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FACULTY OF ENGINEERING & ARCHITECTURE

FALL TERM 2006-07

Name:.......................................

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Master v.1

TEST ID 1000

(EECE 480) Engineering Electromagnetics

CLOSED BOOK (1 ½ HRS)

Programmable Calculators are not allowed
Provide your answers on the computer's card only
Return the computer's card attached to the question sheet
Mark with a pencil your name and your ID-No
Use pencil for marking your answers
When using eraser, be sure that you have erased well

{In all problems below take $\varepsilon_0 = (10^{-9}/36\pi)$}

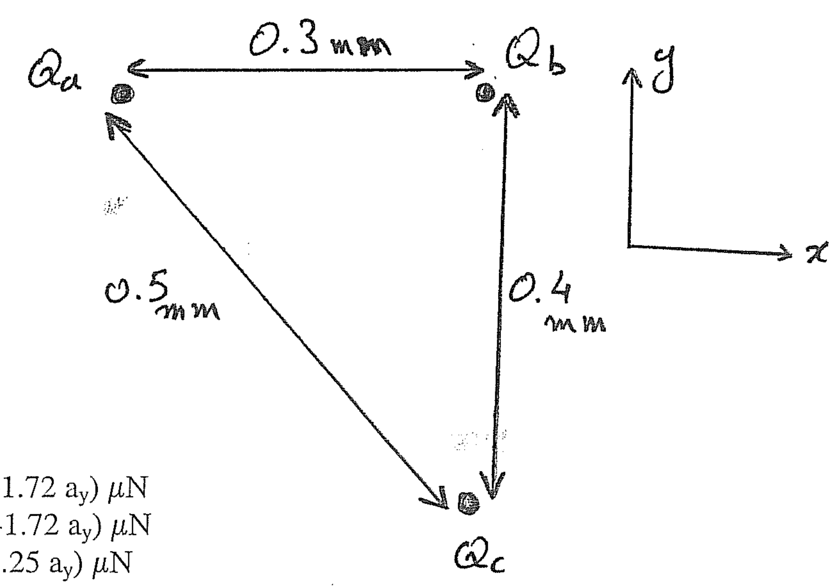
1. A potential field is given by $V=3x^2y-yz$. Which of the following <u>is not true:</u>

   a. At point (1,0,-1), both V and E vanish
   b. $x^2y=1$ is an equipotential line on the xy plane
   c. The equipotential surface V=-8 passes through point P(2,-1,4)
   d. The electric field at P(2,-1,4) is $12a_x - 8a_y - a_z$
   e. None of the above

2. Suppose a uniform electric field exists in the room in which you are working, such that the lines of force are horizontal and at right angles to one wall. As you walk toward the wall from which the lines of force emerge into the room, are you walking toward

   a. Points of lower potential ?
   b. Points of higher potential ?
   c. Points of the same potential?
   d. Insufficient information to determine the direction of higher potential
   e. None of the above

3. Consider in the xy plane the 3 point charges shown below. If $Q_A = 4.10^{-12}$ C and $Q_B = 15.10^{-12}$ C and $Q_C = 15.10^{-12}$ C, find the electric force exerted on Q_A.



   a. $(7.25\ a_x + 1.72\ a_y)\ \mu N$
   b. $(-7.25\ a_x - 1.72\ a_y)\ \mu N$
   c. $(1.72 a_x - 7.25\ a_y)\ \mu N$
   d. $(-7.25\ a_x + 1.72\ a_y)\ \mu N$
   e. None of the above

4. Two charges are arranged in the xy plane as follows:
$Q_1=10^{-9}$ C at (0,1) and $Q_2=-10^{-9}$ C at (0,-1)

Find the electric potential V anywhere along the x-axis.

a. 0 V
b. 3.18 V
c. $(10^{-9}/4\pi\varepsilon_o)$ / [(1/(x-1)) – (1/(x+1))] V
d. $(10^{-9}/4\pi\varepsilon_o)$ / [(1/(x+1)) – (1/(x-1))] V
e. None of the above

