$\triangle$ Dec 13^{th}, 2004 EECE441

Midterm

Problem 4:

Consider two infinitely long parallel wires, each carrying a current I in the z direction, one passing through the point $(x=0, y=-a)$ and the other through $(x=0, y=-a)$.

Find B at $P(x=b, y=0)$

Solution:

Using the result of B field at a distance r from an infinitely long straight wire carrying a current I in the z direction : $B = \hat{\varphi}\mu_0 I / 2\pi r$ we have :

$$B_P = B_1 + B_2$$

$$= \frac{\mu_0 I}{2\pi\sqrt{a^2+b^2}}(\hat{x}\sin\varphi + \hat{y}\cos\varphi) + \frac{\mu_0 I}{2\pi\sqrt{a^2+b^2}}(-\hat{x}\sin\varphi + \hat{y}\cos\varphi)$$

$$= \hat{y}\frac{\mu_0 I b}{\pi(a^2+b^2)}$$

where we have used $\cos\varphi = b(a^2+b^2)^{-1/2}$

Problem 5:

For the current element $I\,dx\,(a_x + a_y)$ situated at the point (1, -2, 2), find the magnetic flux density at the point (2, -1, 3)

. For the point (2, -1, 3)

$$R = (2-1)a_x + (-1+2)a_y + (3-2)a_z$$

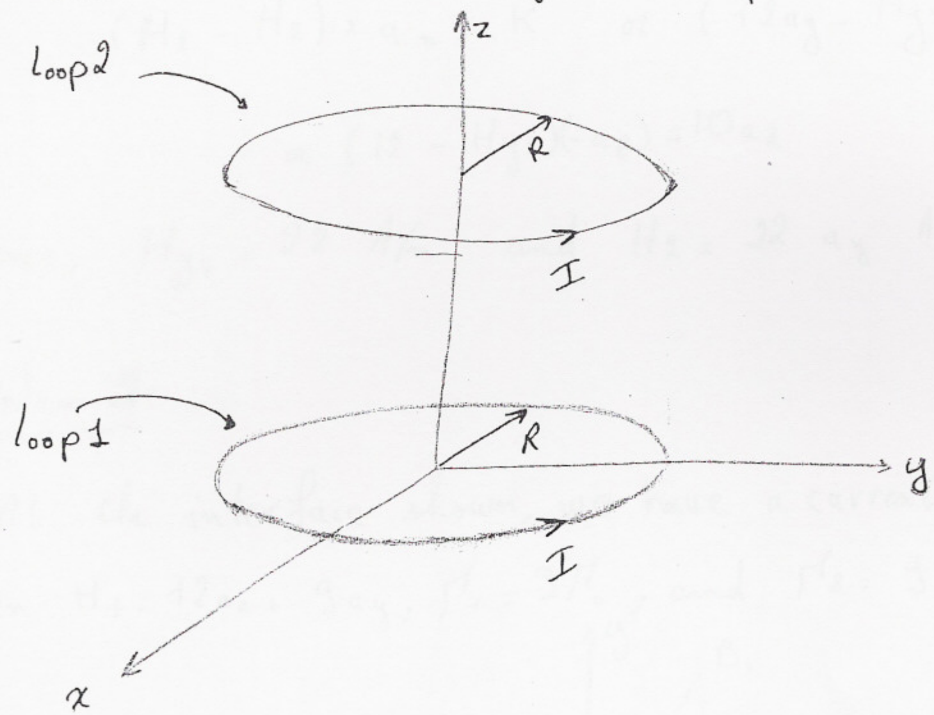
$$= a_x + a_y + a_z$$

$$B = \frac{\mu_0}{4\pi} \frac{I\,dz\,(a_x + a_y) \times (a_x + a_y + a_z)}{(\sqrt{3})^3}$$

$$= \frac{\mu_0 I\,dz}{12\sqrt{3}\pi}(a_x - a_y)$$

Problem 6:

For the loops shown in figure, we have $R = 0.1m$ and $I = 100\,A$, and the separation between the loops is $0.2m$. Calculate the flux density at a point midway between the loop



Solution:

At the mid point, by symmetry,

$$B = \frac{\mu_0 I R^2}{(R^2 + h^2)^{3/2}} a_z = \frac{4\pi \times 10^{-7} \times 100 \times 0.1}{(0.1^2 + 0.1^2)^{3/2}} a_z = 0.444\,a_z \text{ mT}$$

Problem 10:

At the interface between two regions (at $x=0$), there exists a current sheet $K = 10a_z$ A/m. Given $H_1 = 12a_z$ A at $x = 0^-$, determine H_2 at $x = 0^+$.

