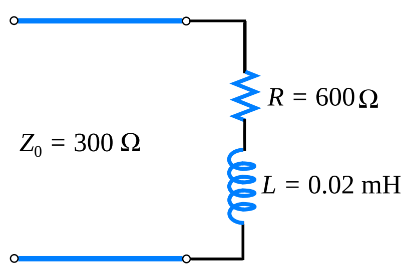
Hence,

$$\alpha = 0.016 \text{ Np/m},$$
$$\beta = 34 \text{ rad/m}.$$

$$u_{\text{p}} = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10^9}{34} = 1.85 \times 10^8 \text{ m/s}.$$

$$\begin{aligned}
Z_0 &= \left[\frac{R' + j\omega L'}{G' + j\omega C'}\right]^{1/2} \\
&= \left[\frac{1049e^{j89.95^\circ}}{1.1e^{j90^\circ}}\right]^{1/2} \\
&= \left[954e^{-j0.05^\circ}\right]^{1/2} \\
&= 31e^{-j0.025^\circ} \simeq (31 - j0.01)\ \Omega.
\end{aligned}$$

Problem 2.58



A 300-Ω lossless air transmission line is connected to a complex load composed of a resistor in series with an inductor, as shown in the figure. At 5 MHz, determine: **(a)** Γ, **(b)** S, **(c)** location of voltage maximum nearest to the load, and **(d)** location of current maximum nearest to the load.

Solution:

(a)

$$\begin{aligned} Z_L &= R + j\omega L \\ &= 600 + j2\pi \times 5 \times 10^6 \times 2 \times 10^{-5} = (600 + j628)\ \Omega. \end{aligned}$$

$$\begin{aligned} \Gamma &= \frac{Z_L - Z_0}{Z_L + Z_0} \\ &= \frac{600 + j628 - 300}{600 + j628 + 300} \\ &= \frac{300 + j628}{900 + j628} = 0.63e^{j29.6^\circ}. \end{aligned}$$

(b)

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.63}{1 - 0.63} = 1.67.$$

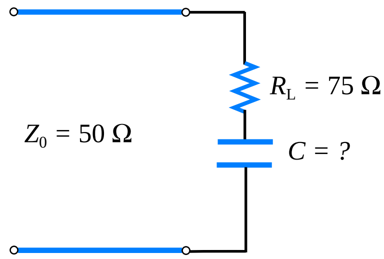
(c)

$$\begin{aligned} l_{max} &= \frac{\theta_r \lambda}{4\pi} \qquad \text{for } \theta_r > 0. \\ &= \left(\frac{29.6^\circ \pi}{180^\circ}\right)\frac{60}{4\pi}, \qquad \left(\lambda = \frac{3 \times 10^8}{5 \times 10^6} = 60 \text{ m}\right) \\ &= 2.46 \text{ m} \end{aligned}$$

(d) The locations of current maxima correspond to voltage minima and vice versa. Hence, the location of current maximum nearest the load is the same as location of voltage minimum nearest the load. Thus

$$\begin{aligned} l_{min} &= l_{max} + \frac{\lambda}{4}, \qquad \left(l_{max} < \frac{\lambda}{4} = 15 \text{ m}\right) \\ &= 2.46 + 15 = 17.46 \text{ m}. \end{aligned}$$

Problem 2.59



A 50-Ω lossless transmission line is connected to a load composed of a 75-Ω resistor in series with a capacitor of unknown capacitance. If at 10 MHz the voltage standing wave ratio on the line was measured to be 3, determine the capacitance C.