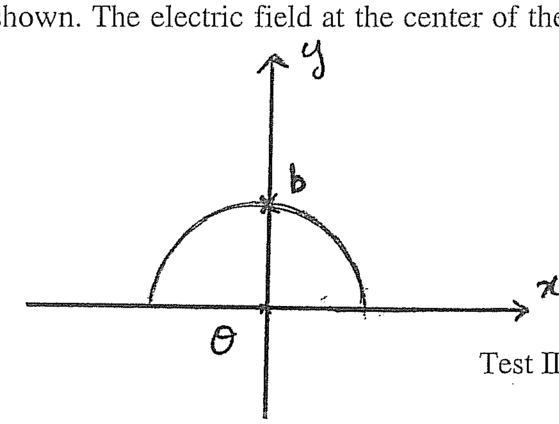
5. Calculate the stored energy in a system of four identical point charges Q = 4 nC located at the corners of a square, each side of which is 1m long.

a. 780 nJ
b. 390 nJ
c. 576 nJ
d. 204 nJ
e. None of the above

6. A spherical capacitor consists of an inner conducting sphere of radius R_i and an outer conductor with a spherical inner wall of radius R_o. The space in between is filled with a dielectric of permittivity ε. The capacitance C is:

a. 4πε(1/Ri – 1/Ro)-1
b. 4πε (1/Ri + 1/Ro)-1
c. 4πε(1/Ri – 1/Ro)
d. 4πε(1/Ri + 1/Ro)
e. None of the above

7. A line charge of uniform density ρ_L= 5 μC/m forms a semicircle of radius b in the upper half of the xy-plane as shown. The electric field at the center of the semicircle is:



a. - (ρL/2πεob) ay
b. -(ρL/4πεob) ay
c. (ρL/2πεob) (ax + ay)
d. (ρL/2πεob) (ax - ay)
e. None of the above

8. The finite sheet $0 \le x \le 1, 0 \le y \le 1$ on the $z = 0$ plane has a charge density $\rho_S = xy\left(x^2 + y^2 + 25\right)^{1.5}$ nC/m^2. Find the total charge on the sheet. [Hint: substitute x dx by (1/2) d(x^2)].

a. 27.26 nC
b. 33.15 nC
c. 42.76 nC
d. 65.81 nC
e. None of the above

9. In problem 8, find the electric field at (0, 0 ,5)

a. (-1/6, -1/6, 5/4) V/m
b. (-3, -3, 33.75) V/m
c. (-2.3, -2.3, 18.45) V/m
d. (-1.5, -1.5, 11.25) V/m
e. None of the above

10. Given that $\vec{D} = z\,r\cos^2\varphi\,\vec{a_z}$ C/m^2, calculate the total charge enclosed by the cylinder of the radius 1m with $-2 \le z \le 2$m.

a. 1.33π, C
b. 4π, C
c. 1.5π, C
d. 12π, C
e. None of the above

11. Given the potential $V = (10\sin\theta\cos\varphi)/r^2$, calculate the work done in moving a 10 μC charge from point A(1, 30°, 120°) to B(4, 90°, 60°)

a. 35.3 μJ
b. 28.12 μJ
c. 18.52 μJ
d. 11.67 μJ
e. None of the above

12. Two extensive homogenous isotropic dielectrics meet on the plane $z=0$. For $z \geq 0$, $\varepsilon_{r1}=4$ and for $z \leq 0$, $\varepsilon_{r2}=3$. A uniform electric field $\overrightarrow{E_1}=5\overrightarrow{a_x}-2\overrightarrow{a_y}+3\overrightarrow{a_z}$ kV/m exists for $z \geq 0$. Find $\overrightarrow{E_2}$ for $z \leq 0$.