Solution:

Since H_1 is purely tangential, $H_{x_2} = H_{n_1} = 0$ and we n no information regarding the permeabilities of the two reg

Now, we Know:

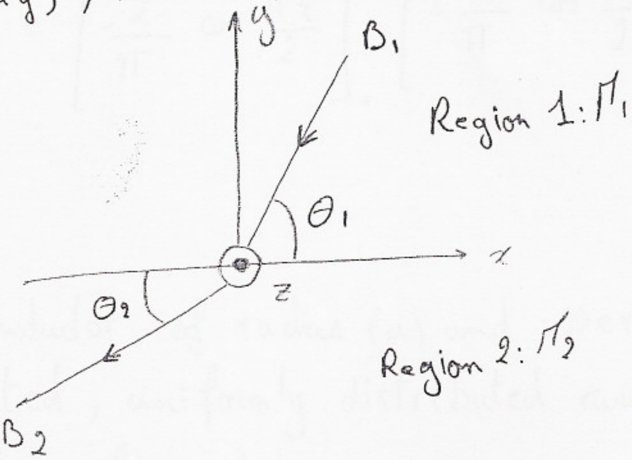
$$(H_1 - H_2) \times a_n = K \quad \text{or} \quad (12a_y - H_{y2}a_y) \times a_x = 10a_z$$

$$\text{or} \quad (12 - H_{y_2})(-a_z) = 10a_z$$

Hence, $H_{y_2} = 22$ A/m and $H_2 = 22\,a_y$ A/m

Problem 9:

At the interface shown, we have a current sheet $K = -8a_z$. Given $H_1 = 12a_x + 9a_y$, $\mu_1 = 3\mu_0$, and $\mu_2 = 9\mu_0$, determine H_2.



Solution:

we have:

$$(H_1 - H_2) \times (-a_y) = K \quad \text{or} \quad (12 - H_{x2})(-a_z) = -8a_z$$

$$\text{or} \quad H_{x2} = 4$$

$(\mu_1 H_1 - \mu_2 H_2) \cdot (-a_y) = 0 \qquad \text{or} \ (27\mu_0 - 9\mu_0 H_{y2})(-1)$

or $\quad H_{y2} = 3$

Consequently, $H_2 = 4a_x + 3a_y = \frac{1}{3} H_1$ (for the particular dat of this problem, there is zero deflection of the field).

Problem 1:

The B-field over the xy plane is given by:

$$B = \sin\left(\frac{\pi x}{2}\right) \sin\left(\frac{\pi y}{2}\right) a_z \quad (T)$$

Calculate the flux passing through the square

$$0 < x, \ y < 2m$$

Solution:

$$\psi = \int B \cdot dS = \int_{x=0}^{2} \int_{y=0}^{2} \left[\sin\left(\frac{\pi x}{2}\right) \sin\left(\frac{\pi y}{2}\right)\right] a_z \, dx\, dy \, a_z$$

$$= \left[-\frac{2}{\pi} \cos\frac{\pi x}{2}\right]_0^2 \left[-\frac{2}{\pi} \cos\frac{\pi y}{2}\right]_0^2 = \frac{16}{\pi^2} = 1.62 \text{ Wb}.$$

Problem 11:

A cylindrical conductor of radius (a) and permeability μ carries a z-directed, uniformly distributed current I. Evaluate th magnetization M, within the conductor.