Solution:

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5$$

$$Z_L = R_L - jX_C, \qquad \text{where } X_C = \frac{1}{\omega C}.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$|\Gamma|^2 = \left[\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)\left(\frac{Z_L^* - Z_0}{Z_L^* + Z_0}\right)\right]$$

$$|\Gamma|^2 = \frac{Z_L Z_L^* + Z_0^2 - Z_0(Z_L + Z_L^*)}{Z_L Z_L^* + Z_0^2 + Z_0(Z_L + Z_L^*)}$$

Noting that:

$$Z_L Z_L^* = (R_L - jX_C)(R_L + jX_C) = R_L^2 + X_C^2,$$

$$Z_0(Z_L + Z_L^*) = Z_0(R_L - jX_C + R_L + jX_C) = 2Z_0 R_L,$$

$$|\Gamma|^2 = \frac{R_L^2 + X_C^2 + Z_0^2 - 2Z_0 R_L}{R_L^2 + X_C^2 + Z_0^2 + 2Z_0 R_L}.$$

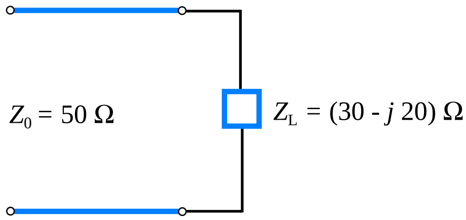
Upon substituting $|\Gamma_L| = 0.5$, $R_L = 75\ \Omega$, and $Z_0 = 50\ \Omega$, and then solving for X_C, we have

$$X_C = 66.1\ \Omega.$$

Hence

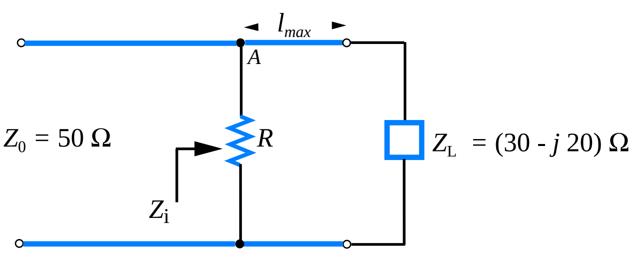
$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 10^7 \times 66.1} = 2.41 \times 10^{-10} = 241 \text{ pF}.$$

Problem 2.60 A 50-Ω lossless line is terminated in a load impedance $Z_L = (30 - j20)\ \Omega$.



(a) Calculate Γ and S.

(b) It has been proposed that by placing an appropriately selected resistor across the line at a distance l_{max} from the load (as shown in the figure below), where l_{max} is the distance from the load of a voltage maximum, then it is possible to render $Z_i = Z_0$, thereby eliminating reflections back to the sending end. Show that the proposed approach is valid and find the value of the shunt resistance.



Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j20 - 50}{30 - j20 + 50} = \frac{-20 - j20}{80 - j20} = \frac{-(20 + j20)}{80 - j20} = 0.34e^{-j121^\circ}.$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.34}{1 - 0.34} = 2.$$

(b) We start by finding l_{max}, the distance of the voltage maximum nearest to the load. Using (2.56) with $n = 1$,

$$\begin{aligned} l_{max} &= \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2} \\ &= \left(\frac{-121^\circ \pi}{180^\circ}\right) \frac{\lambda}{4\pi} + \frac{\lambda}{2} = 0.33\lambda. \end{aligned}$$

Applying (2.63) at $l = l_{max} = 0.33\lambda$, for which $\beta l = (2\pi/\lambda \times 0.33\lambda = 2.07$ radians, the value of Z_{in} before adding the shunt resistance is:

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)$$
$$= 50 \left(\frac{(30 - j20) + j50 \tan 2.07}{50 + j(30 - j20) \tan 2.07} \right) = (102 + j0)\ \Omega.$$

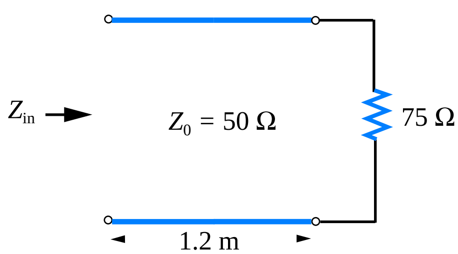
Thus, at the location A (at a distance l_{max} from the load), the input impedance is purely real. If we add a shunt resistor R in parallel such that the combination is equal to Z_0, then the new Z_{in} at any point to the left of that location will be equal to Z_0.

