

Problem 1

Two uniform vector fields are given by $E = -5 a_r + 10 a_\phi + 3 a_z$ and $F = a_r + 2 a_\phi - 6 a_z$

Calculate $|E \times F|$

- a. 35
- b. 67.33
- c. 44.12
- d. 74.06
- e. None of the above

Problem 2

A unit normal vector to the cylindrical surface $r=4$ is:

- a. a_r
- b. a_ϕ
- c. a_z
- d. $a_\phi + a_r$
- e. None of the above

Problem 3

If vector $A = 4 a_r - 3 a_\phi + 5 a_z$. At point $(1, \pi/2, 0)$, the component of A parallel to surface $r=1$ is:

- a. $4 a_r$
- b. $-3 a_\phi + 5 a_z$
- c. $5 a_z$
- d. $5 a_\phi + 3 a_z$
- e. None of the above

Problem 4

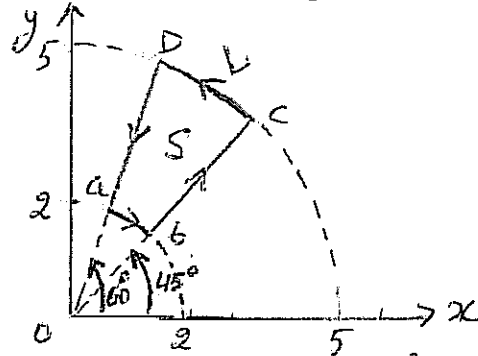
Determine the divergence of the vector: $Q = r \sin \phi a_r + r^2 z a_\phi + z \cos \phi a_z$

- a. $2 \sin \phi + 3 \cos \phi$
- b. $2 \sin \phi + \cos \phi$
- c. $\sin \phi + 2 \cos \phi$
- d. $\sin \phi + \cos \phi$
- e. None of the above

Problem 5

If $A = r \cos\phi \mathbf{a}_r + \sin\phi \mathbf{a}_\phi$, evaluate $\oint A \cdot d\mathbf{l}$ around the path shown in the figure.

- a) $(27/4)(\sqrt{3}-1)$
- b) $(27/2)(\sqrt{2}-1)$
- c) $(27/3)(\sqrt{3}-1)$
- d) $(27/4)(\sqrt{2}-1)$
- e) None of the above



Problem 6

A square plate described by $-2 \leq x \leq 2$, $-2 \leq y \leq 2$, $z=0$ carries a charge $12|y| \text{ mC/m}^2$. Find the total charge on the plate.

- a) 192 mC
- b) 233 mC
- c) 89 mC
- d) 167 mC
- e) None of the above

Problem 7

A point charge of 30 nC is located at the origin while plane $y=3$ carries charge 10 nC/m^2 . Find D at $(0,4,3)$

- a) $2.12\mathbf{a}_y + 0.09 \mathbf{a}_z \text{ nC/m}^2$
- b) $3.14\mathbf{a}_y + 0.02\mathbf{a}_z \text{ nC/m}^2$
- c) $5.076\mathbf{a}_y + 0.0573 \mathbf{a}_z \text{ nC/m}^2$
- d) $1.7\mathbf{a}_y + 0.136 \mathbf{a}_z \text{ nC/m}^2$
- e) None of the above

Problem 8

A charge distribution in free space has $\rho_v = 2r \text{ nC/m}^3$ for $0 \leq r \leq 10\text{m}$ and zero otherwise. Determine E at $r=2\text{m}$.

- a) $15.3 \mathbf{a}_r \text{ V/m}$
- b) $226 \mathbf{a}_r \text{ V/m}$
- c) $354 \mathbf{a}_r \text{ V/m}$
- d) $3.927 \mathbf{a}_r \text{ V/m}$
- e) None of the above

Problem 9

If Electric potential $V=x - y + xy +2z$ (V). Find E at point (1,2,3).

- a) $3a_x - 2a_z$ V/m
- b) $3a_x + 2a_z$ V/m
- c) $-3a_x + 2a_z$ V/m
- d) $-3a_x - 2a_z$ V/m
- e) None of the above

Problem 10

Two homogeneous dielectrics meet on plane $z=0$. For $z \geq 0$, $\epsilon_{r1}=6$ and for $Z \leq 0$, $\epsilon_{r2}=3$. A uniform electric field $E_1= 5a_x - 2a_y +3 a_z$ kV/m exists for $z \geq 0$. Find E_2 for $z \leq 0$.