Solution:

By Amper's Law, within the conductor, $0 < r < a$, we have

$$H_\varphi (2\pi r) = \frac{\pi r^2}{\pi a^2} I \quad \text{or} \quad B_\varphi = \mu H_\varphi = \frac{\mu I r}{2\pi a^2} \quad (T)$$

But,

$$M = M_\varphi a_\varphi = \left(\frac{B_\varphi}{\mu_0} - H_\varphi\right) a_\varphi = \frac{I}{2\pi a^2}\left(\frac{\mu}{\mu_0} - 1\right) r\, a_\varphi \quad (A.$$

Problem 2 and 3:

Given that $H_1 = -2a_x + 6a_y + 4a_z$ A/m in region $y - x - 2 \leq 0$ where $\mu_1 = 5\mu_0$, calculate

1) H_{1n}

2) H_{2n} in region $y - x - 2 \geq 0$ where $\mu_2 = 2\mu_0$

Solution:

Since $y - x - 2 = 0$ is a plane, $y - x \leq 2$ or $y \leq x + 2$ is region 1 in the shown figure. A point in this region may be used to confirm this. For example, the origin $(0,0)$ is in th region since $0 - 0 - 2 < 0$. If we let the surface of the plan be described by $f(x,y) = y - x - 2$, a unit vector normal to the plane is given by:

$$a_n = \frac{\nabla f}{|\nabla f|} = \frac{a_y - a_x}{\sqrt{2}}$$

$$M_1 = \chi_{m_1} H_1 = (\mu_{r1} - 1) H_1 = (5-1)(-2, 6, 4)$$

$$= -8a_x + 24a_y + 16a_z \ \text{A/m}$$

$$B_1 = \mu_1 H_1 = \mu_0 \mu_{r1} H_1 = 4\pi \times 10^{-7} (5)(-2, 6, 4)$$

$$= -12.57a_x + 37.7a_y + 25.13a_z \ \mu\text{Wb/m}^2$$

$$H_{1n} = (H_1 \cdot a_n) a_n = \left[(-2, 6, 4) \cdot \frac{(-1, 1, 0)}{\sqrt{2}}\right] \frac{(-1, 1, 0)}{\sqrt{2}}$$