Hence, we need to select R such that

$$\frac{1}{R} + \frac{1}{102} = \frac{1}{50}$$

or $R = 98\ \Omega$.

Problem 2.61 For the lossless transmission line circuit shown in the figure, determine the equivalent series lumped-element circuit at 400 MHz at the input to the line. The line has a characteristic impedance of 50 Ω and the insulating layer has $\varepsilon_r = 2.25$.



Solution: At 400 MHz,

$$\lambda = \frac{u_p}{f} = \frac{c}{f\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{4 \times 10^8 \sqrt{2.25}} = 0.5 \text{ m}.$$
$$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{0.5} \times 1.2 = 4.8\pi.$$

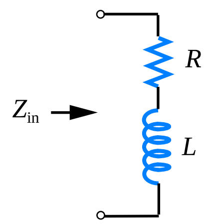
Subtracting multiples of 2π, the remainder is:

$$\beta l = 0.8\pi \text{ rad}.$$

Using (2.63),

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)$$
$$= 50 \left(\frac{75 + j50 \tan 0.8\pi}{50 + j75 \tan 0.8\pi} \right) = (52.38 + j20.75)\ \Omega.$$

Z_{in} is equivalent to a series RL circuit with



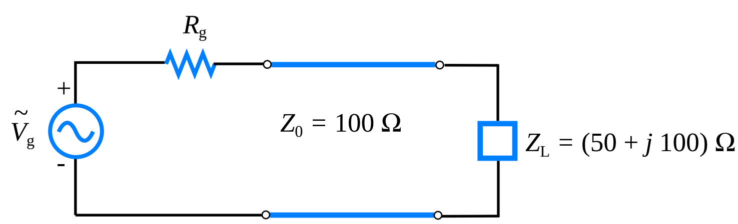
$$R = 52.38\ \Omega$$
$$\omega L = 2\pi f L = 20.75\ \Omega$$

or

$$L = \frac{20.75}{2\pi \times 4 \times 10^8} = 8.3 \times 10^{-9}\ \text{H},$$

which is a very small inductor.

Problem 2.62



The circuit shown in the figure consists of a 100-Ω lossless transmission line terminated in a load with $Z_L = (50 + j100)\ \Omega$. If the peak value of the load voltage was measured to be $|\widetilde{V}_L| = 12$ V, determine:

(a) the time-average power dissipated in the load,

(b) the time-average power incident on the line, and

(c) the time-average power reflected by the load.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j100 - 100}{50 + j100 + 100} = \frac{-50 + j100}{150 + j100} = 0.62e^{j82.9^\circ}.$$

The time average power dissipated in the load is:

$$\begin{aligned} P_{av} &= \frac{1}{2}|\widetilde{I}_L|^2 R_L \\ &= \frac{1}{2}\left|\frac{\widetilde{V}_L}{Z_L}\right|^2 R_L \\ &= \frac{1}{2}\frac{|\widetilde{V}_L|^2}{|Z_L|^2} R_L = \frac{1}{2} \times 12^2 \times \frac{50}{50^2 + 100^2} = 0.29 \text{ W.} \end{aligned}$$

(b)

$$P_{av} = P^i_{av}(1 - |\Gamma|^2)$$

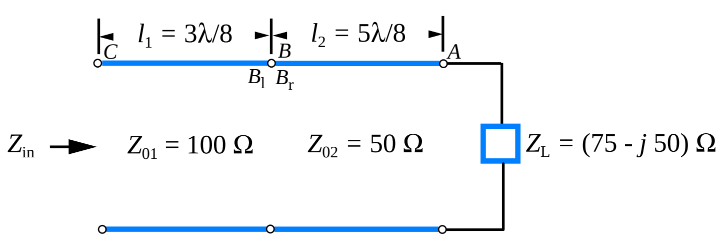
Hence,

$$P^i_{av} = \frac{P_{av}}{1 - |\Gamma|^2} = \frac{0.29}{1 - 0.62^2} = 0.47 \text{ W.}$$