- a) $2a_x - 2a_y +4 a_z$ kV/m
- b) $5a_x - 2a_y +4 a_z$ kV/m
- c) $2a_x +5a_y +3 a_z$ kV/m
- d) $5a_x - 2a_y +6 a_z$ kV/m
- e) None of the above

Problem 11

Region $y \leq 0$ consists of a perfect conductor while region $y \geq 0$ is a dielectric medium with $\epsilon_{r1}=4$. If there is a surface charge of 2 nC/m^2 on the conductor. Determine E at (-4,1,5).

- a) $18\pi a_y$ V/m
- b) $36\pi a_y$ V/m
- c) $2 a_y$ V/m
- d) $2\pi a_y$ V/m
- e) None of the above

Problem 12

If the Charge density increases linearly with distance from the origin such that $\rho_v=0$ at the origin and $\rho_v=10 \text{ c/m}^3$ at $R=2\text{m}$. Find the corresponding variation of D.

- a. $a_r (5/4) R^2 \text{ C/m}^2$
- b. $a_r (2/5) R^2 \text{ C/m}^2$
- c. $a_r (5/3) R^2 \text{ C/m}^2$
- d. $a_r (3/5) R^2 \text{ C/m}^2$
- e. None of the above

Table 2-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, $A =$	$\hat{x}A_x + \hat{y}A_y + \hat{z}A_z$	$\hat{r}A_r + \hat{\phi}A_\phi + \hat{z}A_z$	$\hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$
Magnitude of A, $ A =$	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1,$ for $P(x_1, y_1, z_1)$	$\hat{r}r_1 + \hat{z}z_1,$ for $P(r_1, \phi_1, z_1)$	$\hat{R}R_1,$ for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$ $\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{z} \cdot \hat{x} = 0$ $\hat{x} \times \hat{y} = \hat{z}$ $\hat{y} \times \hat{z} = \hat{x}$ $\hat{z} \times \hat{x} = \hat{y}$	$\hat{r} \cdot \hat{r} = \hat{\phi} \cdot \hat{\phi} = \hat{z} \cdot \hat{z} = 1$ $\hat{r} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{z} = \hat{z} \cdot \hat{r} = 0$ $\hat{r} \times \hat{\phi} = \hat{z}$ $\hat{\phi} \times \hat{z} = \hat{r}$ $\hat{z} \times \hat{r} = \hat{\phi}$	$\hat{R} \cdot \hat{R} = \hat{\theta} \cdot \hat{\theta} = \hat{\phi} \cdot \hat{\phi} = 1$ $\hat{R} \cdot \hat{\theta} = \hat{\theta} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{R} = 0$ $\hat{R} \times \hat{\theta} = \hat{\phi}$ $\hat{\theta} \times \hat{\phi} = \hat{R}$ $\hat{\phi} \times \hat{R} = \hat{\theta}$
Dot product, $A \cdot B =$	$A_x B_x + A_y B_y + A_z B_z$	$A_r B_r + A_\phi B_\phi + A_z B_z$	$A_R B_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, $A \times B =$	$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l} =$	$\hat{x} dx + \hat{y} dy + \hat{z} dz$	$\hat{r} dr + \hat{\phi} r d\phi + \hat{z} dz$	$\hat{R} dR + \hat{\theta} R d\theta + \hat{\phi} R \sin \theta d\phi$
Differential surface areas	$ds_x = \hat{x} dy dz$ $ds_y = \hat{y} dx dz$ $ds_z = \hat{z} dx dy$	$ds_r = \hat{r} r d\phi dz$ $ds_\phi = \hat{\phi} dr dz$ $ds_z = \hat{z} r dr d\phi$	$ds_R = \hat{R} R^2 \sin \theta d\theta d\phi$ $ds_\theta = \hat{\theta} R \sin \theta dR d\phi$ $ds_\phi = \hat{\phi} R dR d\theta$
Differential volume, $dV =$	$dx dy dz$	$r dr d\phi dz$	$R^2 \sin \theta dR d\theta d\phi$