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Quiz 1 - Solution

EECE 480 - Engineering Electromagnetics

Fall 06-0

Problem 1:

A potential field is given by $V = 3x^2y - yz$.
Which of the following is not true:

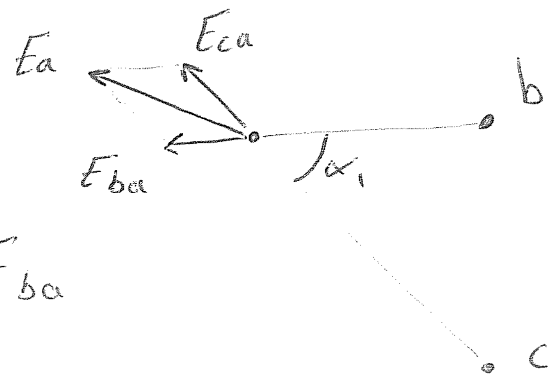
- At point $(1,0,-1)$, both V and E vanish (a)

Problem 2:

Suppose a uniform electric field exists in the
room in which you are working, such that the lines
of force are horizontal and at right angles to
one wall. As you walk toward the wall from
which the lines of force emerge into the room, are
you walking toward

- Points of higher potential (b)

Problem 3:



$F_a = Q_a E_a$

where $\vec{E_a} = \vec{E_{ca}} + E_{ba}$

$$\vec{E_a} = \frac{Q_c \, \vec{a_{ca}}}{4\pi \varepsilon_0 R_{ca}^2} + \frac{Q_b \, \vec{a_{ba}}}{4\pi \varepsilon_0 R_{ba}^2}$$

or $$\vec{E_a} = -\left(\frac{Q_c \cos\alpha_1}{4\pi\varepsilon_0 R_{ca}^2} + \frac{Q_b}{4\pi\varepsilon_0 R_{ba}^2} \right)\vec{a_x} + \frac{Q_c \sin\alpha_1}{4\pi\varepsilon_0 R_{ca}^2}\vec{a_y}$$

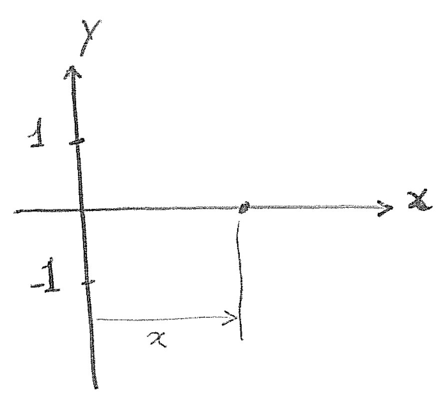
here : $$\cos\alpha_1 = \frac{R_{ba}}{R_{ca}} = \frac{0.3}{0.5} = 0.6$$

$$\sin\alpha_1 = \frac{R_{cb}}{R_{ca}} = \frac{0.4}{0.5} = 0.8$$

So $$\vec{F_a} = (-7.25\,\vec{a_x} + 1.72\,\vec{a_y})$$

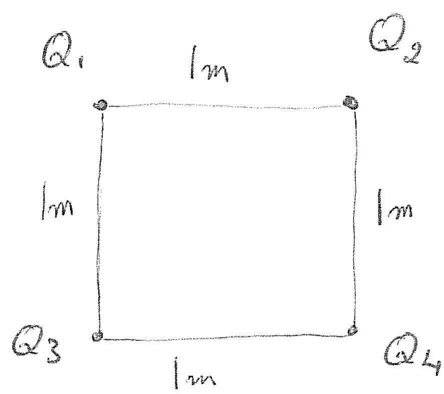
Problem 4:

$$V = \frac{1}{4\pi\varepsilon_0} \sum \frac{Q_K}{|r - r_K|}$$



$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{10^{-9}}{\sqrt{x^2+1}} + \frac{-10^{-9}}{\sqrt{x^2+1}} \right) = 0$$

Problem 5:



$$W_E = \frac{1}{2}\sum_{i=1}^{4} Q_i V_i$$

$$= \frac{1}{2}(Q_1V_1 + Q_2V_2 + Q_3V_3 + Q_4V_4)$$

But $Q_1 = Q_2 = Q_3 = Q_4$

& $$V_1 = V_2 = V_3 = V_4 = \frac{4\times 10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{1} + \frac{1}{1} + \frac{1}{\sqrt{2}}\right] = 97.5 \text{ volt}$$

$$\Rightarrow W_E = 2Q_1 V_1 = 2 \times 4 \times 10^{-9} \times 97.5 = 780\, nJ$$

Problem 6:

Assume charges $+Q$ and $-Q$ on the inner and outer conductors, respectively.

$$\vec{E} = \hat{a}_R E_R = \frac{Q}{4\pi \varepsilon R^2} \hat{a}_R$$

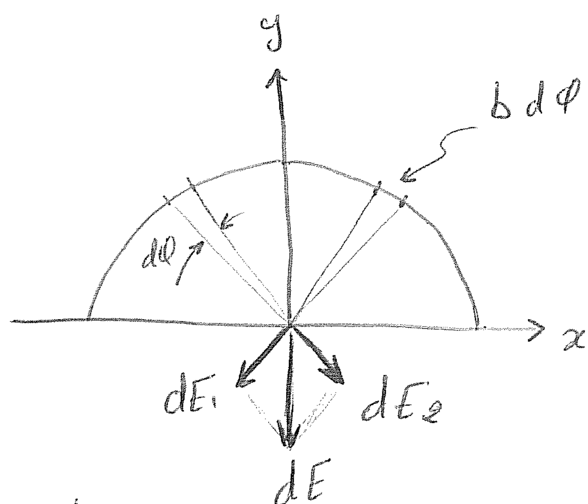
$$V = -\int_{R_o}^{R_i} \vec{E} \cdot \vec{dl} \quad \text{but} \quad \vec{dl} = dR\, \hat{a}_R + R\, d\theta\, \hat{a}_\theta + R \sin\theta\, d\phi\, \hat{a}_\phi$$