a. $5\overrightarrow{a_x}-2\overrightarrow{a_y}+2\overrightarrow{a_z}$ kV/m

b. $6.65\overrightarrow{a_x}-2.66\overrightarrow{a_y}+4\overrightarrow{a_z}$ kV/m

c. $3.75\overrightarrow{a_x}-1.5\overrightarrow{a_y}+3\overrightarrow{a_z}$ kV/m

d. $5\overrightarrow{a_x}-2\overrightarrow{a_y}+4\overrightarrow{a_z}$ kV/m

e. None of the above

Appendix

Table 2-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, A =	$\hat{x}A_x + \hat{y}A_y + \hat{z}A_z$	$\hat{r}A_r + \hat{\phi}A_\phi + \hat{z}A_z$	$\hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$
Magnitude of A, $\|A\|$ =	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1}$ =	$\hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1$, for $P(x_1, y_1, z_1)$	$\hat{r}r_1 + \hat{z}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{R}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$ $\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{z} = \hat{z}\cdot\hat{x} = 0$ $\hat{x}\times\hat{y} = \hat{z}$ $\hat{y}\times\hat{z} = \hat{x}$ $\hat{z}\times\hat{x} = \hat{y}$	$\hat{r}\cdot\hat{r} = \hat{\phi}\cdot\hat{\phi} = \hat{z}\cdot\hat{z} = 1$ $\hat{r}\cdot\hat{\phi} = \hat{\phi}\cdot\hat{z} = \hat{z}\cdot\hat{r} = 0$ $\hat{r}\times\hat{\phi} = \hat{z}$ $\hat{\phi}\times\hat{z} = \hat{r}$ $\hat{z}\times\hat{r} = \hat{\phi}$	$\hat{R}\cdot\hat{R} = \hat{\theta}\cdot\hat{\theta} = \hat{\phi}\cdot\hat{\phi} = 1$ $\hat{R}\cdot\hat{\theta} = \hat{\theta}\cdot\hat{\phi} = \hat{\phi}\cdot\hat{R} = 0$ $\hat{R}\times\hat{\theta} = \hat{\phi}$ $\hat{\theta}\times\hat{\phi} = \hat{R}$ $\hat{\phi}\times\hat{R} = \hat{\theta}$
Dot product, A · B =	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, A × B =	$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l}$ =	$\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$	$\hat{r}\,dr + \hat{\phi}r\,d\phi + \hat{z}\,dz$	$\hat{R}\,dR + \hat{\theta}R\,d\theta + \hat{\phi}R\sin\theta\,d\phi$
Differential surface areas	$ds_x = \hat{x}\,dy\,dz$ $ds_y = \hat{y}\,dx\,dz$ $ds_z = \hat{z}\,dx\,dy$	$ds_r = \hat{r}r\,d\phi\,dz$ $ds_\phi = \hat{\phi}\,dr\,dz$ $ds_z = \hat{z}r\,dr\,d\phi$	$ds_R = \hat{R}R^2\sin\theta\,d\theta\,d\phi$ $ds_\theta = \hat{\theta}R\sin\theta\,dR\,d\phi$ $ds_\phi = \hat{\phi}R\,dR\,d\theta$
Differential volume, $d\mathcal{V}$ =	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Useful Vector Identities

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$$

$$\nabla(\psi V) = \psi\nabla V + V\nabla\psi$$

$$\nabla\cdot(\psi\mathbf{A}) = \psi\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla\psi$$

$$\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} + \nabla\psi\times\mathbf{A}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

$$\nabla\cdot\nabla V = \nabla^2 V$$

$$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\nabla\times\nabla V = 0$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\int_V \nabla\cdot\mathbf{A}\,dv = \oint_S \mathbf{A}\cdot d\mathbf{s} \qquad \text{(Divergence theorem)}$$

$$\int_S \nabla\times\mathbf{A}\cdot d\mathbf{s} = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell} \qquad \text{(Stokes's theorem)}$$

ent, Divergence, Curl, and Laplacian Operations

ian Coordinates (x, y, z)

$$\nabla V = \mathbf{a}_x\frac{\partial V}{\partial x} + \mathbf{a}_y\frac{\partial V}{\partial y} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r\frac{\partial V}{\partial r} + \mathbf{a}_\phi\frac{\partial V}{r\partial\phi} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \mathbf{a}_r\left(\frac{\partial A_z}{r\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R\frac{\partial V}{\partial R} + \mathbf{a}_\theta\frac{\partial V}{R\partial\theta} + \mathbf{a}_\phi\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix} = \mathbf{a}_R\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]$$
$$+ \mathbf{a}_\theta\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right]$$
$$+ \mathbf{a}_\phi\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$