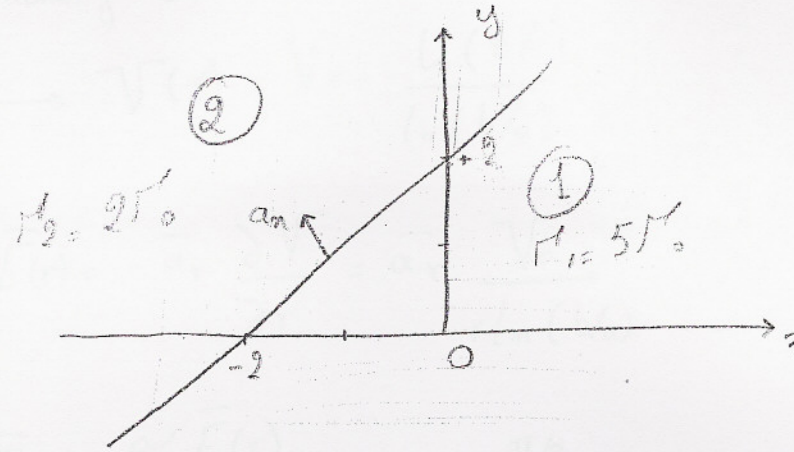
$H_{1n} = -4a_x + 4a_y$

But $H_1 = H_{1n} + H_{1t}$

Hence,

$$H_{1t} = H_1 - H_{1n} = (-2, 6, 4) - (-4, 4, 0)$$

$$= 2a_x + 2a_y + 4a_z$$



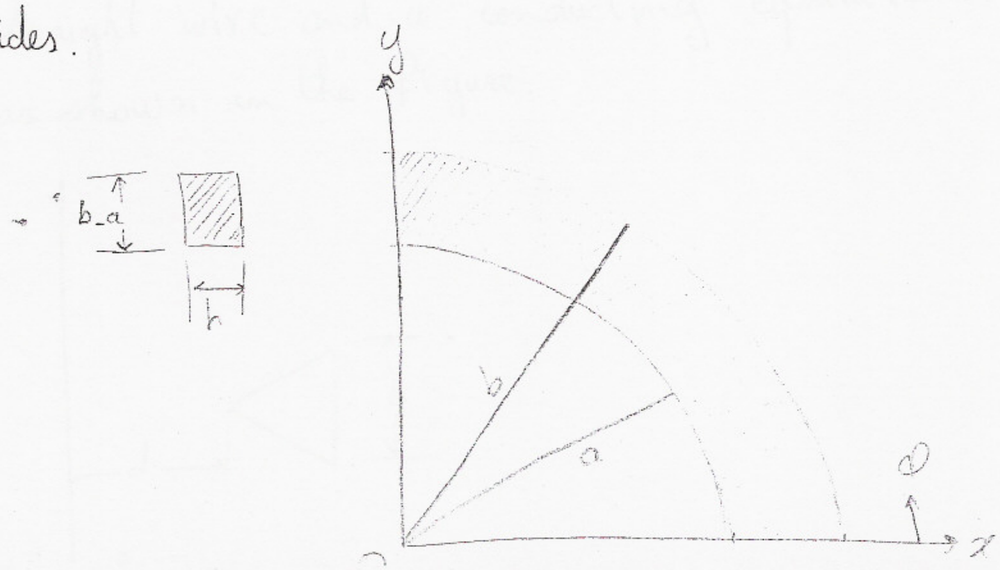
Using the boundary conditions, we have

$$H_{2t} = H_{1t} = 2a_x + 2a_y + 4a_z$$

$$B_{2n} = B_{1n} \longrightarrow \mu_2 H_{2n} = \mu_1 H_{1n}$$

Problem 7:

Refer to the flat conducting quarter-circular washer in the figure below. Find the resistance between the curved sides.



Solution:

Using Laplace's equation in cylindrical coordinates.

$$\bar{\nabla}^2 V = 0 \longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$

$$V(r) = C_1 \ln r + C_2$$

Boundary conditions: $V(a) = V_0$; $V(b) = 0$

$$\longrightarrow V(r) = V_0 \frac{\ln(b/r)}{\ln(b/a)}$$

$$\bar{E}(r) = -\bar{a}_r \frac{\partial V}{\partial r} = \bar{a}_r \frac{V_0}{r\ln(b/a)}$$

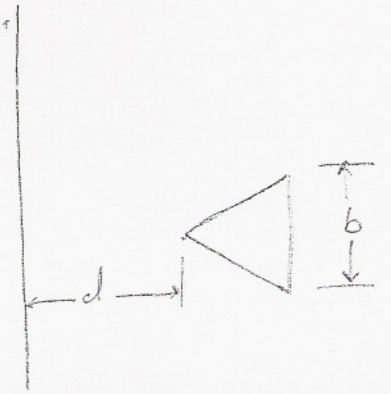
$$\bar{J}(r) = \sigma \bar{E}(r)$$

$$I = \int_S \bar{J} \cdot d\bar{S} = \int_0^{\pi/2} \bar{J} \cdot (\bar{a}_r h r \, d\phi) = \frac{\pi \sigma h V_0}{2\ln(b/a)}$$

$$R = \frac{V_0}{I} = \frac{2\ln(b/a)}{\pi \sigma h}$$

Problem 8:

Determine the mutual inductance between a very long, straight wire and a conducting equilateral triangular loop, as shown in the figure.



Solution:

For I in the long straight wire, $\bar{B} = \bar{a}_\phi \dfrac{\mu_0 I}{2\pi r}$

$$\Lambda_{12} = \int_S \bar{B}\cdot d\bar{S} = \int B_\phi \frac{2}{\sqrt{3}}(r-d)\,dr$$

$$= \frac{\mu_0 I}{\pi\sqrt{3}} \int_d^{d+\frac{\sqrt{3}}{2}b} \frac{(r-d)}{r}\,dr$$

$$= \frac{\mu_0 I}{\pi\sqrt{3}} \left[\frac{\sqrt{3}}{2}b - d\ln\left(1+\frac{\sqrt{3}\,b}{2d}\right)\right]$$

$$L_{12} = \frac{\Lambda_{12}}{I} = \frac{\mu_0}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{3}}\ln\left(1+\frac{\sqrt{3}\,b}{2d}\right)\right].$$