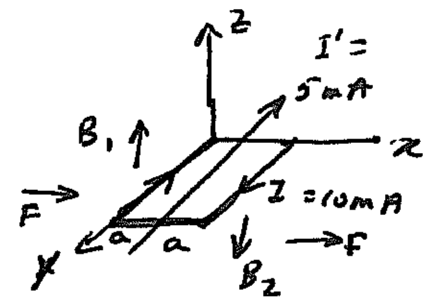
Dec. 2011

EECE 380 -

Quiz #2 - solution.

1. $B_1 = B_2 = \frac{\mu_0 I'}{2\pi a}$



the total force is:

$$F = 2 \times \frac{\mu_0 I I'}{\pi} \vec{a}_x$$

$$= 2 \times \frac{4\cancel{\pi} \times 10^{-7} \times 50 \times 10^{-6}}{\cancel{\pi}} \vec{a}_x = 40 \times 10^{-12}\ N\ \vec{a}_x .$$

2. Regions 1-2 : $\mu_1 \tan\theta_1 = \mu_2 \tan\theta_2$

Regions 2-3 : $\mu_2 \tan\theta_3 = \mu_3 \tan\theta_4$

But $\theta_2 = \theta_3$ so $\mu_1 \tan\theta_1 = \mu_3 \tan\theta_4$

or $\tan\theta_4 = \frac{\mu_1}{\mu_3} \tan\theta_1$

3. $A = (5 \sin\theta)\vec{a}_\theta$; find B at $(5, \frac{\pi}{2}, 0)$

$$B = \text{curl}\, A = \frac{1}{r}\frac{\partial}{\partial r}(5r \sin\theta)\vec{a}_\phi = \frac{5}{r}\sin\theta\, a_\phi$$

at the given point: $B = \frac{\cancel{5}}{\cancel{5}} \sin\frac{\pi}{2}\ \vec{a}_\phi = \vec{a}_\phi$

But $\vec{a}_\phi = -\vec{a}_x \sin\phi + \vec{a}_y \cos\phi$

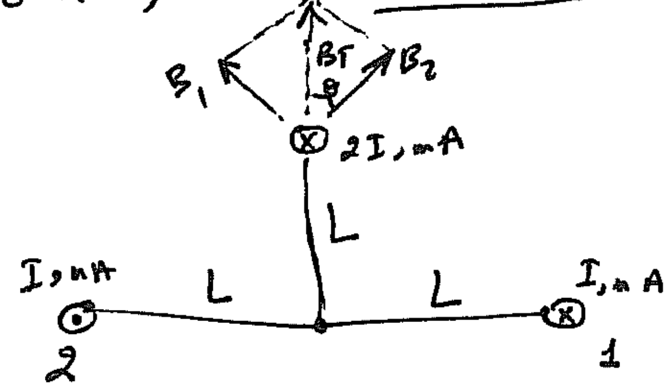
$= -\vec{a}_x\, 0 + \vec{a}_y \cdot 1$ so $\vec{a}_\phi = \vec{a}_y$

4. $B_1 = B_2 = \frac{\mu_0 I}{2\pi\sqrt{2}\, L}$

so $B_T = (B_1 + B_2)\cos\theta$

$$= \frac{\cancel{2}\mu_0 I}{\cancel{2}\pi\sqrt{2}L} \cdot \frac{1}{\sqrt{2}} =$$

$$= \frac{\mu_0 I}{2\pi L}$$



& F on the top wire:

$$F = B_T \cdot L \cdot 2I = \frac{\mu_0 I^2}{\pi} = \frac{4\cancel{\pi}\times10^{-7} \times I^2 \times 10^{-6}}{\cancel{\pi}}$$

$$= 4I^2 \times 10^{-13}\ N.$$

5- the plane surface is defined by:

$$x+y+z-2=0$$

so $$\vec{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\vec{a}_x + \vec{a}_y + \vec{a}_z}{\sqrt{3}}$$