$$\Rightarrow V = -\int_{R_o}^{R_i} \frac{Q}{4\pi \varepsilon R^2} = \frac{Q}{4\pi \varepsilon} \left[\frac{1}{R_i} - \frac{1}{R_o} \right]$$

$$\frac{Q}{V} = C = \frac{4\pi \varepsilon_o \varepsilon_r}{\left[\frac{1}{R_i} - \frac{1}{R_o} \right]}$$

Problem 7:

$$\vec{dE} = \frac{\rho_\ell \cdot d\ell}{4\pi \varepsilon_o r^2} \hat{a}_r$$



The x-components will cancel each other thus ($\hat{a}_r = \hat{a}_x \cos\phi + \hat{a}_y \sin\phi$) will be reduced to $\hat{a}_r = \hat{a}_y \sin$

$$\overrightarrow{dE} = \overrightarrow{dE_1} + \overrightarrow{dE_2}$$

$$= -\frac{\rho_\ell \,(b\, d\phi)}{4\pi \varepsilon_0 b^2} \sin\phi \; \hat{a}_y$$

$$\Rightarrow \vec{E} = \frac{-\rho_\ell}{4\pi \varepsilon_0 b} \int_0^{\pi} \sin\phi \, d\phi \; \hat{a}_y = -\frac{\rho_\ell}{2\pi \varepsilon_0 b} \hat{a}_y$$

<u>Problem 8:</u>

$$Q = \int \rho_s \, dS = \int_0^1 \int_0^1 xy\,(x^2 + y^2 + 25)^{1.5} \, dx\, dy$$

change $x\,dx$ into $\frac{1}{2} d(x^2)$

$$= \frac{1}{2} \int_0^1 y \int_0^1 (x^2 + y^2 + 25)^{1.5} \, dx^2 \, dy$$

$$= \frac{1}{2} \int_0^1 y \, \frac{2}{5} (x^2 + y^2 + 25)^{5/2} \Big|_0^1 \, dy$$

$$= \frac{1}{5} \int_0^1 \frac{1}{2} \left[(y^2 + 26)^{5/2} - (y^2 + 25)^{5/2} \right] dy^2$$

$$= \frac{1}{10} \, \frac{2}{7} \left[(y^2 + 26)^{7/2} - (y^2 + 25)^{7/2} \right] \Big|_0^1 = 33.15 \text{ nC}$$

Problem 9:

$$\vec{E} = \int \frac{\rho_s \, ds}{4\pi\varepsilon_0 r^2} \hat{a}_r = \int \frac{\rho_s \, ds}{4\pi\varepsilon_0 r^3} \vec{r}$$

$$\vec{r} = (0, 0, 5) - (x, y, 0)$$

$$\& \ |\vec{r}| = \sqrt{x^2 + y^2 + 25}$$

$$\Rightarrow E = (-1.5, -1.5, 11.25) \ V/m$$

Problem 10:

$$Q = \int_v \rho_v \, dv$$

$$\rho_v = \vec{\nabla} \cdot \vec{D} = \frac{\partial D_z}{\partial z} = r \cos^2 \phi$$

at $(1, \frac{\pi}{4}, 3)$; $\rho_v = 0.5 \ C/m^3$

Now $Q = \int \rho_v \, dv = \int r \cos^2 \phi \, r \, d\phi \, dr \, dz$

$$= \int_{z=-2}^{2} dz \int_{\phi=0}^{2\pi} \cos^2 \phi \, d\phi \int_{r=0}^{1} r^2 dr = \frac{4\pi}{3} C = 1.33\pi$$

Problem 11:

$$W = -Q \int_A^B \vec{E} \cdot \vec{dl} = Q V_{AB} = Q (V_B - V_A)$$

$$= 10 \left(\frac{10}{16} \sin 90 \cos 60 - \frac{10}{1} \sin 30 \cos 120 \right) \times 10^{-6}$$

$$= 10\left(\frac{10}{32} - \frac{-5}{2}\right) \times 10^{-6} = 28.125 \, \mu J.$$

Problem 12:

$$E_{1n} = \vec{E_1} \cdot \vec{a_n} = \vec{E_1} \cdot \vec{a_z} = 3$$

$$\vec{E_{1n}} = 3\vec{a_z}$$

also : $\vec{E} = \vec{E_n} + \vec{E_t}$

Hence : $\vec{E_{1t}} = \vec{E_1} - \vec{E_{1n}} = 5\vec{a_x} - 2\vec{a_y}$

thus : $\vec{E_{2t}} = \vec{E_{1t}} = 5\vec{a_x} - 2\vec{a_y}$

Similarly : $\vec{D_{2n}} = \vec{D_{1n}} \longrightarrow \varepsilon_{r2} \vec{E_{2n}} = \varepsilon_{r1} \vec{E_{1n}}$

or $\vec{E_{2n}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \vec{E_{1n}} = \frac{4}{3}(3\vec{a_z}) = 4\vec{a_z}$

thus : $\vec{E_2} = \vec{E_{2t}} + \vec{E_{2n}} = 5\vec{a_x} - 2\vec{a_y} + 4\vec{a_z}$ KV/m