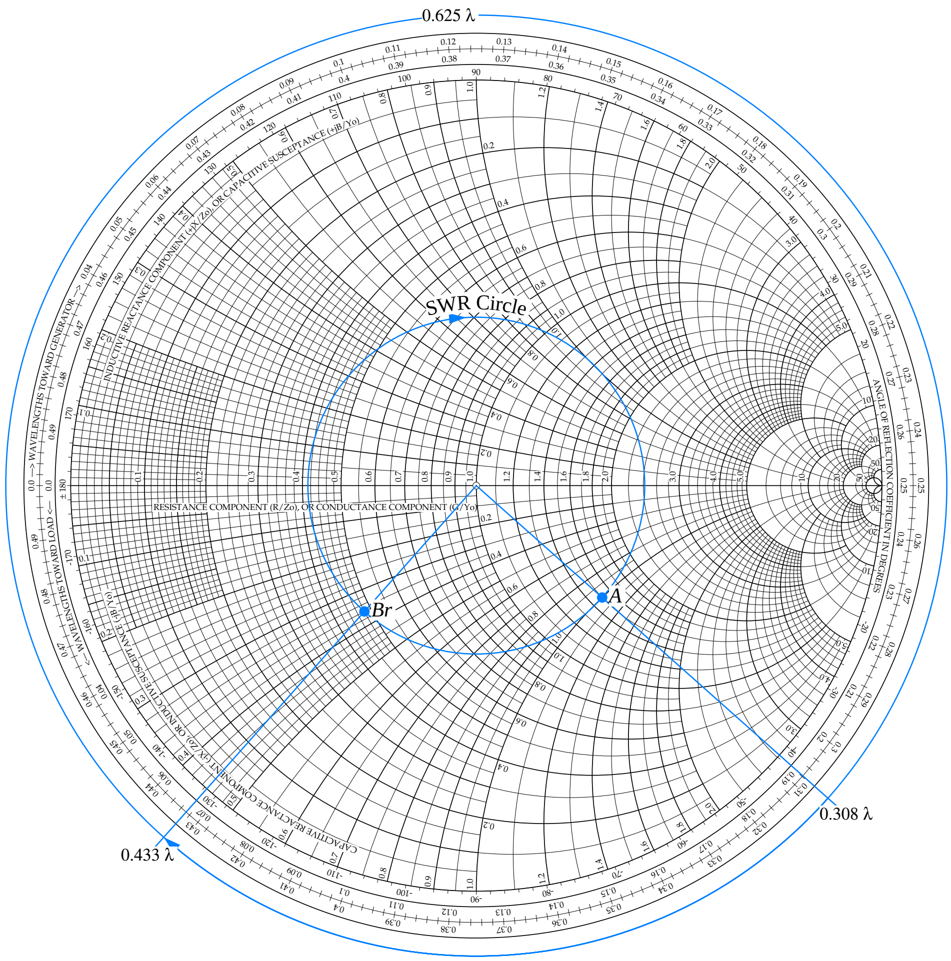
(c)

$$P^r_{av} = -|\Gamma|^2 P^i_{av} = -(0.62)^2 \times 0.47 = -0.18 \text{ W.}$$

Problem 2.63



Use the Smith chart to determine the input impedance Z_{in} of the two-line configuration shown in the figure.



Smith Chart 1

Solution: Starting at point A, namely at the load, we normalize Z_L with respect to Z_{02}:

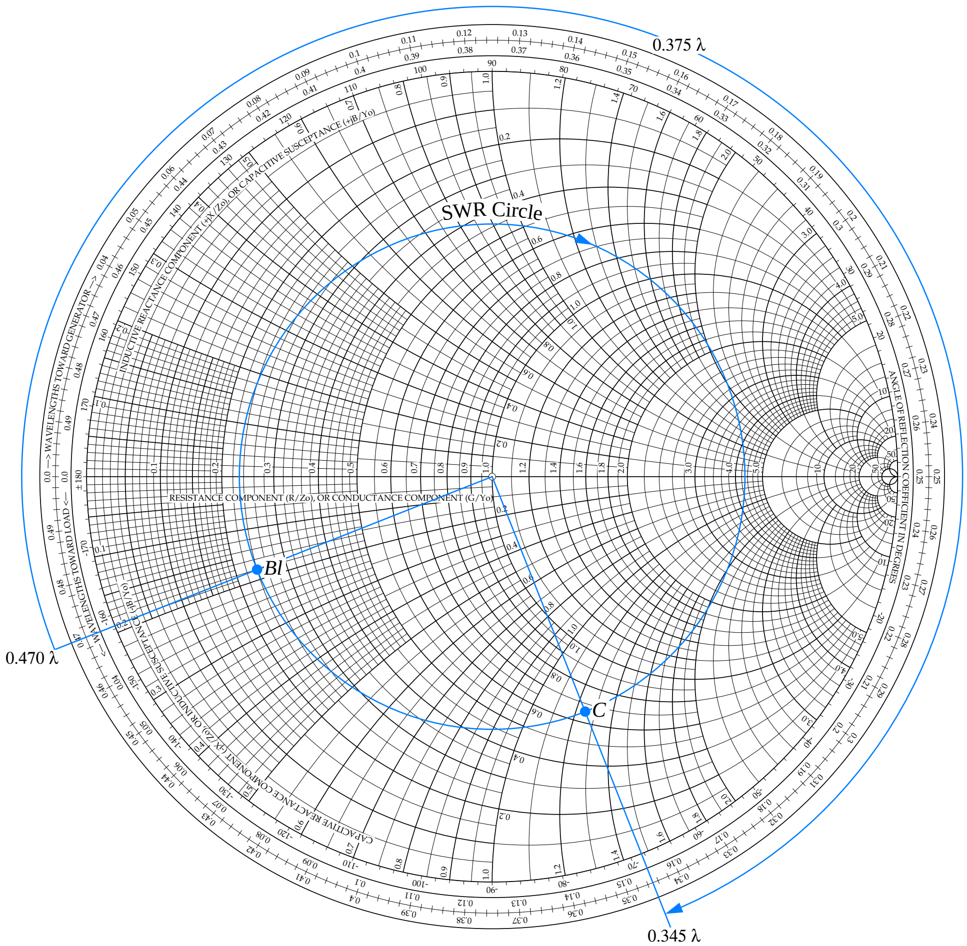
$$z_L = \frac{Z_L}{Z_{02}} = \frac{75 - j50}{50} = 1.5 - j1. \qquad \text{(point } A \text{ on Smith chart 1)}$$

From point A on the Smith chart, we move on the SWR circle a distance of $5\lambda/8$ to point B_r, which is just to the right of point B (see figure). At B_r, the normalized input impedance of line 2 is:

$$z_{in2} = 0.48 - j0.36 \qquad \text{(point } B_r \text{ on Smith chart)}$$

Next, we unnormalize z_{in2}:

$$Z_{in2} = Z_{02} z_{in2} = 50 \times (0.48 - j0.36) = (24 - j18)\ \Omega.$$



Smith Chart 2

To move along line 1, we need to normalize with respect to Z_{01}. We shall call this z_{L1}:

$$z_{L1} = \frac{Z_{in2}}{Z_{01}} = \frac{24 - j18}{100} = 0.24 - j0.18 \qquad \text{(point } B_\ell \text{ on Smith chart 2)}$$

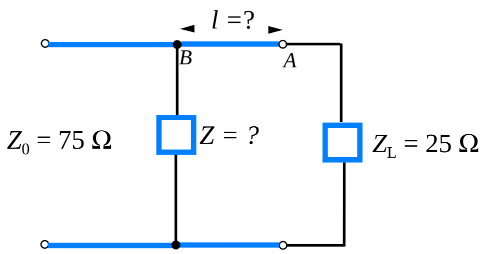
After drawing the SWR circle through point B_ℓ, we move $3\lambda/8$ towards the generator, ending up at point C on Smith chart 2. The normalized input impedance of line 1 is:

$$z_{in} = 0.66 - j1.25$$

which upon unnormalizing becomes:

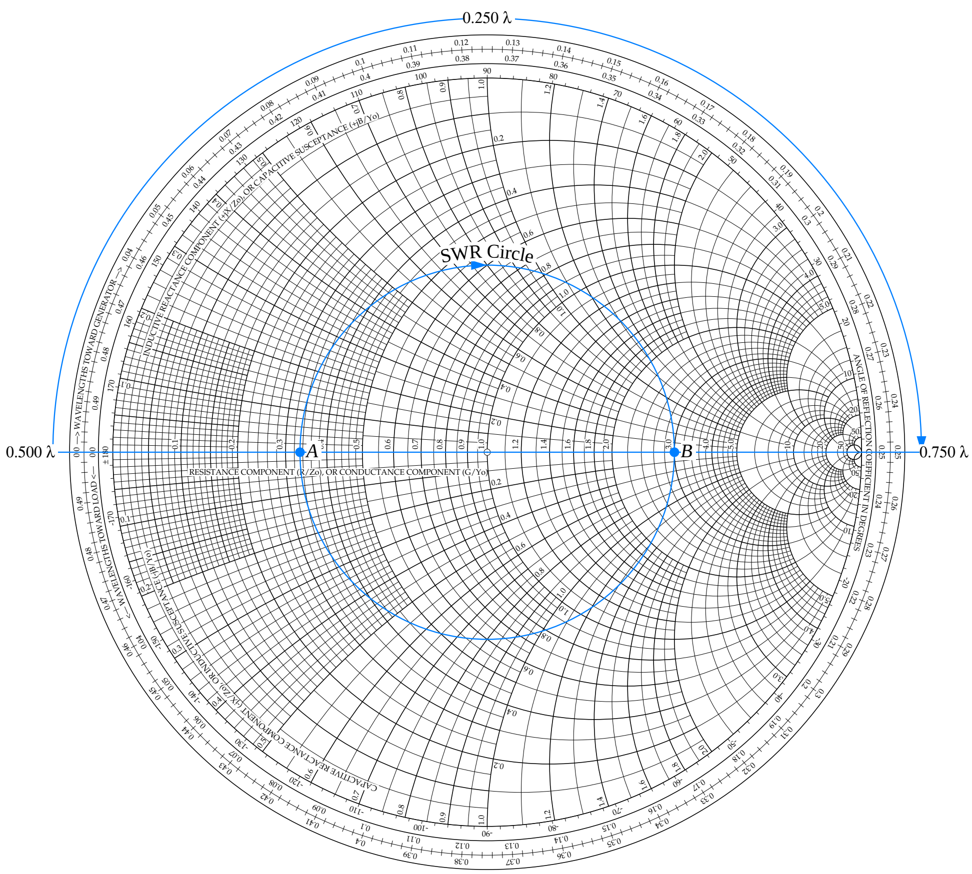
$$Z_{in} = (66 - j125)\ \Omega.$$

Problem 2.64



A 25-Ω antenna is connected to a 75-Ω lossless transmission line. Reflections back toward the generator can be eliminated by placing a shunt impedance Z at a distance l from the load. Determine the values of Z and l.

Solution:



The normalized load impedance is:

$$z_L = \frac{25}{75} = 0.33 \qquad \text{(point } A \text{ on Smith chart)}$$

The Smith chart shows A and the SWR circle. The goal is to have an equivalent impedance of 75 Ω to the left of B. That equivalent impedance is the parallel combination of Z_{in} at B (to the right of the shunt impedance Z) and the shunt element Z. Since we need for this to be purely real, it's best to choose l such that Z_{in} is purely real, thereby choosing Z to be simply a resistor. Adding two resistors in parallel generates a sum smaller in magnitude than either one of them. So we need for Z_{in} to be larger than Z_0, not smaller. On the Smith chart, that point is B, at a distance $l = \lambda/4$ from the load. At that point:

$$z_{in} = 3,$$

which corresponds to

$$y_{in} = 0.33.$$

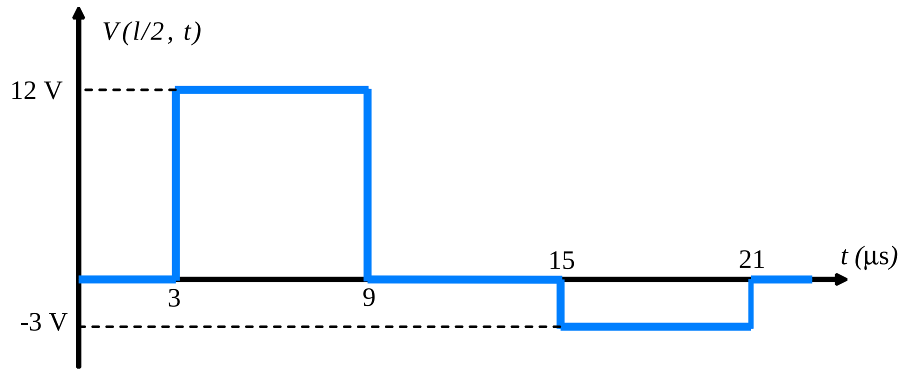
Hence, we need y, the normalized admittance corresponding to the shunt impedance Z, to have a value that satisfies:

$$\begin{aligned} y_{in} + y &= 1 \\ y &= 1 - y_{in} = 1 - 0.33 = 0.66 \\ z &= \frac{1}{y} = \frac{1}{0.66} = 1.5 \\ Z &= 75 \times 1.5 = 112.5\ \Omega. \end{aligned}$$

In summary,

$$\begin{aligned} l &= \frac{\lambda}{4}, \\ Z &= 112.5\ \Omega. \end{aligned}$$

Problem 2.65 In response to a step voltage, the voltage waveform shown in the figure below was observed at the midpoint of a lossless transmission line with $Z_0 = 50\ \Omega$ and $u_p = 2 \times 10^8$ m/s. Determine: **(a)** the length of the line, **(b)** Z_L, **(c)** R_g, and **(d)** V_g.



Solution:

(a) Since it takes 3 μs to reach the middle of the line, the line length must be

$$l = 2(3 \times 10^{-6} \times u_p) = 2 \times 3 \times 10^{-6} \times 2 \times 10^8 = 1200 \text{ m}.$$

(b) From the voltage waveform shown in the figure, the duration of the first rectangle is 6 μs, representing the time it takes the incident voltage V_1^+ to travel from the midpoint of the line to the load and back. The fact that the voltage drops to zero at $t = 9$ μs implies that the reflected wave is exactly equal to V_1^+ in magnitude, but opposite in polarity. That is,

$$V_1^- = -V_1^+.$$

This in turn implies that $\Gamma_L = -1$, which means that the load is a short circuit:

$$Z_L = 0.$$

(c) After V_1^- arrives at the generator end, it encounters a reflection coefficient Γ_g. The voltage at 15 μs is composed of:

$$\begin{aligned} V &= V_1^+ + V_1^- + V_2^+ \\ &= (1 + \Gamma_L + \Gamma_L \Gamma_g) V_1^+ \\ \frac{V}{V_1^+} &= 1 - 1 - \Gamma_g \end{aligned}$$

From the figure, $V/V_1^+ = -3/12 = -1/4$. Hence,

$$\Gamma_g = \frac{1}{4},$$

which means that

$$R_g = \left(\frac{1+\Gamma_g}{1-\Gamma_g}\right) Z_0 = \left(\frac{1+0.25}{1-0.25}\right) 50 = 83.3\ \Omega.$$

(d)

$$V_1^+ = 12 = \frac{V_g Z_0}{R_g + Z_0}$$

$$V_g = \frac{12(R_g + Z_0)}{Z_0} = \frac{12(83.3+50)}{50} = 32\ \text{V}.$$