$$m = I S \vec{a}_n = 10 \times S \times \vec{a}_n$$

$$S = \frac{1}{2} \cdot (2\sqrt{2}) \times (2\sqrt{2}) \sin 60^\circ = 4 \sin 60 = 4\frac{\sqrt{3}}{2} = 2\sqrt{3}$$

so $$m = 10 \times 2\cancel{\sqrt{3}} \times \frac{\vec{a}_x + \vec{a}_y + \vec{a}_z}{\cancel{\sqrt{3}}} = 20(\vec{a}_x + \vec{a}_y + \vec{a}_z).$$

6. the torque on the loop is:

$$T = m \times B$$

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 20 & 20 & 20 \\ 0.6 & 0.4 & 0.5 \end{vmatrix} = 2\vec{a}_x + 2\vec{a}_y - 4\vec{a}_z$$

7. $$H_1 = -2\vec{a}_x + 6\vec{a}_y + 4\vec{a}_z$$

$$H_{1n} = (H_1 \cdot \vec{a}_n)\vec{a}_n = \left[(-2,6,4) \cdot \frac{(-1,1,0)}{\sqrt{2}}\right] \frac{(-1,1,0)}{\sqrt{2}}$$

where; $$\vec{a}_n = \frac{\vec{a}_y - \vec{a}_x}{\sqrt{2}}$$

so $$H_{1n} = -4\vec{a}_x + 4\vec{a}_y$$

& $$H_{1t} = H_1 - H_{1n} = -2\vec{a}_x + 6\vec{a}_y + 4\vec{a}_z - (-4\vec{a}_x + 4\vec{a}_y)$$

$$= 2\vec{a}_x + 2\vec{a}_y + 4\vec{a}_z$$

8. $H_{2t} = H_{1t} = 2\vec{a_x} + 2\vec{a_y} + 4\vec{a_z}$

$B_{2n} = B_{1n} \Rightarrow \mu_2 H_{2n} = \mu_1 H_{1n}$

or $H_{2n} = \dfrac{\mu_1}{\mu_2} H_{1n} \Rightarrow B_{2n} = \mu_2 H_{2n} = \mu_1 H_{1n}$

so $B_{2n} = \mu_1 \left(-4\vec{a_x} + 4\vec{a_y}\right) = 5\mu_0 \left(-4\vec{a_x} + 4\vec{a_y}\right)$
$= \mu_0 \left(-20\vec{a_x} + 20\vec{a_y}\right).$

9- $B = \mu_0 H = \dfrac{3}{r}\cos\phi \; \vec{a_r}$ (T)

$$\Phi = \int_0^1 \int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{3}{r}\cos\phi\right)\vec{a_r} \cdot r\,d\phi\,dz\,\vec{a_r}$$

$= 5.58$

10- Force on the left conductor:

$F_1 = IL\vec{a_y} \times B\vec{a_x} = BIL(-\vec{a_z})$

& $T_1 = \dfrac{w}{2}(-\vec{a_x}) \times BIL(-\vec{a_z}) = BIL\dfrac{w}{2}(-\vec{a_y})$

so total torque : $T = BILw(-\vec{a_y})$.

11- $m = IA\vec{a_n} = 5 \times 0.08 \times 0.04\,\vec{a_n} = 1.6\times10^{-2}\vec{a_x}$

$T = \vec{m} \times \vec{B} = 1.6\times10^{-2}\vec{a_x} \times 0.05\,\dfrac{\vec{a_x}+\vec{a_y}}{\sqrt{2}}$

$= 5.66\times10^{-4}\,\vec{a_z}$ N.m

12- if the coil turns through 45°, the direction of $\vec{m}$ will be $\dfrac{\vec{a_x}+\vec{a_y}}{\sqrt{2}}$ & so $\vec{B}$ & $\vec{m}$ become parallel

so $T = m \times B = 0$.